Dry Rock Elastic Moduli Behavior with Pressure

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Abstract

The knowledge of the pressure dependence of dry rock bulk and shear moduli is essential in time-lapse seismic studies, both on the feasibility and on the interpretation of 4D results. This behavior is accessed only through experimental determination, and there are many different regression laws proposed on the literature to represent it.

We had compared the effectiveness of a set of different laws on large data sets of dry rock velocities measured on the lab. In this paper we focus our attention in three of these laws that are widely used on time-lapse and also seems more reasonable than other relations often applied.

Introduction

Time-lapse seismic technology is quickly evolving from a qualitative study of different seismic volumes to a quantitative and predictive tool to access reservoir saturation and pressure evolution. One of the main issues in obtaining quantitative information from time-lapse volumes lies on the intrinsic coupling between pressure and saturation effects on the seismic behavior of rocks.

Saturation effects are generally modeled by means of the Gassmann relations that are assumed to be valid for the seismic frequency and scale range. The pressure behavior can only be accessed through laboratory measurements since the theoretical models for rock behavior with stress usually fails to reproduce experimental data.

There are many different relations proposed by several authors to reproduce the behavior of seismic velocities or elastic moduli of dry rocks with pressure. Throughout the years, we have compared the success for many of these equations on large data sets of Brazilian rocks, in order to verify its effectiveness in interpolation as well as in extrapolation of rock properties for different pressures.

In this particular paper, we will concentrate our attention on three proposed pressure laws that are widely used on time-lapse studies and that have been proven to be quite reasonable.

One of these laws is the one recently proposed by Collin MacBeth (2004), derived from the elasticity theory of porous media. After MacBeth, the rock moduli are given by:

$$K(P) = \frac{K_\infty}{1+E_K \exp(-P/P_K)}, \qquad (1a)$$

and

$$\mu(P) = \frac{\mu_\infty}{1+E_\mu \exp(-P/P_K)}, \qquad (1b)$$

where K and μ refer to the bulk and shear moduli, respectively. At very large pressures the elastic moduli are simply K_∞ and μ_∞, and for very low pressures the moduli are given by $K_\infty/(1+E_K)$ and $\mu_\infty/(1+E_\mu)$.

David Lumley (2003) and the ***4th Wave Imaging*** group prefers to apply a logarithmic fitting to the experimental data, as follows:

$$K(P) = a_K + b_K \ln(P), \qquad (2a)$$

$$\mu(P) = a_\mu + b_\mu \ln(P). \qquad (2b)$$

They use these laws in order to access the pressure changes through their proprietary pressure and saturation inversion technology. Although the *a* and *b* coefficients are related to the characteristics of the rock, these relations give no insight on the low and high pressure behavior of the rock.

Gary Mavko (2004) and his co-workers proposed the velocity-pressure relation:

$$\frac{V(P)}{V_H} = 1 - \left(\frac{1}{1+a}\right)\exp\left(\frac{-P}{P_H}\right), \qquad (3)$$

which is also widely applied in time-lapse studies.

In order to compare the predictions based on Mavko's relation with those from relations (1) and (2), we could estimate the moduli from the velocities given by (3), simply modifying the equation (3) to:

$$K(P) = K_H\left[1 - b_K \exp\left(\frac{-P}{P_{KH}}\right)\right] \qquad (4a)$$

$$\mu(P) = \mu_H\left[1 - b_\mu \exp\left(\frac{-P}{P_{\mu H}}\right)\right], \qquad (4b)$$

that brings up a good fit to the data and preserves the functional form as well. K_H and μ_H are the high pressure limits of the bulk and shear moduli, respectively, while $K_H(1-b_K)$ and $\mu_H(1-b_\mu)$ are the zero pressure limits.

There are other interesting relations, like those inspired on the Hertz Middlin contact theory, as published by Dvorkin and Nur (1996), Vidal *et al*. (2000) and Landro (2004). We restricted our evaluation to the three models described above since our experience has shown that these are usually the best relations to provide high correlation coefficients and also yields best predicted versus observed values behavior.

Each one of these relations has its assets and drawbacks, the Mavko relation, for instance, can be observed as a heuristic equation that adjusts the natural behavior of velocities reaching an asymptotic value at high pressures while it increases as an exponential-like relation. Nevertheless, it sometimes fails in the presence of "bad" experimental points. On the plus side the "4th Wave" relation fits virtually any experimental data set, however on the minus side the "zero-pressure" and "infinity pressure" behavior is somewhat meaningless. The relation proposed by MacBeth has the clear advantage of its solid physical basis and also suits the general empirical trend, as the Mavko's equation.

