Inverse Problem of Wave propagation in Weakly Lateral Heterogeneous Layered Medium

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Abstract

A wave propagation generated by a boundary source into a weakly lateral heterogeneous medium (WLHM) occupying a half-space $z > 0$ and consisting of two different materials separated by an interface is considered in the acoustic approximation. In this paper WLHM means that the velocity of the wave propagation depends weakly on the horizontal coordinates $x = (x_1, x_2)$ in comparison with the strong dependence on the vertical coordinate z. The density is assumed to be dependent only on z, and the shape of the interface is described by a function weakly dependent on $x = (x_1, x_2)$. We consider the problem of the reconstruction of the velocity and density inside every component of the half-space and the shape of the interface between them from the knowledge of the medium response measured at z=0. We obtain a recurrent system of 1D inverse problems to find the first two terms in the decomposition in the refraction index $n(z, \epsilon x)$ (inverse power of the velocity of wave propagation) and the shape of the interface $z = h(\epsilon x)$ with respect to the small parameter ϵ, the ratio of the horizontal and vertical gradients of the velocity, along with the density. In the zero-order approximation we derive a system of non-linear Volterra integral equations for determination of the density and zero-order term of the refraction index. Next, the first-order approximation the refraction index is determined as a solution of a coupled linear system of Volterra-type integral equations. We demonstrate the effectiveness of the approach in numerical applications to the 2D case of wave propagation.

Introduction

The inverse problems (IP)of geo-exploration imply investigation of domains inside the earth crust which contain gas, oil or other minerals. A large part of the earth crust may be approximated by a layered structure, that is, may be represented as a stack of layers separated with interfaces. Within every layer the properties (velocity of the wave propagation, density, and so on) depend smoothly on z and slightly vary along the horizontal coordinates and have jump discontinuities while crossing an interface. In this paper we concentrate on the reconstruction of a smooth part of the velocity and density as well as a shape of interface in the acoustic approximation in an inhomogeneous half-space consisting of upper layer and semi-infinite bottom.

Applying a boundary point source at the ground surface $z = 0$, we analyze a response of the medium at $z = 0$ which provides the data for the inverse problem. We study this problem assuming that the velocity of the wave propagation depends weakly on the horizontal coordinates, $x = (x_1, x_2)$ comparing to the strong dependence on the vertical coordinate, z, i.e. we deal with a weakly lateral heterogeneous medium (WLHM). Thus, we have a small parameter ϵ characterizing the ratio of the horizontal and vertical gradients of $c(z, x)$. Introducing the refraction index,

$$n(z, x) = \frac{1}{c(z, x)},$$

and a function describing the shape of interface $z = h(\epsilon x)$, we assume

$$n^2(z, \epsilon x) = n_0^2(z) + \epsilon < x, \bar{n}(z) > +O(\epsilon^2), \qquad (1)$$

$$\bar{n}(z) = (n_1(z), n_2(z)),$$

$$z = h(\epsilon x) = h_0 + \epsilon < \bar{h}, x > +..., \quad \bar{h} = (h_1, h_2), \qquad (2)$$

where $<,>$ means a scalar product. The importance of WLHM is now well-understood in theoretical and mathematical geophysics and there are currently numerous results on the wave propagation in WLHM (Borovikov et al., 1979, Cervenu, 1987). However, to the best of our knowledge, the only paper on inverse problems making use of this type of dependence of velocity on the vertical and horizontal coordinates is the publication by Bube K.P., 1985, based on quite a different approach to the one of the paper by Blagovestchenskii A.S., 1971, where $c = 1$ and there is a potential term $q(z, \epsilon x)$.

We develop a perturbation-type algorithm of the reconstruction of the main coefficients $n_0(z)$, $n_{1,2}(z)$ of the refraction index as well as the density $\rho(z)$ and the coefficients h_0, $h_{1,2}$. We note that the method can be extended to higher-order approximations. We, however, do not do this not only for extra technicalities, but also taking into the account the reality of geophysical measurements. As the method reduces the inverse problem to a recurrent system of 1D inverse problems, it provides, for multi-dimensional inverse problems occurring in important practical applications, a technique which inherits some practically useful properties of the one-dimensional inverse problems, e.g. robustness and rapid convergence of corresponding numerical algorithms.

