Compensation for Q-losses revisited - A more stable approach using SVD

Adelson S. de Oliveira, PETROBRAS S/A, Brazil

This paper was prepared for presentation at the 9th International Congress of the Brazilian Geophysical Society held in Salvador, Brazil, 11-14 September 2005.

Abstract

This paper discusses an improved approach for Q-compensation. The approach is based on a singular value decomposition of a matrix that represents earth's attenuation and dispersion processes according to Futterman's model. It is shown that the proposed approach may provide a much broader frequency band recover of signals under severe attenuation than methods supported by usual stabilizing techniques. It also briefly discusses a very common misleading thought of time-varying processes as a succession of stationary filters. Synthetic and real data examples are shown.

Introduction

Any implementation of a Q-compensation algorithm as predicted on Futterman's model (1962) has to deal with the stabilization problem related to extremely small amplitudes at higher frequencies. Most available algorithms make use of stabilization procedures that are not optimal. A non-optimal stabilization scheme usually restricts more than necessary the effectiveness with which one could safely recover information. As a result, seismic processing geophysicists very often have to undercorrect for these losses due to the amount of noise and artifacts the employed technique bursts out. In most cases, only phase corrections are applied.

Singular value decomposition (SVD) allows for a very suitable discrimination of the (quasi)null space of any linear transformation. This makes it possible to isolate ill-behaved subspaces and, consequently, a pseudo-inverse may be defined with a more appropriate degree of stability constrained only by the present signal/noise ratio.

According to Futterman's model, the frequency dependent selective attenuation of energy in seismic data (and in many more other physical phenomena) may be described as an exponential amplitude decay with respect to time and frequency. A phase is associated with the model so as to comply with causality requirements. To any given travel time is associated a function that represents the impulsive response of the earth at that time. Mathematically, this attenuation is time-variant and cannot be described as a convolution. Hence, many pessimistic expectations about the recoverability of information in severely attenuating media may be misjudgment. In fact, the attenuation may be severe out of a very narrow frequency band for some travel times without affecting the reversibility of the process as a whole. However, although the process may be invertible in principle, it is also generally ill-posed. Therefore, a low signal/noise ratio is the main limiting factor and an optimized procedure to push the limits of recoverability as far as possible is quite desirable.

Most Q-compensation algorithms used in industry today were conceived to accomplish a small cost/benefit ratio. The cost estimation is now quite different due to the availability of more efficient computers. New approaches may be affordable today. An example of a clever implementation of a cost effective Q-compensation algorithm can be found in Varela et al (1993). This algorithm is currently implemented in cascades with a good stability control based in a Q/t ratio limitation. This procedure is adopted here as the representative of current methods for the sake of comparison with the proposed SVD approach.

A brief discussion on the invertibility of time-varying processes

Stationary band limited processes are not reversible because an infinite number of possible functions would satisfy the associated convolutional equation. Non-stationary filters or, in a more precise form, general linear transformations may not be diagonalizable by Fourier Transforms. This means that a usual frequency domain analysis of the behavior of the transformation is not as fruitfull as in the stationary case. Particularly, band limitations on Fourier spectra of temporarily isolated components of the transformation does not necessarily imply irreversibility of the whole transformation.

Let's consider, for example, a transformation that acts like a sinc of progressively smaller band as time increases. To avoid limitations on discrete representations of the sinc function, the *discrete* sinc is considered. The transformation is represented by an $N \times N$ matrix **A** with elements given by,

$$\mathbf{A}(i,j) = \begin{cases} N & j = 1 \\ \frac{\sin(\pi(N-j+1)(i-j)/N)}{\sin(\pi(i-j)/N)}, & j = 2,4,6,\ldots \\ \frac{\sin(\pi(N-j+2)(i-j)/N)}{\sin(\pi(i-j)/N)}, & j = 3,5,7,\ldots \end{cases} \quad (1)$$

where i varies from 1 to N and the indetermination for $i = j$ has to be evaluated. Under the application of **A**, a trace containing a single spike at a given position, turns into a *discrete* sinc function, a band limited trace. Moving the spike up and down one gets traces with broader and narrower bandwidths, respectively. However, the inverse of **A** seems to exist to a great degree of accuracy. Given the invertibility of **A**, the original trace may be recovered without any loss. Thus, it may be stated that despite the amount of information apparently lost from the input trace to the output filtered one, it is possible to compensate for the lack of information and get the original trace back. In other words, there has been no effective loss of information. Figure 1 shows three traces: a train of spikes, the result after the application of the process represented by **A**, and the result after inverting the process via $\mathbf{A}^{-1}$.