The main goal in this paper is to verify the efficiency of each one of these equations to adjust with experimental data, interpolating and extrapolating elastic moduli values, predicting values that were not measured. Although we carried out comparisons using a great number of Brazilian fields, in this paper we describe the results for only two particular Brazilian reservoirs, on which velocities were measured from the ambient pressure to values greater than 6000 psi (41.37 MPa). One of the data sets refers to a tight gas sand reservoir on an onshore Brazilian basin (47samples), while the other data set refers to an offshore unconsolidated turbidity oil reservoir (33 samples).

Method

We measured compressional and shear wave velocities of dry core plugs by the ultrasonic transmission technique. The confining pressure was increased from 1000 to 6000psi, while the pore pressure was kept on the ambient pressure. All samples were also submitted to basic petrophysical measurements.

The results of dry rock bulk and shear moduli were fitted with the different equations discussed above. In order to check the efficiency of each formula we suited also incomplete data sets, making a kind of "blind test" for the equations.

We had compare the predicted versus observed values for each equation, using both data sets (complete and incomplete), and it was compared the errors in each estimation as well. In the case of the Mavko and MacBeth relations, it is interesting to compare the predictions of low and infinity pressure limits of bulk and shear moduli.

Results

This abstract will concentrate on the prediction examples using regressions for incomplete data sets with only 4 pressure points per sample, fitting the moduli measured from 1000 to 4000 psi (6.89 to 27.58 MPa) and using the equations to predict bulk and shear moduli at 6000 psi (41.37 MPa).

It is slightly correct to relate the quality of a data fitting to its regression coefficient R. We observed that the R coefficients obtained are generally higher than 0.95, although there are some exceptions for the 4th wave relation. The MacBeth's and Mavko's relations give greater R values for either moduli, bulk and shear.

On figure 1, we present an example for an unconsolidated sand sample showing the bulk and shear moduli with different predictions obtained using four pressure points (from 1000 to 4000 psi, or 6.89 to 27.58 MPa). In this case, the bulk modulus is well predicted by the MacBeth equation even for the points not used on the regression, while the shear modulus is better predicted by the 4th wave relation. Figure 2 shows another example of incomplete data fitting to a tight sand sample. For this particular sample, the Mavko equation predicts precise both the bulk and shear moduli for the data points not used on the regression. However, the 4th wave regression fails for these two moduli. It is important to point out that these results are not general since it will depend on the pressure behavior of each sample.

In figures 3 and 4 are presented the values for predicted versus observed measurements regarding the bulk and shear moduli at 6000psi of the unconsolidated sand data set, using the relations fitted to incomplete data sets. Figures 5 and 6 shows the same sort of plots for the tight sand case.

On figures 7 and 8 we illustrate bar plots of the relative frequency errors on the prediction of the bulk and shear moduli for the unconsolidated sand data set, using the relations obtained with the "incomplete data set". Figures 9 and 10 illustrate the error in the case of the tight sand. These errors are simply the difference between the prediction and the measured value at 6000psi divided by the measured value.

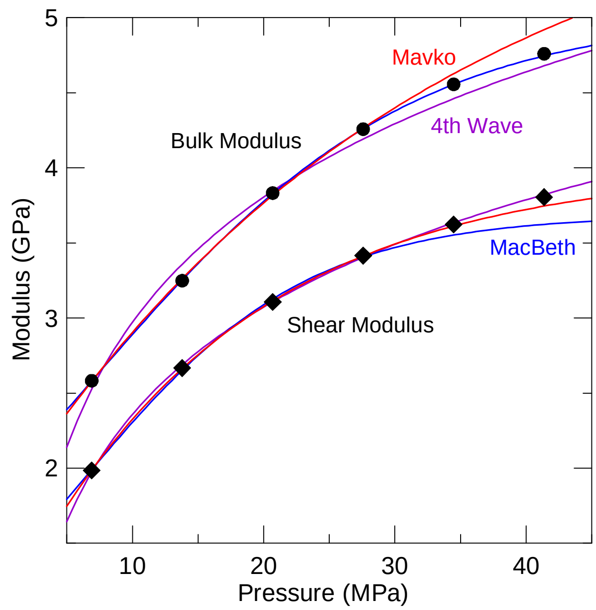

Figure 1 – Example of "incomplete" data fitting for the bulk (circles) and shear (diamonds) moduli with the equations from Mavko (red), MacBeth (blue) and Lumley (purple) in the case of an unconsolidated sand sample.