The zero-order approximation, $n_0(z)$, $\rho(z)$ and h_0 are found from the solution of a 1D inverse problem for the wave equation in a layered medium which is well studied (Bube et al., 1983). At this stage we employ a method of coupled non-linear Volterra-type integral equations similar to the method developed by Blagovestchenskii (Blagovestchenskii 1971, 2001). Our approach based on the Fourier transform of the wave equation with respect to the horizontal coordinates. This makes possible to utilize the dependence of the resulting problems on ξ, with ξ being a dual to x. The next order terms of the refraction index are obtained as solutions to some linear coupled Volterra integral equations. We develop a numerical algorithm based on these integral equations which has proved to be quite robust and effective. Having said so, we note that the integral equations for higher order unknowns contain higher and higher order derivatives of the previously found terms, thus increasing ill-posedness of the inverse problem. This is hardly surprising taking into the account the well-known strong ill-posedness of the multi-dimensional inverse problems (Colton et al., 1998 and Engle et al.,1996). What is, however, interesting is that, within the model considered in this paper, there is just a gradual increase of instability adding two derivations at each stage in the reconstruction algorithm.

Formulation of the problem

Let us consider the wave propagation into the inhomogeneous half-space described by the wave equation

$$n^2(z,\epsilon x)u_{tt} - \rho(z)\mathrm{div}(\frac{1}{\rho(z)}\nabla u) = 0, \qquad x = (x_1, x_2). \tag{3}$$

Above and below the interface $z = h(\epsilon x)$ we have

$$n(z,\epsilon x) = \begin{cases} n^{(1)}(z,\epsilon x), & 0 < z < h(\epsilon x), \\ n^{(2)}(z,\epsilon x), & z > h(\epsilon x), \end{cases}$$

$$\rho = \begin{cases} \rho_1(z), & 0 < z < h(\epsilon x), \\ \rho_2(z), & z > h(\epsilon x). \end{cases}$$

We assume that $u = 0$ for $t \leq 0$, and the boundary condition is

$$u\bigg|_{z=0} = \delta(x)f(t)\sqrt{\frac{\rho_1(0)}{n_0(0)}}, \quad f(t) = \delta(t) \ \text{ or } \ f(t) = \theta(t), \tag{4}$$

where $\delta(x)$ and $\theta(t)$ are δ-function and Heaviside function, correspondingly. On the interface between two layers we assume the following continuity conditions hold

$$[u]\bigg|_{z=h(\epsilon x)} = 0, \quad [\rho^{-1}\frac{\partial u}{\partial n}]\bigg|_{z=h(\epsilon x)} = 0, \tag{5}$$

where the square brackets mean a jump across the interface - the difference between lower and upper limiting values. We see seek solution to the direct problem in the form

$$u(z,x,t) = \frac{1}{4\pi^2}\int\limits_{R^2} e^{-i<\xi,x>}U(z,\xi,t)d\xi, \qquad \xi = (\xi_1, \xi_2), \tag{6}$$

$$U(z,\xi,t) = \sum_{m=0}^{\infty}(i\epsilon)^m U^{(m)}(z,\xi,t), \tag{7}$$

where all functions $U^{(m)}(z,\xi,t)$ are real.

Our goal is to reconstruct the refraction index and the shape of interface, namely, the functions $\rho(z)$, $n_0(z),\ n_{1,2}(z)$ and constants $h_0,\ h_{1,2}$, knowing the inverse data collected during $0 < t < 2T$ in seismic measurements which are the values of a response given by

$$\frac{\partial u}{\partial z}\bigg|_{z=0} = R(x,t,\epsilon),$$

and

$$R(x,t,\epsilon) = \sum_{m=0}^{\infty}\epsilon^m R_m(x,t). \tag{8}$$

Decompositions (1), (2), (7) and (8) give rise to the recurrent system of problems for $U^{(m)}$. In particular, the zero-order problem is

