Contribution of the relative velocity between source and receivers to primary electromagnetic fields in the sea

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Abstract

The analysis of primary EM fields caused by a finite length line source below the sea level is important for submarine applied geophysics as well as for protecting ships from the threat of sea mines in both shallow and deep sea water environments. The surveys employ a source moving with a constant velocity whereas the receivers remain fixed at the sea floor. So an accurate determination of the primary field taking into account the relative velocity between the source and the receivers is necessary in order to optimize the interpretation of both TDEM and FDEM data.

Introduction

Analysis of primary EM fields caused by active sources below the sea level has several applications. It supports the interpretation of submarine geophysical data for environmental, deep crustal, and exploratory research, and it is also useful for protecting ships from the threat of sea mines. During the last two decades practical and theoretical efforts have led to the development of controlled source electromagnetic induction (CSEMI) techniques for surveying the sea substractum (Constable and Cox, 1996; Eidesmo et al., 2002).

The surveys consist of towing a transmitting cable near the sea bottom while the receivers remain fixed at the sea bottom. Besides the conductivity of the fluid this procedure has a fundamental difference to ground and airborne surveys, where transmitters and receivers are on the same frame of reference. In this paper we investigate the role that the relative velocity between the transmitter and the receivers has on the variation of the primary electromagnetic field. A precise knowledge of the primary field is important because the scattered field usually represents a fraction of it. So, even minor differences in the primary field caused by the differential velocity may render inadequate inverse modeling and interpretation of EMI data. We will neglect displacement currents.

Development of the solution

For the fundamental aspects of the EM theory in applied geophysics we refer to Wait (1982) and Ward and Hohmann (1988). The magnetic induction field, $\mathbf{b}(x, y, z, t)$, and the magnetic vector potential, $\mathbf{a}(x, y, z, t)$, relate to each other according to

$$\mathbf{b} = \nabla \times \mathbf{a}, \tag{1}$$

whereas $\mathbf{a}$, obeys the inhomogeneous wave equation of pure electric conduction (Sommerfeld, 1949):

$$\nabla^2 \mathbf{a} - \mu_0 \sigma \frac{\partial \mathbf{a}}{\partial t} = -\mu_0 \mathbf{j}_s. \tag{2}$$

If the source is an electric dipole along the x direction and situated at a point (x_c, y_0, z_0) of a homogeneous infinite medium, such that $x_c = x_0 + vt$, the electric current density is given by:

$$\mathbf{j}_s(\mathbf{R}, t) = \mathcal{I}(t)\, dx_0\, \delta(x - x_c)\, \delta(y - y_0)\, \delta(z - z_0)\, \mathbf{i}\,. \tag{3}$$

Both the electric dipole and the primary potential have only an x component. Employing Green's method, Fourier and Laplace transforms (Papoulis, 1962), and Sommerfeld integral (Sommerfeld, 1949) we arrive at the following expression for the solution of Equation 2 for a causal current source.

$$\mathbf{a}(\mathbf{R}, t) = \frac{\sqrt{\mu_0^3\, \sigma}\, dx_0}{8\sqrt{\pi^3}} \int_0^t \frac{\mathcal{I}(t-\xi) e^{-\frac{\mu_0 \sigma \left(R(t-\xi)\right)^2}{4\xi}}}{\sqrt{\xi^3}} d\xi, \, , \tag{4}$$

where $R(t) = \sqrt{(x - x_0 - vt)^2 + (y - y_0)^2 + (z - z_0)^2}$. If $\mathcal{I}(t) = C\delta(t)$ Equation 4 yields a solution in which R is not a function of t or v. Therefore, for a pulse source the velocity of the source doesn't affect the spatial and temporal description of the electromagnetic field, because the signal is transmitted at a single instant. However we have to be careful in the interpretation of the convolution integral in the case of a moving source with an arbitrary current waveform. We will obtain a wrong result if we just convolve the solution for a current pulse with the given arbitrary current function. The problem resides in the exponential term, because it contains both the dummy ξ, and the time shifted $t - \xi$, dependences. The matter becomes more complex for a line of dipoles, because it is necessary to integrate in x_0. Expanding that exponent in Taylor's series helps to understand the inherent difficulty.

$$e^{-\frac{\mu_0 \sigma (R(t-\xi))^2}{4\xi}} = e^{-\frac{\mu_0 \sigma ((y-y_0)^2+(z-z_0)^2}{4\xi}}$$

$$\sum_{n=0}^{\infty} (-1)^n \frac{\mu_0^n \sigma^n}{4^n\, n!\, \xi^n} (x - x_0 - v(t-\xi))^{2\,n} .$$