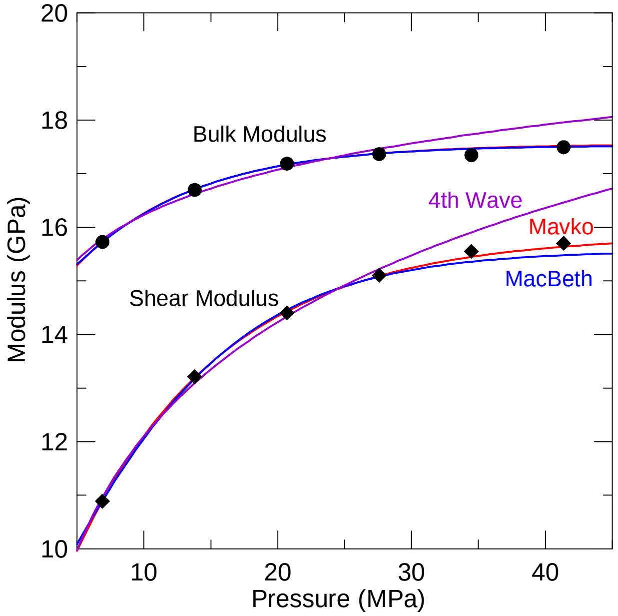

Figure 2 – Example of "incomplete" data fitting for the bulk (circles) and shear (diamonds) moduli with the equations from Mavko (red), MacBeth (blue) and Lumley (purple) in the case of tight sand sample.

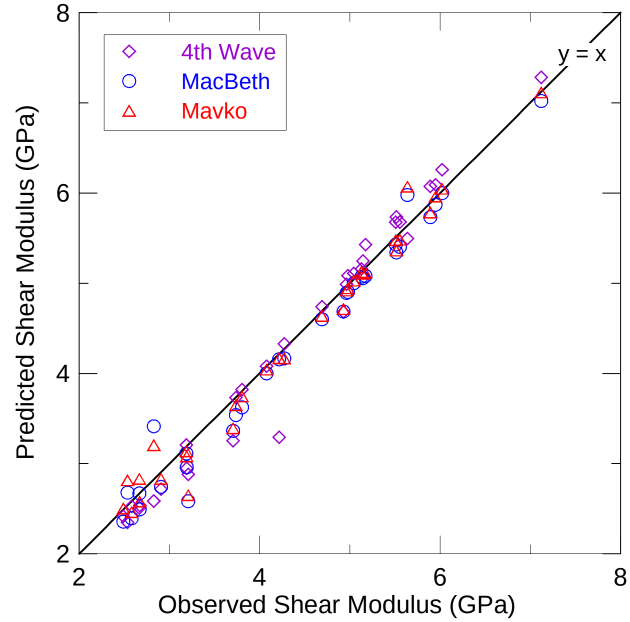

Figure 4 – Predicted versus observed shear modulus at 6000psi for the unconsolidated sand data set.

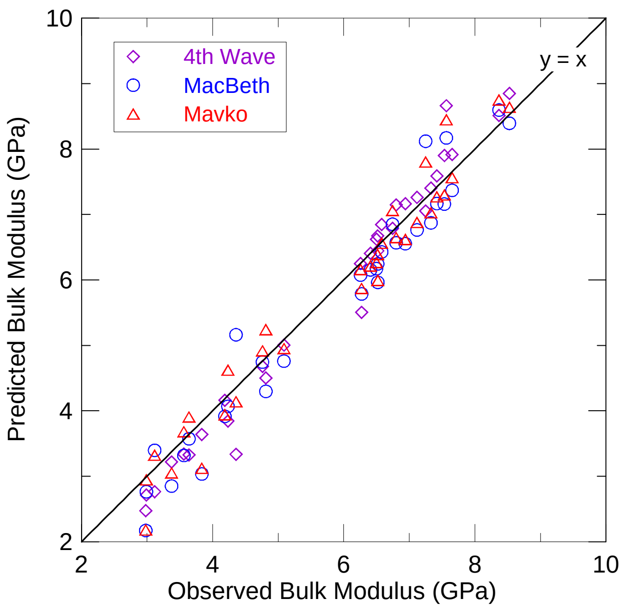

Figure 3 – Predicted versus observed bulk modulus at 6000psi for the unconsolidated sand data set.