$$n_0^2(z)U_{tt}^{(0)} - U_{zz}^{(0)} + |\xi|^2 U^{(0)} = 0, \qquad |\xi|^2 = \xi_1^2 + \xi_2^2, \tag{9}$$

$$U^{(0)}\bigg|_{z=0} = f(t)\sqrt{\frac{\rho_1(0)}{n_0(0)}},$$

$$[U^{(0)}]\bigg|_{z=h_0} = 0, \qquad [\frac{1}{\rho}U_z^{(0)}]\bigg|_{z=h_0} = 0,$$

with the inverse data given by

$$U_z^{(0)}\bigg|_{z=0} = r_0(t,\xi) = \int\limits_{R^2}\cos{(<\xi,x>)}R_0(x,t)dx.$$

The first-order problem is

$$n_0^2(z)U_{tt}^{(1)} - U_{zz}^{(1)} + |\xi|^2 U^{(1)} = <\bar{n}, \nabla_\xi U_{tt}^{(0)}>, \tag{10}$$

$$U^{(1)}\bigg|_{z=0} = 0,$$

$$\left[U^{(1)} - <\bar{h}, \nabla_\xi U_z^{(0)}>\right]\bigg|_{z=h_0} = 0,$$

$$\left[\frac{1}{\rho}\left\{U_z^{(1)} - <\bar{h}, \nabla_\xi U_{zz}^{(0)}> + U^{(0)} <\xi, \bar{h}>\right\}\right]\bigg|_{z=h_0} = 0,$$

with the inverse data given by

$$U_z^{(1)}\bigg|_{z=0} = r_1(t,\xi) = \int\limits_{R^2}\sin{(<\xi,x>)}R_1(x,t)dx.$$

In this paper we confine ourselves to the reconstruction of only $\rho,\ n_0,\ \ \bar{n}$ and $h_0,\ \bar{h}$. The reconstruction of the higher order terms is, in principal, possible using the same ideas as for $\bar{n}$ and $\bar{h}$, although is more technically involved. Moreover, in practical applications in geophysics the measured data make possible to find the inverse data only for $\rho,\ n_0,\ \bar{n}$ and $h_0,\ \bar{h}$. Clearly, in real measurements R is not given as a power series. However, as R_{2m} is even and R_{2m+1} is odd with respect to x, we have

$$r_0(t,\xi) = \int\limits_{R^2}\cos{(<\xi,x>)}R(x,t)dx + O(\epsilon^2),$$

$$r_1(t,\xi) = \epsilon^{-1}\int\limits_{R^2}\sin{(<\xi,x>)}R(x,t)dx + O(\epsilon^2).$$

Algorithm for solving the IP in the zero-order approximation

In this section we briefly describe an algorithm of solving the inverse problem of the zero-order approximation for the inhomogeneous half-space, that is description of an algorithm to solve IP of determination of $n_0(z)$, $\rho(z)$ and h_0 by means of the response function $r_0(\xi, t)$. Let us introduce two new independent variables

$$y = \int_0^z n_0(z)dz, \quad \sigma(y) = \frac{n_0(z(y))}{\rho(z(y))}.$$

The function $\sigma(y)$ is called acoustic rigidity. Then, we have

$$U_{tt}^{(0)} - \frac{1}{\sigma(y)}\frac{\partial}{\partial y}\left(\sigma(y)U_y^{(0)}\right) + \frac{|\xi|^2}{n_0^2}U^{(0)} = 0. \quad (11)$$

Let us change the dependent variable

$$\psi_1(y,t) = \sqrt{\sigma(y)}U^{(0)}, \quad \psi_2(y,t) = \frac{\partial \psi_1}{\partial t} + \frac{\partial \psi_1}{\partial y}, \quad (12)$$

thus, reducing the second order PDE for $U^{(0)}$ to a system of two PDE of the first order

$$\begin{cases} \psi_{1t} + \psi_{1y} &= \psi_2 \\ \psi_{2t} - \psi_{2y} &= q(y)\psi_1, \end{cases} \quad (13)$$

where

$$q(y,\xi) = -\frac{(\sqrt{\sigma(y)})''}{\sqrt{\sigma(y)}} - \frac{|\xi|^2}{n_0^2}. \quad (14)$$

