# Optimal aperture in Kirchhoff common-angle time migration

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## Summary

Recently, much attention has been given in obtaining angle gathers and also amplitude versus angle (AVA) curves during the migration process. An attractive way to do this is provided by (time or depth) common-angle Kirchhoff migration (CAKM), which considers travel-time curves and weights that refer to the common-angle (instead of the conventional common-offset or common-shot) configuration. As occurs with any Kirchhoff-type procedure, the CAKM output strongly depends on the choice of the migration aperture, not only for reduction of computational costs, but also for the accuracy of results. In this work we shortly review the CAKM theory and address the problem of optimal selection of migration aperture. In this context, the concept of projected Fresnel Zone, which plays a prominent role in the estimation of an optimal migration aperture, is also reviewed and applied.

## Introduction

As compared to the conventional common-offset Kirchhoff migration, CAKM is experiencing growing interest in seismic processing, imaging and inversion due to the following reasons: (a) the ability to more easily deal with multiple paths and (b) better access to amplitude versus angle (AVA), instead of amplitude versus offset (AVO) curves. The latter require a further inversion to convert to AVA curves, better suited to reservoir characterization purposes.

In the 2D or 2.5D cases, the CAKM operator is characterized by a 3D curve in the midpoint-offset-time domain. This is a drawback, since one has to load all data in memory to be accessed by that curve. In the 3D case, its usage can be prohibitive. In this way, the derivation of efficient implementation of CAKM is still an open problem.

Santiago (2004) provides a description and also discusses a number of practical aspects of true-amplitude CAKM in time domain. True amplitude means that, for a primary reflection, the migration output automatically compensates for geometrical-spreading loss. In this way, the obtained amplitudes can be considered as a measure of the corresponding reflection coefficients. In particular, it is explained how to estimate an optimal migration aperture with the help of the concept of the projected Fresnel Zone (PFZ). In this paper, we review the PFZ definition and its application to derive an optimal migration aperture.

### 2.5D Kirchhoff common-angle time migration

The 2.5D Kirchhoff common-angle time migration can be represented by the following integral along a certain travel-time curve, $\tau_D$,

$$V(g,M)=\int_A da.W(a,M)D[U(a,t=t_D(a,M))], \qquad (1)$$

where, $V(g,M)$ is the migrated common reflection angle, $g$, output, $W(a,M)$ is a weighting function and $a$ is the migration dip. In the case of PP data, the migration dip $a$ is simply the angle that the normal to the line that bisects the angle defined by the rays that connect source to the migrated point and migrated point to the receiver. Moreover, $D$ represents the half-derivative operator, $U(a,t)$ is the seismic data, represented by its analytical signal, to be migrated and $A$ is the migration aperture that defines the extension of the migration operator. If the amplitudes to locate in $V(g,M)$ are supposed to represent the reflection coefficients, the weighting function, $W(a,M)$, has to take into account the geometrical spreading compensation along the incoming and outcoming rays from the image point.

In this migration scheme, the travel-time curve is located in the 3D space ($x$, $h$, $t$), where $x$ is the mid-point coordinate, $h$ is the half-offset between source and receiver and $t$ is time in the input domain. Under the assumption of a homogeneous medium (which is the usual case for time migration), the travel-time expression is described by 3 parameters, namely the cosine of the migration dip, the sine of the reflection angle and the time in the output domain, $t_m$. For a given a point, $M(x_m,t_m)$, in the migrated domain, a smooth RMS velocity field, $v(M)$, and a fixed reflection angle, $g$, one can calculate the travel-time based on sine and cosine rules. The variation of the migration dip, $a$, implies varying offsets, as well as CMP coordinates. The CAKM travel-time, half-offset and midpoint coordinates can be determined by the following equations (Fomel and Prucha, 1999).

$$t=t_m\frac{\cos(a)\cos(g)}{\cos^2(a)-sen^2(g)} \qquad (2a)$$

$$h=\frac{vt_m}{2}\frac{sen(g)\cos(g)}{\cos^2(a)-sen^2(g)} \qquad (2b)$$

$$x-x_m=\frac{vt_m}{2}\frac{sen(a)\cos(a)}{\cos^2(a)-sen^2(g)}. \qquad (2c).$$

According to Santiago (2004), the true-amplitude weighting function, $W(a,M)$, is given by

$$W(a,M)=\frac{v}{2}\frac{\sqrt{t_m^3\cos a.\cos g}}{(\cos^2 a-\sin^2 g)}. \qquad (3)$$

Up to now we provided equations for the staking line and true-amplitude weighting function. In the next section we show how to determine the migration aperture, so as to get more efficient and accurate amplitudes.