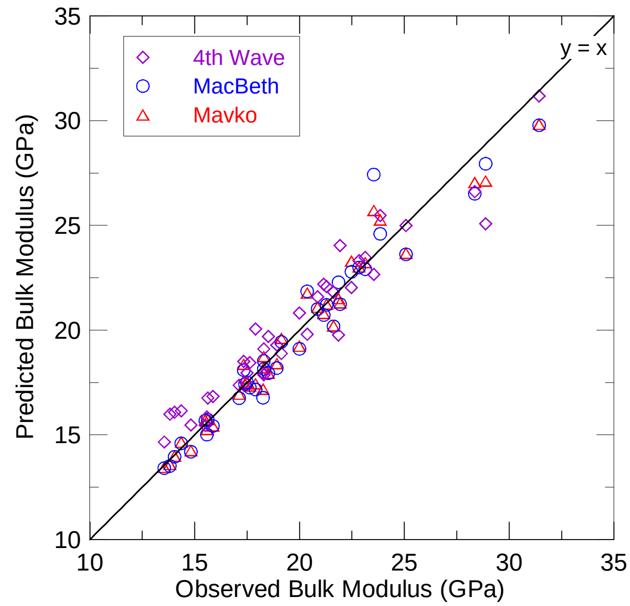

Figure 5 – Predicted versus observed bulk modulus at 6000psi for the tight sand data set.

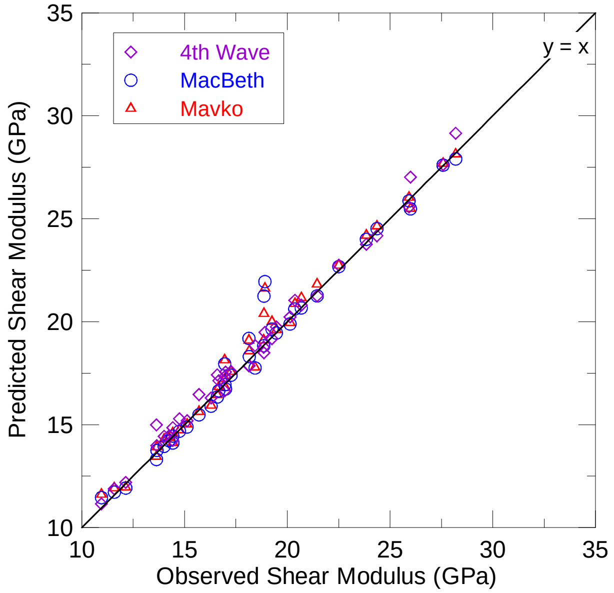

Figure 6 – Predicted versus observed shear modulus for the tight sand data set.

Discussion

Comparing the results for the two different data sets it is important to take into account the characteristics of each rock.

The unconsolidated sands have little or no cementation. As we subject these types of rocks to pressure in the lab, it experiences a process of compaction and even re-arrangement of the grains. This artificial compaction process is reflected on the elastic behavior of the sands, therefore its velocities and moduli increases continuously and in general does not reach a plateau value, even at pressures as high as 8000 psi (55.16MPa). Further pressure increase may lead to velocity or modulus stabilization. Afterwards at very high pressures we may also destroy the characteristics of pore space and even crush the grains.

On the other hand, the tight sand velocities and moduli grows due to the closing of micro fractures and in general reach a stationary value at pressures around 30MPa. Probably the majority of these micro fractures are generated owing to the stress relief and by the coring process.