Let $f(t) = \delta(t)$. Then, the closed nonlinear Volterra system

$$\begin{cases} \psi_1(y,t) &= \int_0^y \psi_2(\eta, t+\eta-y)d\eta, \\ \psi_2(y,t) &= -\int_0^y q(\eta,\xi)\psi_1(\eta, t-\eta+y)d\eta + g(t+y,\xi), \\ q(y,\xi) &= -2\int_0^y q(\eta,\xi)\psi_1(\eta, 2y-\eta)d\eta + 2g(2y,\xi) \end{cases} \quad (15)$$

allows us to determine the function $q(y,\xi)$ for $0 < y < T$ knowing the response $g(t,\xi)$

$$g(t,\xi) = \delta'(t) + \frac{n_0'(0)}{2n_0(0)}\delta(t) + \frac{r_0(t,\xi)}{\sqrt{\rho_1(0)n_0(0)}} \quad (16)$$

for $0 < t < 2T$. Let us assume that discontinuity takes place at $y = L$. Having solved this system for two different values ξ_1 and ξ_2, we determine the refraction index in the zero-approximation for $0 < y < L$ by means of formula

$$n_0(y) = \sqrt{\frac{|\xi_2|^2 - |\xi_1|^2}{q(y,\xi_1) - q(y,\xi_2)}},$$

which must be used together with

$$z = \int_0^y \frac{dy}{n_0(y)}.$$

The density $\rho(y)$ may be determined by solving the ODE for $w(y) = \sqrt{\sigma(y)}$

$$w'' + w\left(q(y,\xi) + \frac{|\xi|^2}{n_0^2(y)}\right) = 0,$$

which follows from (14), assuming that $\sigma(0)$ and $\sigma'(0)$ are known.

In case of two layers problem consider a wave propagation through the interface applying WKB singularity analysis for the reflected and transmitted waves. Then, for the sum of the incident and reflected waves we obtain

$$U^{(0)} = \frac{\delta(t-y)}{\sqrt{\sigma(y)}} + \frac{\theta(t-y)}{2\sqrt{\sigma(y)}}Q_0(y) +$$
$$d_0\frac{\delta(t+y-2L)}{\sqrt{\sigma(y)}} + \frac{\theta(t+y-2L)}{2\sqrt{\sigma(y)}}(d_1 + Q_1(y)) + \dots \ ,$$
$$0 < y < L, \ \ L < t < 2L.$$

For the transmitted wave WKB asymptotics is given by

$$U^{(0)} = s_0\frac{\delta(t-y)}{\sqrt{\sigma(y)}} + \frac{\theta(t-y)}{2\sqrt{\sigma(y)}}(s_1 + Q_1(y)) + \dots \ ,$$
$$y > L, \ \ L < t < 2L,$$

where

$$Q_0(y) = \int_0^y q(\eta,\xi)d\eta, \ \ Q_1(y) = \int_L^y q(\eta,\xi)d\eta.$$

Unknown quantities d_0, d_1 and s_0, s_1 are the reflection and transmission coefficients. Using the interface continuity conditions

$$[U^{(0)}]\Big|_{y=L} = 0, \quad [\sigma(y)U_y^{(0)}]\Big|_{y=L} = 0,$$

leads to a system of four linear equation with respect to unknown d_0, d_1 and s_0, s_1. Its solutions are

$$d_0 = \frac{1 - \frac{\sigma_+}{\sigma_-}}{1 + \frac{\sigma_+}{\sigma_-}}, \quad s_0 = \sqrt{\frac{\sigma_+}{\sigma_-}}\frac{2}{1 + \frac{\sigma_+}{\sigma_-}},$$

$$d_1 = \frac{\sigma_-'(1+d_0) + Q_0(L)(\sigma_- - \sigma_+) - s_0\sigma_+'\sqrt{\frac{\sigma_-}{\sigma_+}}}{\sigma_+ + \sigma_-},$$

$$s_1 = \sqrt{\frac{\sigma_+}{\sigma_-}}\frac{\sigma_-'(1+d_0) + 2Q_0(L)\sigma_- - s_0\sigma_+'\sqrt{\frac{\sigma_-}{\sigma_+}}}{\sigma_+ + \sigma_-},$$

where $\sigma_\pm$ and $\sigma_\pm'$ are the limiting values of $\sigma(y)$ and $\sigma'(y)$ at $y \to L-0$ and $y \to L+0$, correspondingly.

