Migration with Gaussian beams

Mikhail M.Popov*, Alexandr P.Kachalov, Sergei A.Kachalov and Petr M.Popov, St.-Petersburg Branch of Steklov Mathematical Institute, St.-Petersburg, Russia.

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Abstract

A migration algorithm and computer codes based on the Gaussian beam method are developed for zero-offset and single short pre-stack configurations of the input data. Our approach preserves the main features of the classical Kirchhoff type migration (Schneider, 1978): for backward propagation of the recorded wave field the Green formula is used (in the time domain it is called the Kirchhoff formula); the image condition is based on the fact that the direct and backward fields are coherent in time on a reflector. Main point of our approach consists in replacing the corresponding Green function by its high-frequency asymptotics in terms of Gaussian beams (Popov, 1982).

Due to advantages of the Gaussian beam method, our approach does not face the caustic and two point problems in ray tracing, it automatically includes late arrivals and maximum amplitude of the wave field in the imaging process. As the requirement for the wave field to be coherent on a reflector is necessary but not sufficient condition, the stacking procedure is essential because it enables to remove random coherency in the migration domain.

Introduction

It is now well-known that the Gaussian beam method (Popov, 1982, Babich and Popov, 1989) provides technological and efficient algorithm for the computations of wave fields both in the frequency and time domain compared to the ray method. Besides, in the latter case summation of space-time Gaussian beams, or quasi-jets, is preferable, (Kachalov and Popov, 1988, Popov, 1987). So the idea to adjust the Gaussian beam method to migration and imaging problems seems to be natural.

A migration algorithm and computer codes which involve Gaussian beams were developed earlier (Hill,1990). One of the main point of his approach consists in decomposition of the input data into plane waves and further decomposition of each plane wave into set of Gaussian beams in order to propagate the recorded field backward. However, we cannot consider that procedure to be adequate and efficient for complicated velocity models and geometrical shape of reflectors. We prefer to use classical scheme of Kirchhoff type migration where the corresponding Green function is replaced by its Gaussian beam asymptotics.

The migration problem which we solve by our algorithm can briefly be described as follows. Assume we fix a reflection event in the recorded data. Then, for given velocity model, we propagate it backward in time and fix it in such a position in the migration domain where it coincides, or satisfies boundary conditions, with the direct wave field generated by the source. Maximum of coherency between the fields indicates then position of the reflector for the pre-stack input data. In the case of zero-offset, the backward propagated reflection event indicates position of the reflector itself at certain moment of time.

There is a popular and attractive approach to migration problems based on the Born approximation and subsequent inversion of the corresponding elliptic Fourier integral operator. In this case the output data describe the variation of the velocity in the migration domain directly. However, this method contains, in general, mathematical contradiction. Indeed, the Born approximation sets up an upper limit on the circular frequency. In comes out from the fact that in the case of compact support of the velocity variation the initial integral equation belongs to the Fredholm's type of integral equations. The iteration procedure for this equation converges within a frequency domain restricted by the first root of the Fredholm's determinant, i.e. the first eigenvalue of the homogeneous integral equation. At the same time, the inversion of the Fourier integral operator requires the circular frequency to tend to infinity. Note that the classical Kirchhoff type migration does not contain a mathematical contradiction.

Mathematical foundation.

We assume that the wave field U($\vec{x}$,t), $\vec{x}$=(x,y,z), to be migrated and modeled, satisfies the acoustic wave equation

$$LU \equiv (\Delta - \frac{1}{C^2(\vec{x})}\frac{\partial^2}{\partial t^2})U = 0, \qquad (1)$$

where Δ is the Laplace operator and C=C($\vec{x}$) is the velocity. The seismic surface is a part of the plane z=0. Seismograms recorded on the seismic surface are denoted by $U^{(0)}(x,y,t) = U(x,y,0,t)$ and they are the input data for zero-offset migration. In case of non-zero offset, a point source which generates the wave field is located on the seismic surface too.