Numerical analysis

Let us illustrate the situation by comparing the values of b_y in nanotesla between a moving and a fixed line of electric dipoles. Applying Equation 1 in Equation 4 and integrating the result in x_0 we obtain the following expression for $-L \leq x_0 \leq 0$:

$$b_y(\mathbf{R}, t) = -\frac{(z-z_0)\, \mu_0^2\, \sigma}{16\, \pi}$$

$$\int_0^t \frac{e^{-\frac{\mu_0\, \sigma\, \left((y-y_0)^2+(z-z_0)^2\right)}{4\, \xi}}}{\xi^2} \mathcal{I}(t-\xi) \times$$

$$\left\{ erf\left(\sqrt{\frac{\mu_0\, \sigma}{4\, \xi}}(x + L - v(t-\xi)\right)\right.$$

$$\left. - \, erf\left(\sqrt{\frac{\mu_0\, \sigma}{4\, \xi}}(x - v(t-\xi)\right)\right\} d\xi. \qquad (5)$$

In Equation 5 we will keep a constant value for the following parameters: $\sigma = 3$ siemens/m; $\mu_0 = 4\,\pi \times 10^{-7}$ henry/m; $y - y_0 = 20$ m; $z - z_0 = 20$ m; and the length of the source cable $L = 300$ m. We will compute b_y for three values of v: 0, 5 m/s, and 10 m/s. We will employ two current functions and two conditions for the longitudinal distance between the receiver and the closer extremity of the line of dipoles: $x = 20$ m and $x - vt = 20$ m.

The curves of Figures 1(a) and 1(b) are typical of TDEM data. Both of them show a 1 s transient behaviour of b_y after turning off a current $\mathcal{I}_1(t) = 500$ A, for $0 < t < 4$ s and $\mathcal{I}_1(t) = 0$ otherwise. In Figure 1(a) the receiver point is fixed. Because of the source velocity there is a large difference of the field values as a function of the relative displacement in time between source and receiver. Figure 1(b) shows the same variation assuming a constant longitudinal distance between transmitter and receiver. In the present case the effect of the relative velocity between the source and the receiver yields a small but measurable difference larger than 100 pT for transient times less than 10 ms. The change in sign between the two figures is caused by a difference in compensation due to the displacement during the *on time* in Figure 1(a) and the lack of it in Figure 1(b). The curves of Figures 2(a) and 2(b) are typical of FDEM data. Both of them show a 10 s cyclical behaviour of b_y for a current $\mathcal{I}_2(t) = 500 \sin(0.5\,\pi\, t)$ A, for $0 \leq t \leq 10$ s and $\mathcal{I}_2(t) = 0$, for $t \leq 0$. In Figure 2(a) the receiver point is fixed. There is a large difference of the field values as a function of the relative displacement in time between source and receiver for this type of current too. Furthermore, the amplitude of each one of the two curves related to a mobile source varies with time. Therefore, the r.m.s value of the two curves due to a mobile source is a function of the number and the range of cycles employed in its determination. Figure 2(b) shows the same variation assuming a constant longitudinal distance between transmitter and receiver. The effect of the relative velocity between the source and the receiver yields a difference with an amplitude larger than 200 pT.

Conclusion

Because the voltages measured by induction coils allow to measure the magnetic field with a precision of 1 picotesla, our results show that the velocity of the source affects in a measurable manner the primary field value whether we consider the observation point at a constant or a variable distance from the source. Consequently the velocity will also affect the secondary field. So this fact has to be taken into account in the separation of the secondary field and in the identification of its actual space-time position. Otherwise one may do a gross interpretation error.

