Kirchhoff modeling and migration in weakly anisotropic medium using the pWFC (perturbed WaveFront Construction method)

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Abstract

The presence of velocity anisotropy in the layers or in the overburden leads to mispositioning of reflector structures and degradation the image quality, when an isotropic migration processing is performed on seismic data. These distortions occur even for weakly anisotropic layers, which are the most common type of rocks in geologic models for seismic applications.

In this paper we present a method where a first-order perturbation is applied to the wavefront construction algorithm which generates a traveltime table that fits the traveltime of a weakly anisotropic medium with arbitrary of symmetry. We call this method the perturbed WaveFront Construction (pWFC). These traveltime tables are used in traditional Kirchhoff modeling and migration algorithm. The pWFC allows an approximated computation of traveltime tables for weakly anisotropic medium efficiently, avoiding a more intensive computation by a full anisotropic ray-tracing algorithm.

To illustrate the difference when considering or not weak anisotropy in the seismic processing, two common-offset (100m and 1100m) data were modeled using WFC isotropic and using pWFC method. The 1100m dataset was migrated twice using a Kirchhoff algorithm once using an isotropic traveltime tables and the other a traveltime tables computed by the pWFC method.

Introduction

Kirchhoff migration has a great sensitivity on the velocity model, mainly concerning to the lateral variations, reflecting on the quality of the migrated images and on the accuracy of positioning the geologic structures. When anisotropy is present in rocks the Kirchhoff migration isotropic algorithm fails in dealing with the anisotropic effects on the migrated image.

Weak anisotropy, the most common case in geologic models for seismic propagation, according to Thomsen (1986), allows the use of a first-order perturbation method to correct the isotropic propagation rays equations to fit the anisotropic ones (Psencík & Gajewski, 1998).

These correct ray equations can be used in the wavefront construction (WFC) method, which was introduced by Vinje et al. (1993). We call this method, which we introduce in this paper, as perturbed wavefront construction (pWFC) method. In this work, we propose to include pWFC method for computing the traveltime tables in a traditional Kirchhoff modeling and a kinematics Kirchhoff migration algorithm, allowing applications to weakly anisotropic medium.

Method

The traditional Kirchhoff modeling and migration method is based on the result of the Kirchhoff-Helmholtz (KH) integral, that describes the reflected wavefield as an integral along the reflector (Tygel et al., 2000). The KH integral needs traveltime tables to compute the modeling or migrated output data, which can be computed by the WFC method.
The kernel of isotropic WFC method is performed by numerical integration of a system of ray-tracing equations:

$$\frac{d\vec{x}}{dt} = v^2 \vec{p} \,, \quad (1a)$$

$$\frac{d\vec{p}}{dt} = -v^{-1}\nabla v \,, \quad (1b)$$

where v is the isotropic P or S velocity field, $\vec{x}$ and $\vec{p}$ are the position and slowness vector.
In order to extend the WFC to weakly anisotropic medium the equation (1a) will be change by:

$$\frac{d\vec{x}}{dt} = \vec{v}_g \,, \quad (2a)$$

$$\frac{d\vec{p}}{dt} = -v^{-1}\nabla v \,, \quad (2b)$$

on this new system of equations v is redefined as a reference isotropic velocity field of the perturbed method and $\vec{v}_g$ is the anisotropic group velocity field given by:

$$\vec{v}_g = \left|\vec{v}_g\right|(\cos\phi, sin\phi) \,, \quad (3a)$$

$$\left|\vec{v}_g\right|^2 = f^2 + \left(\frac{\partial f}{\partial \theta}\right)^2 \,, \quad (3b)$$

where the perturbed equation is present in the function f ; the phase velocity of the anisotropic medium, and ϕ is the group direction angle, which is function of the f and θ (phase direction angle), given by:

$$\phi = \theta + \tan^{-1}\left(\frac{1}{f}\frac{\partial f}{\partial \theta}\right). \qquad (4)$$

In this work we specify the pWFC method to VTI medium, then the qP-phase velocity f of a VTI medium, expressed in terms of Thomsen parameters ε and δ, formally $f \equiv f(\vec{p}, v, \varepsilon, \delta)$, is given by:

$$f = v\left(1 + \delta\cos^2(\theta)sin^2(\theta) + \varepsilon\, sin^4(\theta)\right), \qquad (5)$$

where:

$$sin(\theta) = \frac{p_x}{|\vec{p}|}\ ;\ \cos(\theta) = \frac{p_z}{|\vec{p}|}.$$