To derive the basic formulae for migration, let Ω be a compact domain in 3D bounded by a smooth and closed surface $\partial\Omega$ and consider the following problem in Ω

LU=0, $\vec{x} \in \Omega$, $0 \le t \le T$; U=$U^{(0)}(\vec{x},t)$ on $\partial\Omega$;

U=0 and $\dfrac{\partial U}{\partial t} = 0$ for t=T. (2)

Introduce also the Green function $G(\vec{x},t;\vec{x}_0,t_0)$ which is solution of the following initial problem in domain Ω

$$LG(\vec{x},t;\vec{x}_0,t_0) = \delta(t-t_0)\delta(\vec{x}-\vec{x}_0);$$

$$G = 0 \text{ and } \frac{\partial G}{\partial t} = 0 \text{ for } t = t_0. \tag{3}$$

So far we do not impose any boundary conditions on $\partial\Omega$ for the Green function. By employing the Green function G, we are able to express wave field U($\vec{x}_0,t_o$) at any point $\vec{x}_0 \in \Omega$ at any moment t = t_0 via the boundary values of the field U and its normal derivative

$$-U(\vec{x}_0,t_0) = \int_{t_0}^{T} dt \iint_{\partial\Omega} dS_x \Bigg(G(\vec{x},t;\vec{x}_0,t_0)\frac{\partial}{\partial n_x}U^{(0)}(\vec{x},t)$$
$$-U^{(0)}(\vec{x},t)\frac{\partial}{\partial n_x}G(\vec{x},t;\vec{x}_0,t_0)\Bigg) \tag{4}$$

where $\dfrac{\partial}{\partial n_x}$ means the derivative along outgoing normal to $\partial\Omega$ on $\vec{x} = (x,y,z)$ coordinates and the integration dS_x is performed over the surface $\partial\Omega$; coordinates $\vec{x}_0 = (x_0, y_0, z_0)$ and t_0 are fixed.

For the purpose of migration we consider t_0<T, i.e. we perform backward propagation in time of the wave field recorded on the boundary. Under $\partial\Omega$ we understand only a part of surface z = 0, the seismic surface. In order to exclude the term with normal derivative $\dfrac{\partial}{\partial n_x} = -\dfrac{\partial}{\partial z}$ of the input data, we assume that the Green function satisfies the Dirichlet boundary condition on the seismic surface and fulfill it within the Kirchhoff approximation. Thus, the final and already approximate formula for the backward propagated wave field takes the form

$$U(\vec{x}_0,t_0) = -2\int_{t_0}^{T} dt \iint_{z=0} dxdy$$
$$U^{(0)}(x,y,t)\frac{\partial}{\partial z}G(x,y,z,t;\vec{x}_0,t_0) \tag{5}$$

Consider now the exploding reflector model. According to the corresponding imaging condition, positions of all possible reflectors in the migration domain are fixed at the moment $t_0 = 0$ if velocity C of wave propagation is replaced by $1/2$ C in the Green function. Thus the final formula for migration in this case reads

$$U(\vec{x}_0,0) = -2\int_{0}^{T} dt \iint_{z=0} dxdy$$
$$U^{(0)}(x,y,t)\frac{\partial}{\partial z}G(x,y,z,t;\vec{x}_0,0) \tag{6}$$

Consider further non-zero offset. In this case we have to introduce direct wave field generated by a point source located on the seismic surface. To propagate direct wave field $U^{(D)}(\vec{x},t)$ we have to solve the following problem

$$(\Delta - \frac{1}{C^2}\frac{\partial^2}{\partial t^2})U^{(D)}(\vec{x},t,\vec{x}_M) = f(t)\delta(\vec{x}-\vec{x}_M),$$
$$U^{(D)}(\vec{x},t) \equiv 0 \quad \text{for } t<0, \tag{7}$$

where $\vec{x}_M$ indicates position of the source and $f(t)$ is the time pulse at the source.

If we introduce Green function $G(\vec{x},\vec{x}_M;\omega)$ for the wave equation in frequency domain which corresponds to the non-stationary problem (7), then the direct wave field can be presented as follows

$$U^{(D)}(\vec{x},t \mid \vec{x}_M) = \frac{1}{\pi}\mathrm{Re}\int_0^\infty d\omega$$
$$e^{-i\omega t}F(\omega)\ G(\vec{x},\vec{x}_M;\omega) \tag{8}$$

where $F(\omega)$ is the spectrum of time pulse $f(t)$.

To identify position of a reflector in the migration domain, we introduce an imaging condition based on the fact that the incident and reflected fields satisfy a boundary condition on the reflector identically with respect to time. In physical terms, it means that the incident and backward propagated fields are coherent at each point of the reflector.