Analyzing the incoming singularities at point $y = 0$ at the time $t = 2L$, we will be able to measure L, d_0 and d_1. This will give us σ_+ and σ_+'. Thus,

$$h_0 = \int_0^L \frac{dy}{n_0(y)}.$$

The next steps is the reconstruction of velocity at a large depth for $L < y < T$, which is illustrated in the Fig. 1. This reconstruction requires the data observed for the segment $0 < t < 2T$. At this stage we again apply the non-linear Volterra system of integral equations (15) in the new frame (y', A, t') ($y = y' + L, \ \ t = t' + L$). This requires knowledge of $\psi_1(0, t' - y')$ and $\psi_2(0, t' + y')$ at the vertical segment AB, that is the right-hand side limits $\psi_{1,2}^+$. First, we determine

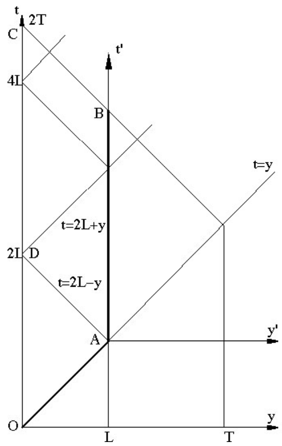

Figure 1: Characteristic lines in the case of two layers.

$\psi^-_{1,2}$, the left-hand side limits, using the first two equations of the system (15) as we know $q(y,\xi)$ for $0<y<L$. This may also be done, for example, by using the finite difference method applied to the system (13). Employing the interface continuity conditions at $y=L$, we obtain the required data of $\psi_1(0,t'-y')$ and $\psi_2(0,t'+y')$ at the segment AB.

Algorithm for solving the IP for the first-order approximation

In this section we describe even more briefly an algorithm to solve IP of determination of $\bar{n}(z)$ and $\bar{h}$ by means of the response function $r_1(\xi,t)$. We first integrate all the time dependent functions of the zero-order problem with respect to t such that

$$U^{(0)}\Big|_{y=0}=\theta(t)\sqrt{\frac{\rho_1(0)}{n_0(0)}}.$$

In the case of layer and bottom separated with interface the formulation of the corresponding problem using y and $\sigma(y)$ variables is given by

$$U^{(1)}_{tt}-\frac{1}{\sigma(y)}\frac{\partial}{\partial y}\left(\sigma(y)U^{(1)}_y\right)+\frac{|\xi|^2}{n_0^2}U^{(1)}=\frac{1}{n_0^2}<\bar{n},\nabla_\xi U^{(0)}_{tt}>, \tag{17}$$

$$U^{(1)}\Big|_{y=0}=0,\quad U^{(1)}_y\Big|_{y=0}=\frac{r_1(\xi,t)}{n_0(0)}.$$

$$\left[U^{(1)}-\rho\sigma(y)<\bar{h},\nabla_\xi U^{(0)}_y>\right]\Big|_{y=L}=0,$$

$$\left[\sigma(y)U^{(1)}_y-\left(\rho\sigma(y)^2<\bar{h},\nabla_\xi U^{(0)}_{yy}>+\right.\right.$$

$$\left.\left.\rho\sigma(y)\sigma(y)'<\bar{h},\nabla_\xi U^{(0)}_y>-U^{(0)}\frac{<\xi,\bar{h}>}{\rho}\right)\right]\Big|_{y=L}=0.$$