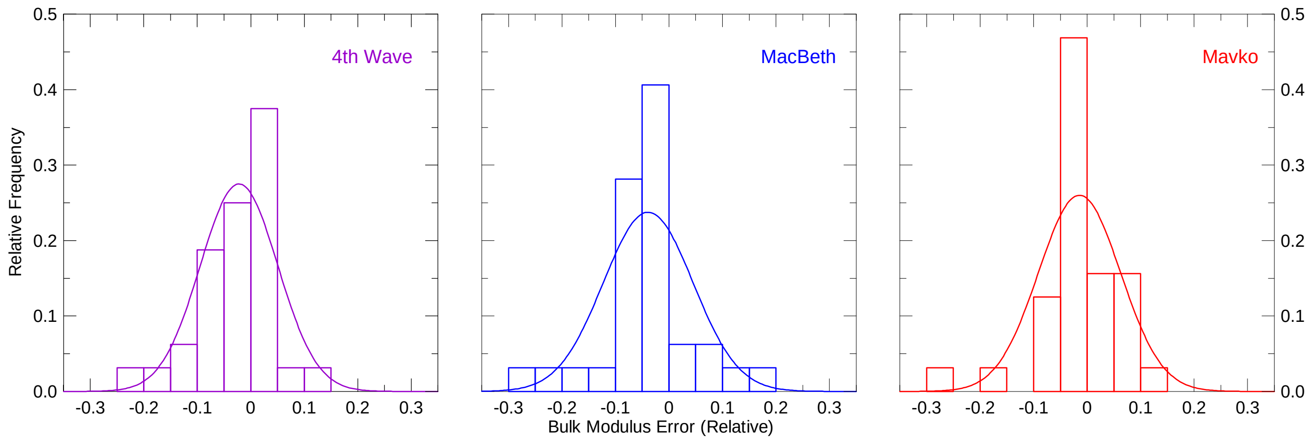

Figure 7 – Error on bulk modulus predictions for unconsolidated sands.

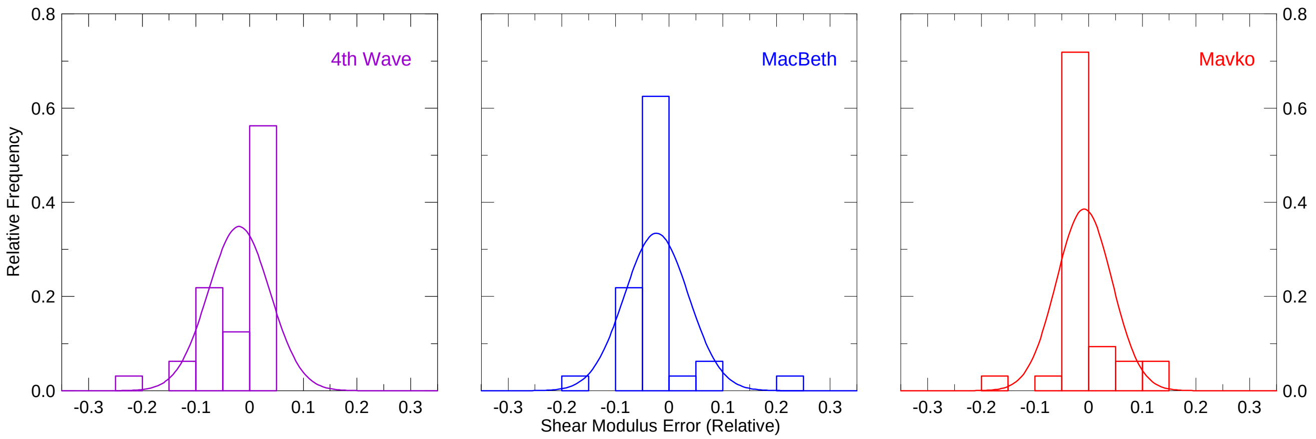

Figure 8 – Error on shear modulus predictions for unconsolidated sands.

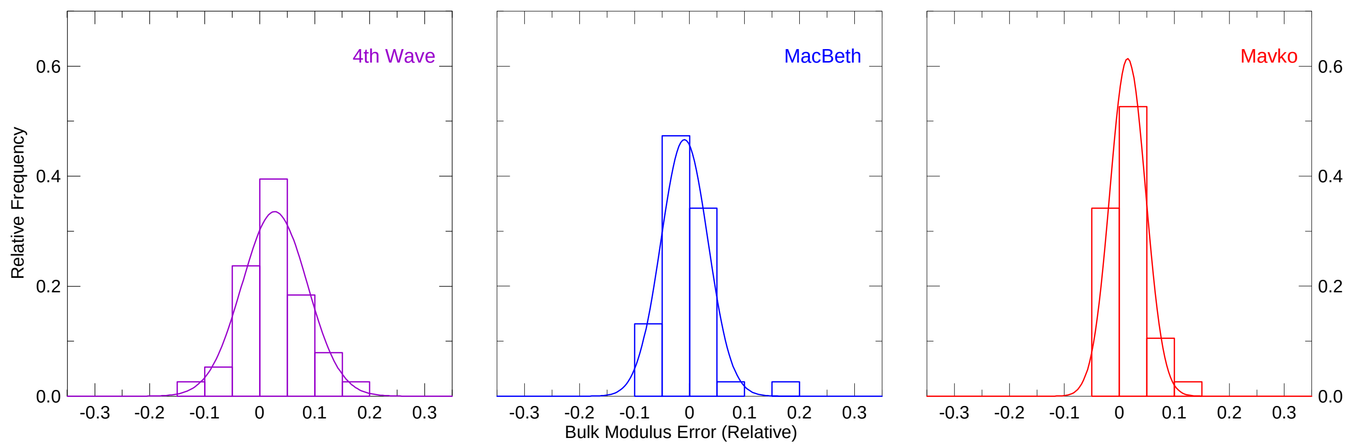

Figure 9 – Error on bulk modulus prediction for tight sands.