Essential feature of our approach consists in replacing the Green function in previous formulae by its Gaussian beam asymptotics. Denote by $G_{GB}(\vec{x},\vec{x}_M;\omega)$ the asymptotics of the Green function in frequency domain in terms of Gaussian beams, then for the Green function in time domain we obtain the following approximate formula

$$G(\vec{x},t \mid \vec{x}_0,t_0) \cong \frac{1}{\pi}\mathrm{Re}\int_0^\infty d\omega$$
$$e^{-i\omega(t-t_0)}G_{GB}(\vec{x},\vec{x}_0;\omega) \tag{9}$$

To obtain expression for $G_{GB}(\vec{x},\vec{x}_0;\omega)$, we have to integrate Gaussian beams U_{GB} on the spherical angles ϑ and φ (ray parameters) with the appropriate initial amplitude $\Phi(\vartheta,\varphi;\omega)$

$$G_{GB}(\vec{x},\vec{x}_0;\omega)=\int d\vartheta\int d\varphi$$

$$\Phi(\vartheta,\varphi;\omega)\ U_{GB}(\vec{x},\vec{x}_0,\omega) \qquad (10).$$

Formula (10) provides the high-frequency approximation $G_{GB}(\vec{x},\vec{x}_0;\omega)$ of the Green function $G(\vec{x},\vec{x}_M;\omega)$, for details see Popov (1982).

Numerical examples

Numerical experiments with 2D migration codes were carried out on PC and therefore we had to diminish the migration domain.

Zero-offset input data were simulated by an "exploding sine" described by the expression $z=H-A\sin(\gamma\ x)$ with H, A and γ being the parameters. This reflector was immersed in homogeneous and inhomogeneous (constant gradient) velocity models. Fig.1 shows relatively complicated structure of the ray field and the image of a concave period of the "exploding sine".

Pre-stack depth migration code was tested on standard 2D Marmousi model with constant density $\rho=1$. In this case we choose two migration subdomains. The first one contains two shallow wedges located at 4000m <x<5500m, 300m<z<900m. For imaging, we used 400 Gaussian beams uniformly emanated from the source with interval 0,9 degrees in between the central rays. In this case a lot of rays do not reach the seismic line and therefore the number of Gaussian beams could be reduced. The image was obtained by stacking of 28 shots located from x=4200m to x=5550m with interval 50m in between. We would like to remind that the diffraction problem for a penetrable wedge remains unsolved in the theory and neither ray, nor Gaussian beam methods describe the wave field in this domain perfectly. However, the image of the wedges is not bad. Fig.2 exhibits a typical ray field emanated from a shot and the image of the domain.

The second subdomain includes the hydrocarbon deep lens located at 5800m<x< 7175m and 2300m<z<2713m. For imaging, we used 360 Gaussian beams emanated in the upper semicircle with interval 0,5 degrees in between. The image was obtained by stacking of 39 shots located on the seismic line from 6300m to 8200m with interval 50m. Fig.3 demonstrates typical ray field generated by a shot and the image of the subdomain.

Conclusions

Numerical examples prove that the proposed method operates for complex ray fields which include caustics, multicoveraged data, late arrivals etc., without additional efforts. It overcomes the two point problem in the ray tracing though requires to have relatively dense fan of central rays in the target domain. The approach remains mathematically transparent and technologically invariant with respect to complexity of ray fields unlike the ray based migration, compare e.g. with Operto et all, (2000).

Application of the ray or Gaussian beam methods requires the velocity to be slowly varying on the dominant wave length in order to meet applicability conditions of the methods. It can be easily fulfilled for artificial models but not for those which are broadly in use in Geophysics. For instance, Marmousi model contains a number of thin layers with big gradients of the velocity. And though the images look reasonably nice, it remains doubtful whether wave field amplitudes are sufficiently accurate. We are sure that all that holds true for finite difference methods as well unless special and non-trivial efforts are undertaken.

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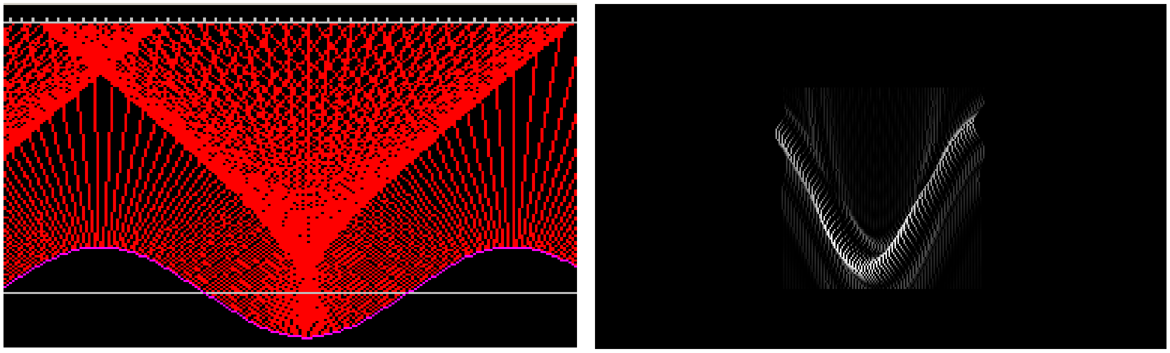

Figure 1 – “Exploding Sine”, structure of rays and image of concave period of the sine.