Therefore, at each time iteration the algorithm numerically integrates equations 2a and 2b, producing new wavefront points that fit the anisotropic exact ones. Figure 1 shows the comparison of pWFC with an analytical computation and the ANRAY software (Gajewski & Psencík, 1990). The proposed method can be applied to arbitrary weakly anisotropic medium by changing the function f. The examples of applications in this work are done in 2D, as was stated in (5), but the algorithm can be expanded to 3D, when f is written in 3D applying in the general equations 3a and 3b.

Examples

A) pWFC Kirchhoff modeling

In this example a homogeneous VTI medium is considered, above a structure, which has vertical velocity 2.0 km/s and the following Thomsen parameters: $\varepsilon = 0.18$ and $\delta = 0.08$, and the same structure was considered below a homogeneous isotropic model with the same velocity of the vertical velocity of the VTI medium describe before.
A Kirchhoff modeling using WFC isotropic and pWFC were applied. Figure 2a and 2b, illustrates the superposition of the model with the commons-offset modeled data.
Figure 2a shows the common-offset 100 m of isotropic modeling and Figure 2b shows the common-offset 100m with VTI pWFC modeling. Observe the results on both are similar, even in t_0 quantities, but the diffractions are slightly mispositioned horizontally and in time. That difference is greater if the offsets are larger, as is showed in Figure 3a (isotropic) and Figure 3b (VTI), for common-offset 1100m, where the horizontal position and time of the structure are changing very much, even for weakly anisotropic VTI, which was considered.

B) pWFC Kirchhoff migration

The Kirchhoff migration applied was a PSDM (Pre-Stack Depth Migration) where each output trace is computed using the traveltime tables performed by pWFC (Portugal et al., 2003), and each sample is spread out along the isochrones, corrected for weak anisotropy.
In this work we applied the Kirchhoff migration pWFC on the modeled data showed in Figure 3b, and for comparison we applied the isotropic WFC traveltime tables.
Figure 4a shows a common-offset 1100 m migrated with pWFC and Figure 4b the results for an isotropic migration algorithm WFC. Clearly, in the result of isotropic migration the diffractions were not well treated and the true position of the structure was not correct positioned in depth, in contrast, the pWFC result (Fig. 4b) was quite right at depth positioning and the diffractions were well collapsed.

Conclusions

In this paper is shown that pWFC works very well for Kirchhoff modeling and migration for weakly anisotropic VTI media. The method has advantages in avoiding computation of a full anisotropic ray tracing to generate traveltime tables for Kirchhoff method. Therefore the pWFC is an efficient and low cost method to deal with imaging problems in weakly anisotropic media and can be extended to 3D medium with arbitrary symmetry.

Acknowledgments

The authors wish to thank PETROBRAS for supporting and giving permission to publish this work.