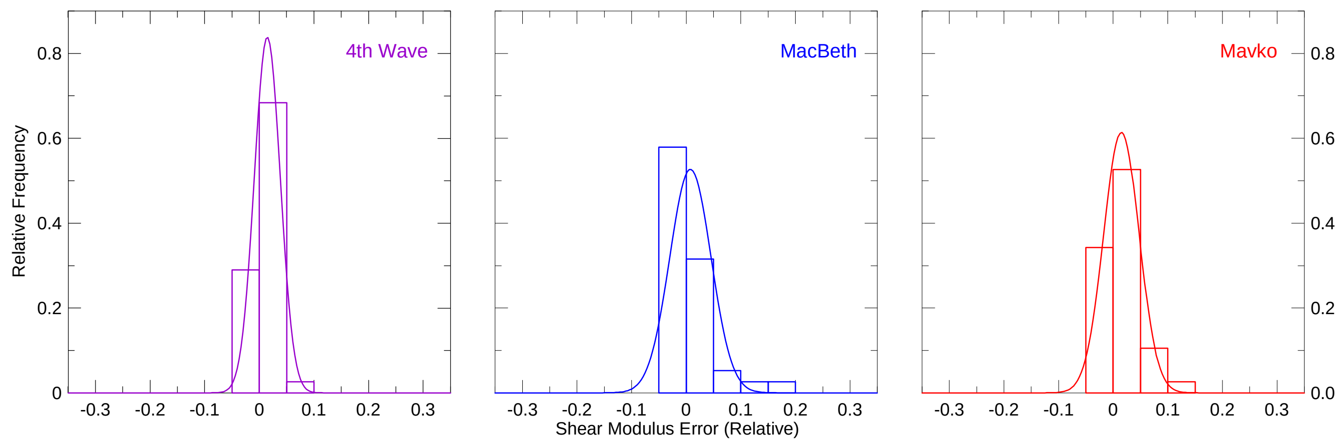

Figure 10 – Error on shear modulus prediction for tight sands.

We observe that, usually, the predicted moduli for the unconsolidated sands have a slight bias to values lower than the actual value. This is a result of the frame characteristics of such rocks. There is virtually no bias on the predictions for the tight sand case, since its velocities tends to reach a plateau.

The error distribution for the unconsolidated sands is broader than for the tight sand (maximum absolute error about 30% for unconsolidated and 15% for tight sands).

All three equations bring up good results in both cases. Concerning the logarithm functional characteristics, the 4^{th} wave relation may be more indicated to unconsolidated rocks. The equations proposed by Mavko and MacBeth are suited for consolidated as well as for unconsolidated rocks.

Since these equations are used on time-lapse studies, one question may arise regarding its impact on elastic attributes variation with pressure. We estimate that, in the worst cases (loose sand) the errors on the dry bulk modulus may lead to errors of the order of 10% for the compressional velocity. This error is comparable to the time-lapse effects that we are looking for. Nevertheless, such a large error is related to velocity extrapolation, while usually these studies involve velocity interpolation, which has much smaller associated errors.

In practice the impact of velocity variation estimation may be minimized with careful measurements on a reasonable sized set of representative rock samples.

We had tested these relations in carbonate, metamorphic and igneous rocks and observed similar results. The elastic moduli behavior is generally related to the degree of consolidation and characteristics of the pore space.

Although all relations works quite well in elastic moduli interpolation and extrapolation, someone may consider to apply the relations proposed by MacBeth or Mavko to access limit values for null and infinity pressures, that is physically related to rock characteristics.

Conclusions

It was compared in this work the efficiency of three different relations commonly used on time-lapse seismic studies to mimic the reservoir elastic behavior dependence on pressure variations.

All of the three relations give reasonably good results even for predicting the elastic properties beyond the pressure range effectively measured on the lab. The impact of the errors involved may be of minor significance in carefully conducted studies.

Acknowledgments

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