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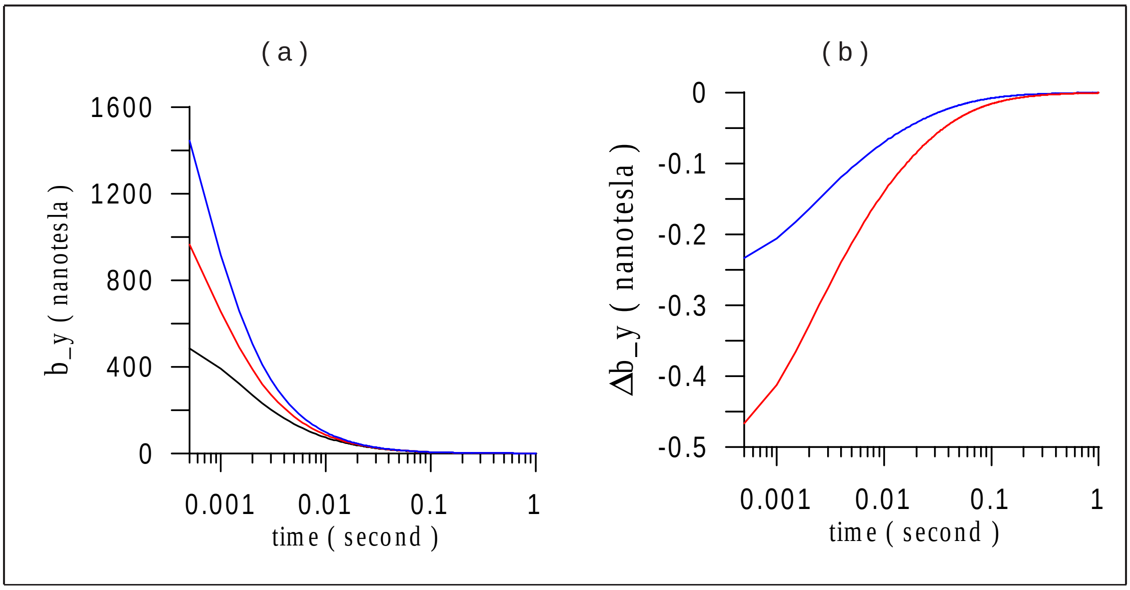

Figura 1: Time counted after turning off a square wave source current lasting 4 s. (a) Transient variation of b_y as a function of time at $x = 20$ m: blue line for $v = 10$ m/s; red line for $v = 5$ m/s; and black line for $v = 0$. (b) Transient variation of the difference of b_y between a moving and a fixed source as a function of time at $x - vt = 20$ m: red line for source moving at $v = 10$ m/s; blue line for source moving at $v = 5$ m/s.

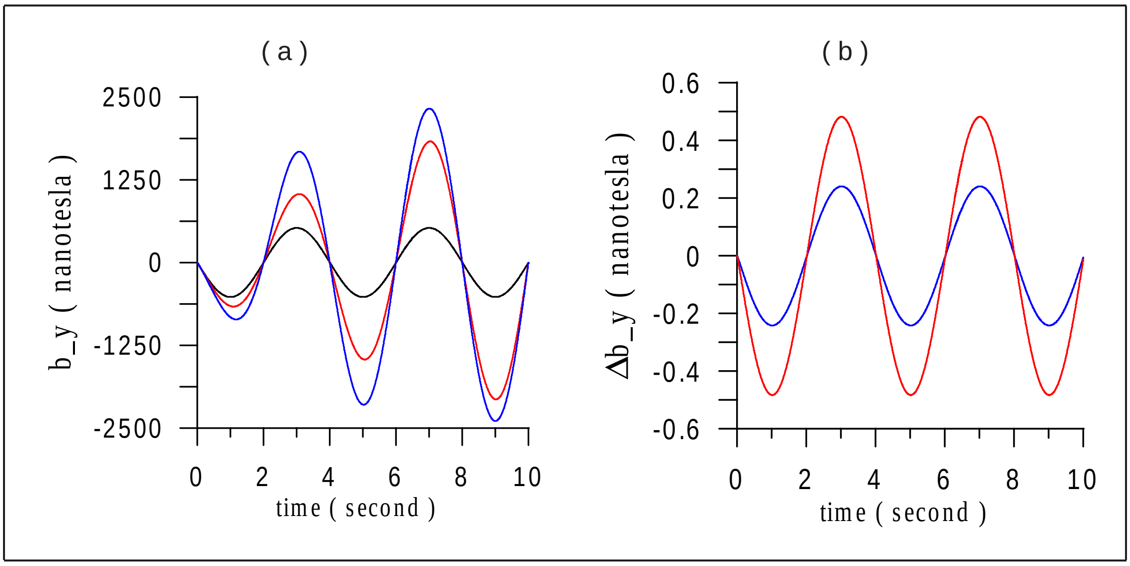

Figura 2: Time counted for a causal sine wave with a period of 4 s. (a) Variation of b_y as a function of time at $x = 20$ m: blue line for $v = 10$ m/s; red line for $v = 5$ m/s; and black line for $v = 0$. (b) Variation of the difference of b_y between a moving and a fixed source as a function of time at $x - vt = 20$ m: red line for source moving at $v = 10$ m/s; blue line for source moving at $v = 5$ m/s.