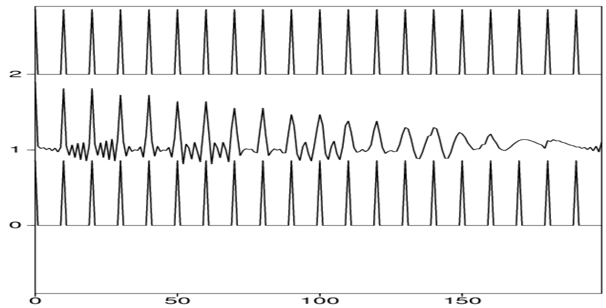

Figure 1: Trace 0 is the input trace with a train of spikes, trace 1 has the result of matrix **A** applied on trace 0, and trace 2 has the result of the inverse of **A** applied on trace 1. All spikes were recovered.

The apparent inconsistency in recovering what was presumably lost may be fixed considering that it is the band limited character of any event for different positions that implies irreversibility in a stationary band limited process. Variations of the bandwidth with time in a time-varying process are a different situation. Events may be fully preserved at certain ranges of time bounding the uncertainty to restricted narrow intervals in the trace. Since the uncertainty about the position and amplitude of a single event convolved with a known signature is zero, there may be full reversibility if the process is not stationary.

Nonetheless, if the process is ill-posed, a small amount of noise makes reversibility of **A** useless. In other words, noise is the only guaranteed cause for irreversibility of time-varying processes.

Futterman's Model for Attenuation and Dispersion

A matrix representing the time-varying process described by Futterman's model is made up of columns containing the time domain version of the well known exponentially decaying formula given by,

$$A(t,f) \;=\; \exp^{-\frac{\pi f t}{\mathrm{Q}}\left[1+\imath\ln\left(\frac{f}{f_r}\right)\right]} \qquad (2)$$

where A, t, f, Q, and f_r are, respectively, the amplitude response of the earth filter, the time, the frequency, the so called Quality Factor, and a reference frequency. Losses in a small Q environment may take higher frequency amplitudes as down as zero from a practical point of view. Nonetheless, as discussed in the previous section, there may be an inverse for the attenuation process and, at least in principle, losses may be reverted. The signal/noise ratio rules the amount of information one really may count with. In figure 2, a Q value as small as 20 was considered. The earth filter was applied to a synthetic trace containing a train of spikes and some (pseudo)random noise was added at a signal/noise ratio of 10^3, just enough to make A^{-1} not usefull. A pseudo-inverse to the earth filter was applied to the filtered trace and the result is plotted against the input trace and the trace resulting from the application

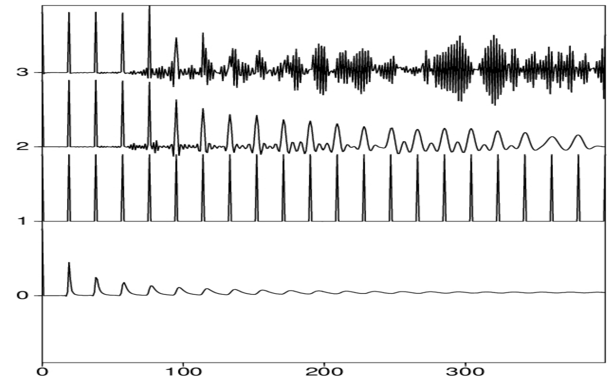

Figure 2: Trace 0 is the input trace with a train of spikes under earth's Q-losses (Q=20,reference frequency=126Hz), trace 1 is the input train of spikes, trace 2 is the SVD pseudo-inverse applied to trace 0, and trace 3 is the result with the regular Q-losses compensation approach.

of the standard Q-compensation algorithm(1993) in figure 2. Note that, although a better recover of the initial trace was not possible with the proposed algorithm (pseudo-inversion), it was possible to resolve for the spikes much better than what was achieved with the conventional approach.