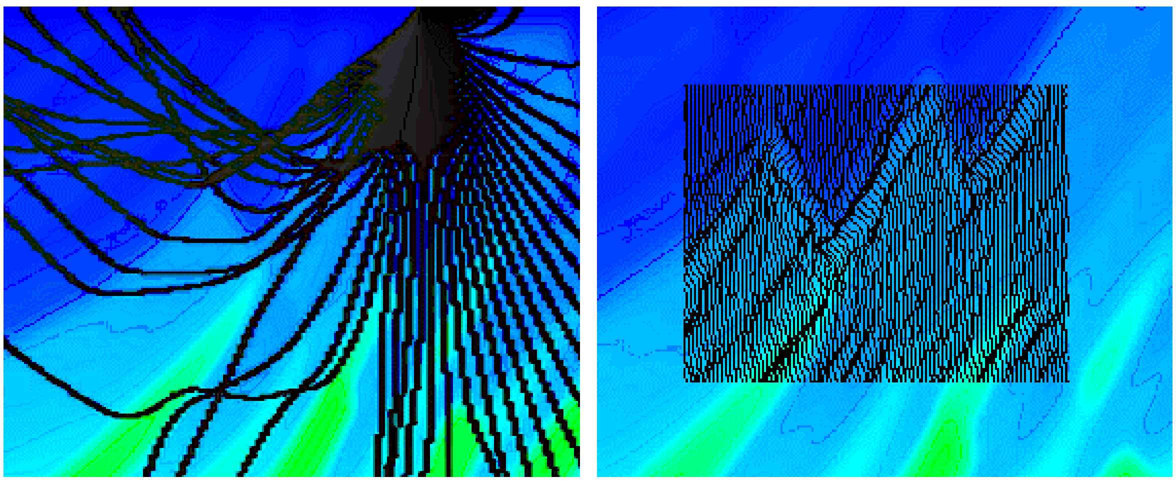

Figure 2 – Pre-stack migration. Typical structure of rays and image of two shallow wedges for Marmousi model.

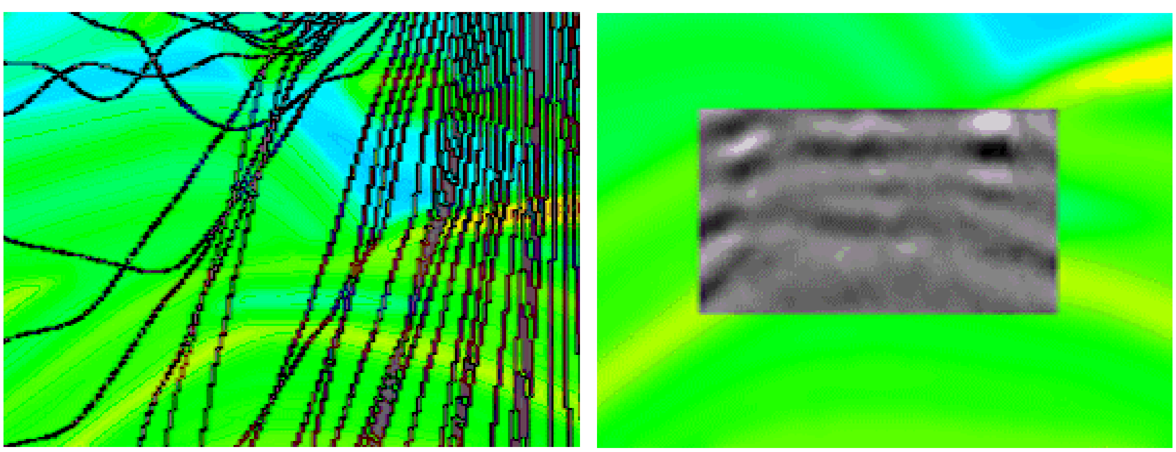

Figure 3 – Pre-stack migration. Typical ray field and image of hydrocarbon deep lens for Marmousi model.