Introducing a new notation for the unknown functions

$$\bar{\varphi}(y)=(\varphi_1(y),\varphi_2(y))=\bar{n}(y)p(y),$$

the following linear Volterra system was obtained (Blagovestchenskii et al., 2005) to solve the corresponding IP for $0<t<2T$ (before discontinuity)

$$a\varphi_1(y)=2\int_0^y d\eta G_1(y,\eta,\eta-y,\xi_1)\frac{<\bar{\varphi}(\eta),\nabla_\xi U^{(0)}_{tt}(\eta,2y-\eta,\xi_1)>}{n_0^2(\eta)p(\eta)}$$

$$-\frac{2}{n_0(0)}G_1(y,0,-y,\xi_1)r_1(2y,\xi_1)+$$

$$\int_0^y d\eta\int_\eta^{2y-\eta}d\tau G_2(y,\eta,y-\tau,\xi_1)\frac{<\bar{\varphi}(\eta),\nabla_\xi U^{(0)}_{tt}(\eta,\tau,\xi_1)>}{n_0^2(\eta)p(\eta)}$$

$$-\frac{1}{n_0(0)}\int_0^{2y}d\tau G_2(y,0,y-\tau,\xi_1)r_1(\tau,\xi_1), \tag{18}$$

$$a\varphi_2(y)=2\int_0^y d\eta G_1(y,\eta,\eta-y,\xi_2)\frac{<\bar{\varphi}(\eta),\nabla_\xi U^{(0)}_{tt}(\eta,2y-\eta,\xi_2)>}{n_0^2(\eta)p(\eta)}$$

$$-\frac{2}{n_0(0)}G_1(y,0,-y,\xi_2)r_1(2y,\xi_2)+$$

$$\int_0^y d\eta\int_\eta^{2y-\eta}d\tau G_2(y,\eta,y-\tau,\xi_2)\frac{<\bar{\varphi}(\eta),\nabla_\xi U^{(0)}_{tt}(\eta,\tau,\xi_2)>}{n_0^2(\eta)p(\eta)}$$

$$-\frac{1}{n_0(0)}\int_0^{2y}d\tau G_2(y,0,y-\tau,\xi_2)r_1(\tau,\xi_2), \tag{19}$$

where we have introduced

$$G_1(y,\eta,t-\tau,\xi)=\sqrt{\sigma(y)}G(y,\eta,t-\tau,\xi),$$

$$G_2(y,\eta,t-\tau,\xi)=G_{1t}(y,\eta,t-\tau,\xi)+G_{1y}(y,\eta,t-\tau,\xi),$$

the Green's function $G(y,\eta,t)$ satisfying

$$G_{tt}-\frac{1}{\sigma(y)}\frac{\partial}{\partial y}\left(\sigma(y)G_y\right)+\frac{|\xi|^2}{n_0^2}G=\delta(t)\delta(y-\eta), \tag{20}$$

$$G=0\ \text{ if }\ y<\eta,$$

and

$$p(y)=-\frac{1}{2n_0^2(y)}\int_0^y\frac{d\eta}{n_0^2(\eta)}.$$

The system was obtained for arbitrary ξ_1 and ξ_2 which are not equal to zero.

Taking into account the discontinuity, the interface between upper layer and bottom, the WKB singularity asymptotic analysis for the sum of the incident and reflected waves gives

$$U^{(1)}=\frac{\theta(t-y)}{\sqrt{\sigma(y)}}\int_0^y<\bar{n},\xi>p(\eta)d\eta+$$

$$\frac{\theta(t+y-2L)}{\sqrt{\sigma(y)}}\left(\int_L^y<\bar{n},\bar{p}_1>d\eta+D\right)+\dots\ ,$$

$$0<y<L,\quad L<t<2L,$$

$$\bar{p}^{(i)}_1(y)=\frac{1}{2n_0^2(y)}\left(\frac{1}{2}\frac{\partial}{\partial\xi_i}d_1-\xi_i\int_L^y\frac{d\eta}{n_0^2(\eta)}\right),\qquad 0<y<L.$$