Real Data examples

Real data have noise which demands a less favorable trade off on the amount of recovered information given an acceptable level of spurious events. The trade off will be more favorable if a more adequate method for handling the (quasi)null space is adopted. This section has two real data examples on the application of the technique described in Varela et al(1993) and the proposed one that shows considerable benefits coming from the use of the latter. In the first example, a small window of a seismic section is shown in four different versions: the original, the Q-compensated with the SVD approach, Q-compensated with the regular algorithm, and the result with the regular algorithm plus filter, respectively, in Figures 3, 4, 5, and 6. Constant values for $\mathrm{Q} = 70$ and $f_r = 126$Hz were used so as to obtain a broader spectrum with the smaller amount of spurious events coming up from the SVD approach. The conventional technique was applied with the same pair (Q, f_r) for comparison. Although this value of Q may be artificially small, it serves to emphasize the importance of a more stable procedure when applying Q-compensation in real seismic data. In the conventional approach a minimum Q/t ratio of 30Hz was used as the optimum choice to avoid spurious events. For the SVD approach the stability was achieved by limiting the amount of accepted singular values so that 98% of the whole energy were fully used.

A better definition of events may be observed in the SVD case. The use of the filter on the regular Q-compensated section to increase the similarity with the SVD case did not achieved the expected result. Differences between the sections shown in Figures 4 and 5 cannot be easily explained by Fourier spectra analysis. On the other hand, a reduction of the minimum Q/t ratio allowed on the regular method has revealled a horizontal noise (not shown) not

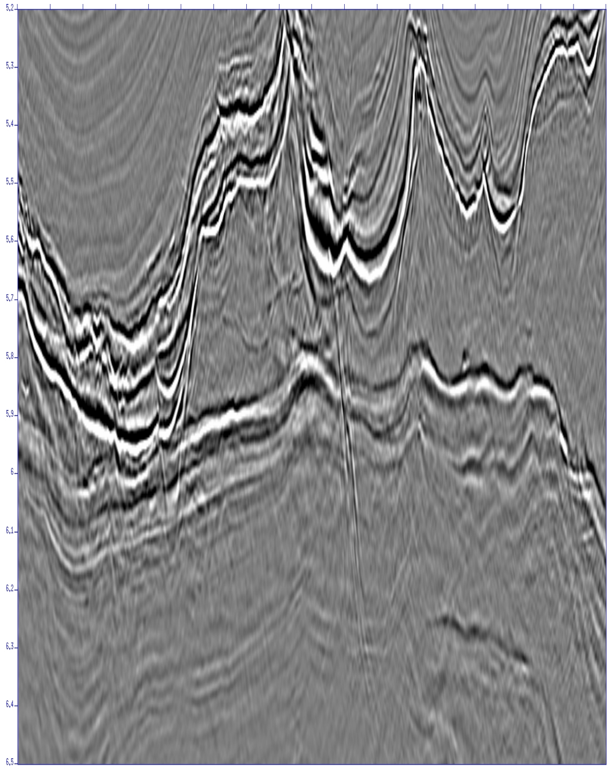

Figure 3: A small window of a seismic section with a low dominant frequency.