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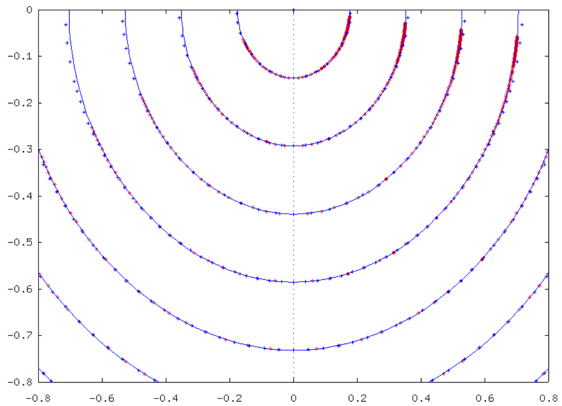
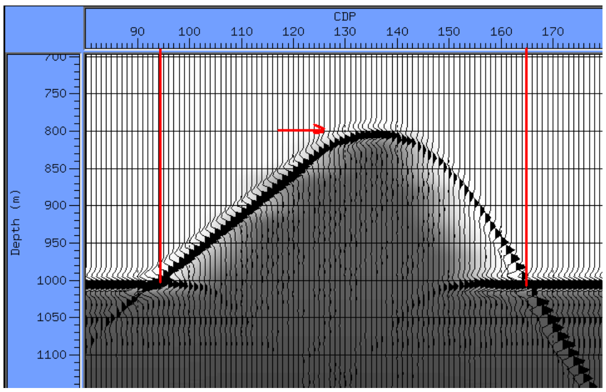
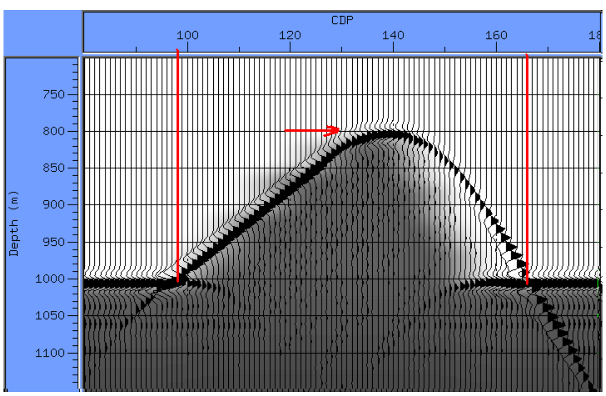


Figure 1: Wavefronts for a homogeneous VTI media: Continuous Line analytically computation, Red points from ANRAY, Crosses from pWFC. Observe that pWFC presents slightly misplaced results only for larger apertures.

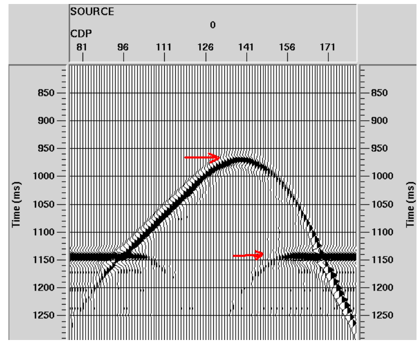


(a)



(b)

Figure 2: Kirchhoff modeling in common-offset 100m (a) isotropic WFC and (b) VTI pWFC. Observe that diffractions appear at slightly different position and time.



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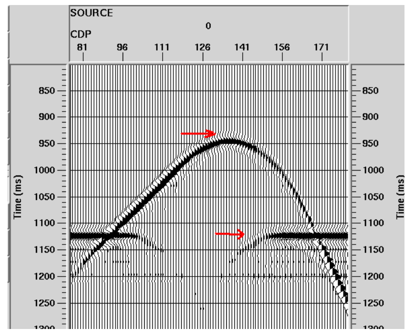
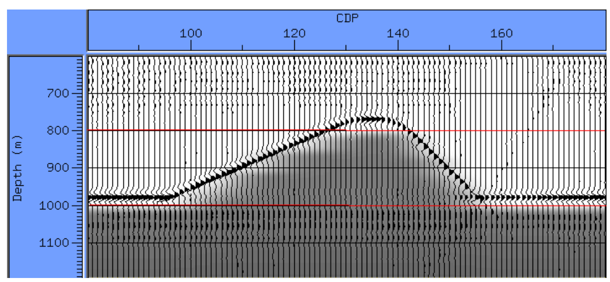
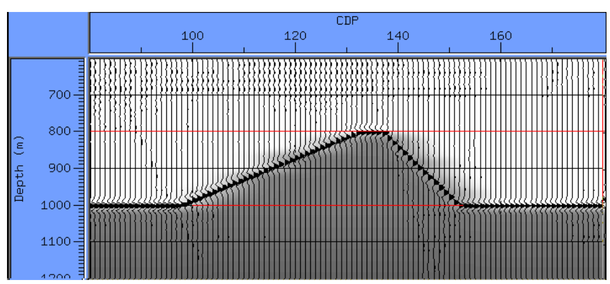


(b)

Figure 3: Kirchhoff modeling in common-offset 1100m (a) isotropic WFC and (b) VTI pWFC. Observe that diffractions appear at larger different position and time.



(a)



(b)

Figure 4: Migration results of common-offset 1100m section from figure 3b, (a) isotropic migration and (b) anisotropic migration pWFC. Observe that diffractions were better treated in anisotropic migration pWFC, and the position in depth has a good fit with an actual one.