Here is the transmitted wave

$$U^{(1)} = \frac{\theta(t-y)}{\sqrt{\sigma(y)}} \left(\int\limits_L^y < \bar{n}, \bar{p}_2 > d\eta + S \right) + ... \ , \qquad (21)$$

$$y > L, \ \ L < t < 2L,$$

$$\bar{p}_2^{(i)}(y) = \frac{1}{2n_0^2(y)} \left(\frac{1}{2}\frac{\partial}{\partial \xi_i} s_1 - \xi_i \int\limits_L^y \frac{d\eta}{n_0^2(\eta)} \right), \ \ y > L.$$

Substituting these asymptotic expressions for $U^{(1)}$ into the two interface continuity relations, we obtain a system of two equations with respect to two unknowns D and S, reflection and transmission coefficients. Measuring at $y = 0$ the amplitude of arriving wave at the moment of time $t = 2L$, we may estimate D, and then, knowing D for $\xi_1 = (a, 0)$ and $\xi_2 = (0, a)$, we obtain the values for $\bar{h} = (h_1, h_2)$

$$h_i = \left[D(\xi_i) \left(1 + \frac{\sigma_+}{\sigma_-} \right) - \left(1 - \frac{\sigma_+}{\sigma_-} \right) \int\limits_0^L < \bar{n}, \xi_i > p(y) dy \right]$$

$$\left[2a\rho_1^- \frac{\sigma_+ - \sigma_-}{\sigma_+ + \sigma_-} \int\limits_0^L \frac{d\eta}{n_0^2(\eta)} \right]^{-1},$$

where ρ_1^- is the limiting value of $\rho_1(y)$ as $y \to L - 0$. The problem of determination of $\bar{n}(z)$ in the second layer by means of the response function $r_1(t, \xi)$ is reduced to a linear Volterra system (Blagovestchenskii et al., 2005) which is similar to (18) and (19) if we consider again the corresponding problem in the new coordinates y', t' related with the frame (y', A, t') ($y = y' + L$, $t = t' + L$, see Fig. 1).

Numerical results

In this section we apply the above method to solving numerically the described inverse problem. Numerical results were obtained for the 2D case problem with coordinates (z, x), and

$$n^2 = n_0^2(z) + \epsilon x n_1(z), \qquad \rho(z) = \frac{n_0(z)}{\sigma(z)}.$$

The algorithms described above to solve the zero and first-order problems of finding $\rho(z)$, $n_0(z)$ and $n_1(z)$, h_0 and h_1 were implemented into a computer code. To generate the inverse data, $r_0(\xi, t)$ and $r_1(\xi, t)$, we use a finite-difference method to solve the initial boundary value problems (9) and (10). In Fig. 2, a) and b), the chosen profiles of $n_0(z)$, $\sigma(z)$ are given by trigonometric polynomials

$$n_0(z) = \begin{cases} 1 + 0.4 \sin z, & 0 < z < h_0, \\ 1 - 0.2 \sin 1.4z, & z > h_0, \end{cases}$$

$$\sigma(z) = \begin{cases} 1 + 0.2 \sin 0.7z, & 0 < z < h_0, \\ 1 - 0.2 \cos 1.2z, & z > h_0. \end{cases}$$

In Fig. 3 for two cases of $n_1(z)$ we chose

$$n_1(z) = \begin{cases} -0.37 \sin 1.2z - 0.04 \sin 2.4z, & 0 < z < h_0, \\ -0.25 \sin z + 0.04 \sin 2z, & z > h_0, \end{cases}$$

$$n_1(z) = \begin{cases} 0.37 \sin 1.2z - 0.04 \sin 2.4z, & 0 < z < h_0, \\ -0.25 \sin 1.5z + 0.04 \sin 3z, & z > h_0. \end{cases}$$

The reconstructed profiles in Fig. 2 and 3 are presented to compare with the corresponding original profiles. It may be seen that we have a good agreement between both types of data. The step in the discretized problem, δy is taken to be 0.08. The values $\xi_{1,2} = 0.1, 0.11$ and $T = 10$ $(L = 5)$ were chosen for all graphs.