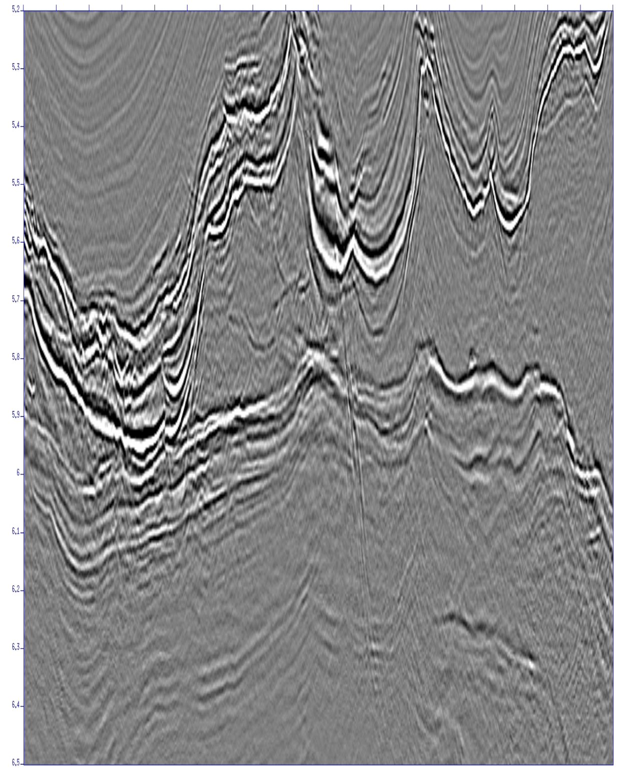

Figure 5: The same seismic window shown in Figure 3 after Q-compensation with the regular approach.

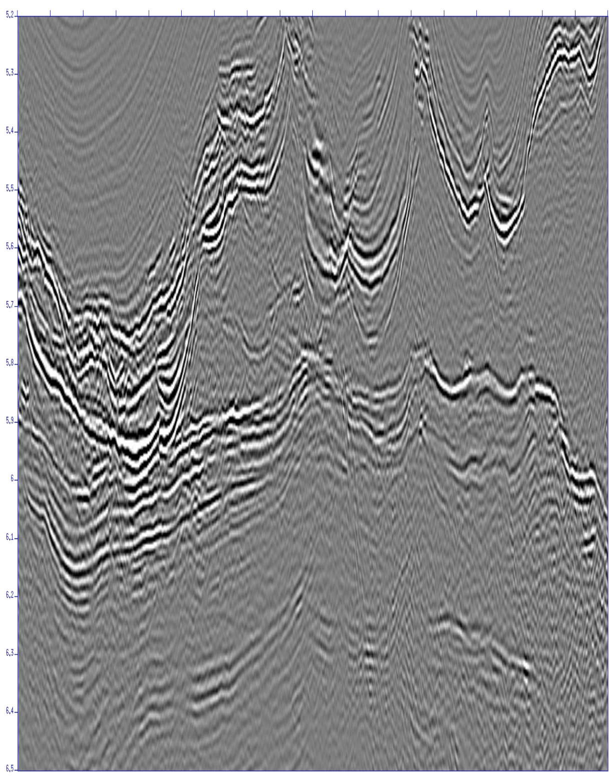

Figure 4: The same seismic window shown above after Q-compensation using the SVD approach.

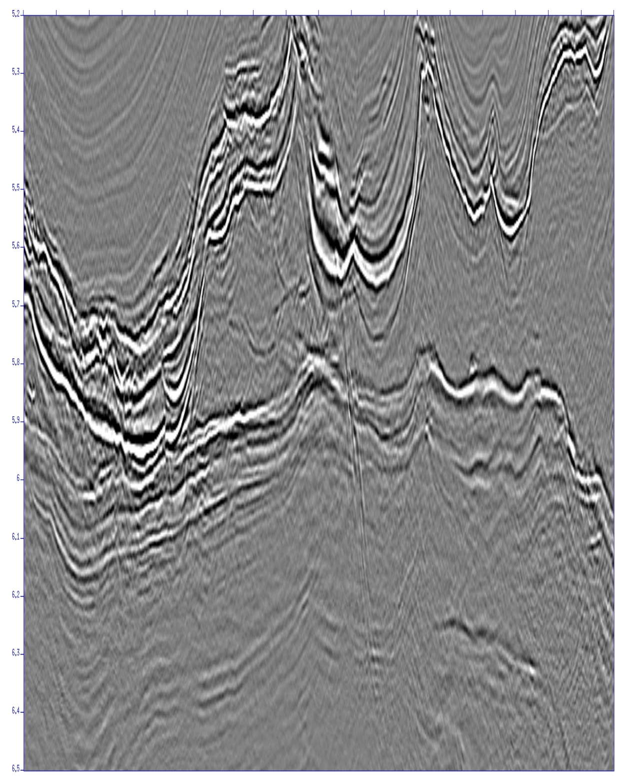

Figure 6: The same seismic window shown in Figure 3 after Q-compensation with the regular approach plus band-pass filter.

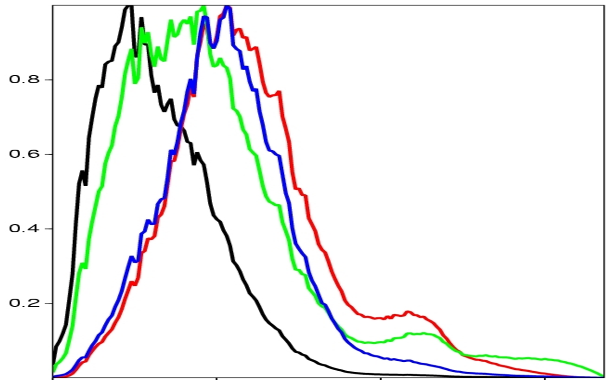

Figure 7: The normalized average spectra of the seismic sections shown in figures 3, 4, 5, and 6. In black, the original section spectra; In green, the regular approach result; in blue, the result with the SVD approach, and in red, the spectrum obtained with the regular approach after a band-pass filter.

seen on the SVD case at the level of stabilization used to generate Figure 4. Hence, it is very likely that a seismic processing geophysicist would take section 5 as the optimized result with the conventional Q-compensation approach if a value of Q as small as 70 was to be used.

Figure 7 has the normalized average Fourier amplitude spectra of the sections displayed in Figures 3, 4, 5, and 6. The original spectrum corresponds to the black line, the one obtained with the conventional method is represented in green, the new approach is displayed in blue, and the application of a band-pass filter to the conventional result is shown in red. The filter made the conventional and SVD approaches quite similar at the low frequency range. However, as observed above, despite the greater similarity between average spetra, the use of the band-pass filter did not made the conventional result look similar to the SVD one. Also, it can be seen that the new approach spectrum has its main contribution localized in higher frequencies than the conventional one. The conventional approach shows an increase in energy around 80Hz where most of the data is noisy.

The second real data example is a well-tie exercise. Figure 8 presents an example of a well/seismic data correlation (shrinking/stretching was not considered). The seismic data was split and the synthetic was introduced (repeated 5 times). Figure 9 has the same data after a painstaking choice of $\mathrm{Q}(t)$ and $f_r(t)$ functions to improve the fit. The agreement is much better. Also, a very small percentage of random noise have been brought up.

Since there are many sources of mismatch between seismic data and synthetics, it cannot be said that the obtained $\mathrm{Q}(t)$ and $f_r(t)$ functions are reliable measurements of the earth attenuation properties. What this exercise shows is that it is possible to go much further with the study of earth attenuation properties than we have gone with conventional stabilizing techniques.

Summary and Comments

This paper discusses the possibility to reverse time-varying processes under appropriate levels of input signal/noise