The method has shown to be quite stable, fast and accurate. When solving Volterra-type integral equations, both non-linear and linear, the iteration processes need just a few iterations (for all graphs the number of iterations was chosen 10). Numerous computer experiments have shown that for a better accuracy and fast convergence of the iteration process it is reasonable to use for the chosen profiles the segment $|\xi| < 0.5$. In applications to geophysics, the unity of the refraction index corresponds to the average speed of the wave propagation $c =$3km/sec.

Summary and Conclusions

We have described a new method of the reconstruction of the velocity of the wave propagation and density in WLHM assuming that the density depends only on depth and developed a corresponding numerical algorithm. The demonstrated method of solving the inverse problem proved to be efficient in computer analysis.

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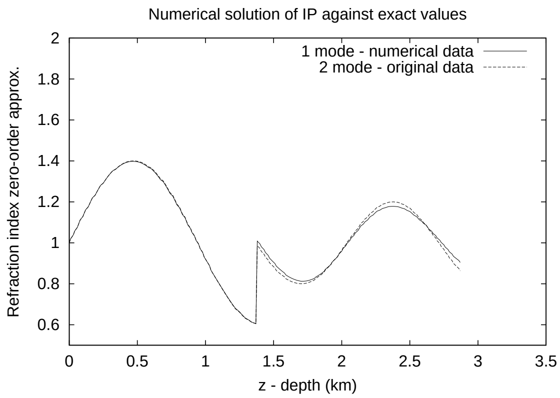
(a)



(b)

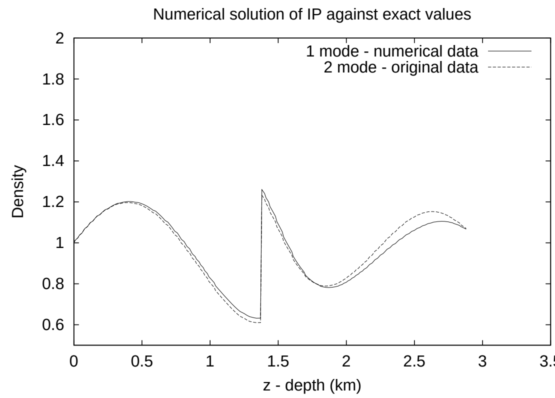
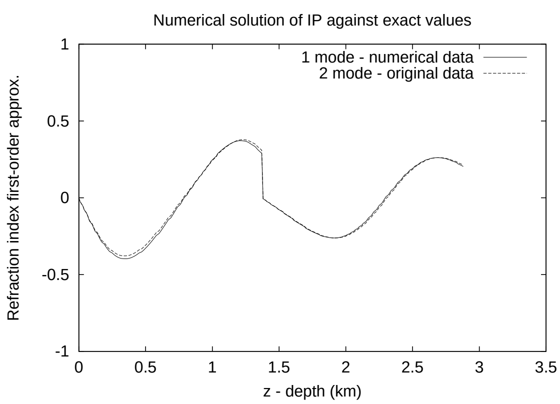


Figure 2: Numerical values of the refraction index $n_0(z)$ - (a) and density $\rho(z)$ - (b) against original profile .

(a)



(b)

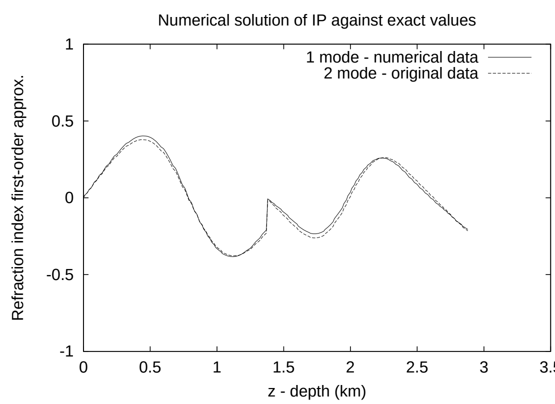


Figure 3: Numerical values of the refraction index first-order approximation $n_1(z)$ - (a) and (b) against original profile .