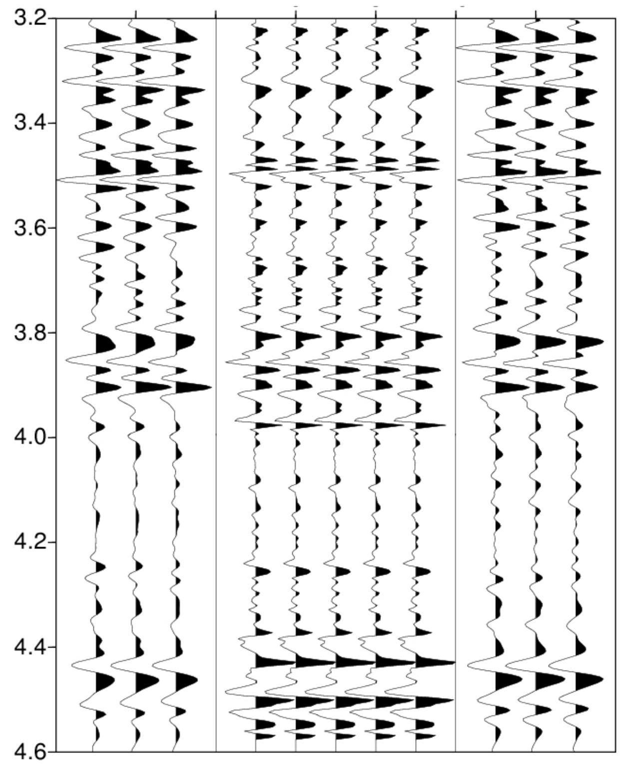

Figure 8: A fit between seismic data (first and final three traces) and synthetic well-log trace (fifth to ninth traces)

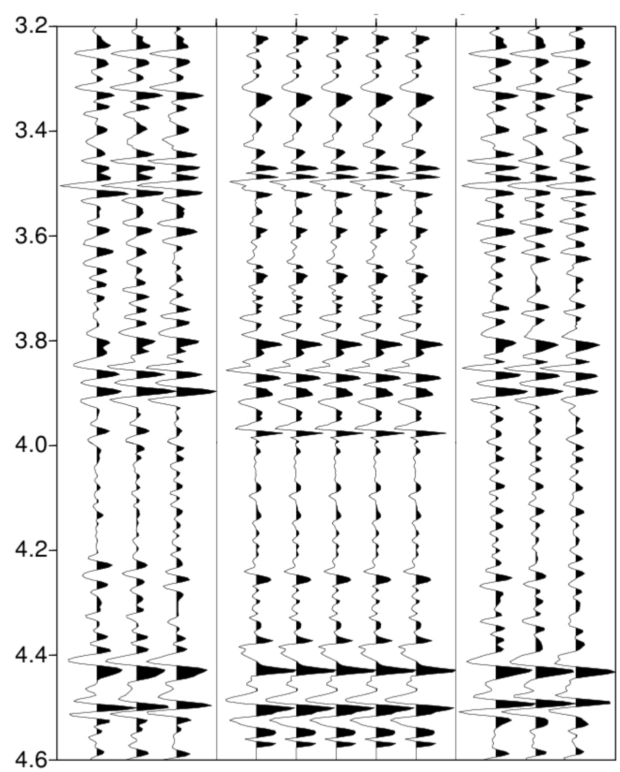

Figure 9: The Q-compensation via SVD case. A fit between seismic data (first and final three traces) and synthetic well-log trace (fifth to ninth traces)

ratio. It also discusses the use of SVD as a means to build pseudo-inverses that optimizes the resulting signal/noise ratio, and how this approach tends to produce a much more stable algorithm than usual techniques for Q-compensation.

The benefits of adopting a SVD approach for Q-compensation was shown to be related to gains in resolution of recovered events, reduced amount of random noise and the possibility to study the attenuation phenomena via better match between seismic and well logs.

References

Futterman, W. I., 1962. Dispersive Body Waves. Journal of Geophysical Research, **67**, p. 5279-5291.

Varela, C. L., Rosa, A. L. R. and Ulrych, T. J., 1993, Modeling of attenuation and dispersion: Geophysics, Soc. of Expl. Geophys., **58**, p. 1167-1173.

Acknowledgments

I would like to thank Fernando Malheiros Roxo da Motta for his interest, enthusiasm and helpfull suggestions on the study that supported this paper. I also would like to thank Armando Lopes Farias for helpfull discussions, suggestions and help with real data processing of the sections shown. Thanks to João Carlos Guedes Logrado for providing the real data-well log fit example.

Thanks to Carlos Lopo Varela and André Luis Romanelli Rosa for discussions and suggestions on the fundamentals and application of the standard Q-loss compensation algorithm.

Special thanks for the reading and corrections made by Fernando Malheiros Roxo da Motta, Ricardo Augusto Rosa Fernandes, and Carlos Lopo Varela.

Finally, I would like to thank PETROBRAS for permission to publish this work.