**Migration aperture**

While submitting a migration job, geophysicists have a few parameters to set up. Routinely, they perform tests to analyze S/N migration ratio and spatial resolution before running final migration. One of the tested parameters is the migration aperture that dictates the extent of the migration operator and has impact on both S/N ratio and spatial resolution.

In Kirchhoff migration algorithms, parameters defining migration aperture have great impact on the quality of the final image and on the execution time. If the aperture size is too small, a low-cost, low-frequency, low-dip and mixed-aspect unmigrated image is obtained. On the other hand, if the aperture is taken too large, the migrated image will be high-cost, high-dip, with a high-resolution appearance, but sometimes with a low signal to noise (S/N) ratio. Therefore, the search for an optimal aperture is a crucial procedure in all Kirchhoff migration processes. In routine processing, aperture tests are systematically carried out to evaluate the continuity and positioning of dipping reflections (salt flanks, faults), S/N-ratio, spatial resolution and execution time.

Migration apertures are routinely centered on the migrated point. In the simple case of a purely kinematical migration – reflectors well positioned, but with wrong amplitudes – the least possible migration aperture radius is that of the (horizontal) distance between the migrated point and its corresponding stationary point. For the situation of a planar dipping reflector depicted in Figure 1, that distance is represented by the segment $r_{ab}$. Using basic principles of geometry, one can see that the expression of $r_{ab}$ is nothing more than the right-hand side of Equation (2c), for which the fixed reflector dip coincides with the migration dip, $a=q$.

To obtain more precise migration results, apertures should be centered, not at the migration point, but on the stationary point, since the largest contribution to the migrated amplitude comes from a region in the vicinity of that point. A key concept for the determination of optimal migration apertures is the one of the n-th Fresnel Zone, denoted by $FZ_{(n)}$ and defined as the collection of points on the reflector for which (see Cervený and Soares, 1992)

$$\left|t_D - t_R\right| < \frac{nT_w}{2} . \tag{4}$$

In the above equation, $\tau_D$ is diffraction time, $\tau_R$ is the reflection time (at the stationary point) and $T_w$ is the dominant period of the seismic wavelet. Physically, the first Fresnel Zone, $FZ_{(n)}$, for a certain source-receiver pair, describes the region at the reflector which most influence the reflection response for that pair. As a consequence, that region should be the one chosen for optimal aperture in migration. The n-th Fresnel Zone (for n = 2, 3,…) represent enlargements of the first Fresnel Zone that, in principle, should be used for tapering purposes. For actual computation purposes, Fresnel Zones, in particular the first Fresnel Zone, $FZ_{(1)}$, are not of practical value since they are located on the reflector in the depth domain. To obtain corresponding migration apertures in the trace domain, Schleicher et al. (1997) introduce the concept of a projected Fresnel Zone, $PFZ_{(n)}$, which is nothing more than the projection of the original n-th Fresnel Zone onto the trace domain according to the measurement configuration. Figure 1 shows the first and second Fresnel and projected Fresnel Zones for the common-angle configuration.

Based on the work of Hertweck et al. (2003) for zero-offset (poststack) time migration, Santiago (2004) prescribes to increase the migration aperture radius by a quantity of, at least, $PFZ_{(1)}$ – the projected first Fresnel Zone radius, in order to get reliable amplitudes to perform AVA analysis after migration. The initial experiments in Santiago (2004) indicate that the procedure guarantees the summation of all the constructive energy related to the reflection coming from migrated point.

The construction of the n-th Fresnel Zone in the common-angle configuration for a point M(x,z) on a dipping plane reflector of angle $q$ with a homogeneous overburden can be obtained as follows: Referring to Figure 2, we consider M(x,z) as a reflection point of traveltime $\tau_R$ and specular rays $l_S$ and $l_G$. We now consider the diffraction point $M_n$ also at the reflector and with diffraction rays, $d_1$ and $d_2$, such that the diffraction traveltime $\tau_D$ satisfies

$$\left|t_D - t_R\right| = \frac{nT_w}{2} . \tag{5}$$

The distance $FZ_{(n)} = M_n - M$ represents the radius of the n-th Fresnel Zone. According to Equation (4) and considering the Taylor series up to the second order for the diffraction traveltime $t_D$, one obtains for the n-th Fresnel Zone radius (see Santiago (2004))

$$FZ_{(n)} = \frac{v}{2}\sqrt{\frac{nT_w t_m}{\cos(q)\cos^3(g)}} . \tag{6}$$

Once defined the size of the Fresnel Zone, it is now necessary to project it onto the measurement surface to determine the traces that constitute the n-th projected Fresnel Zone. These traces are the ones that will contribute to the output migrated amplitude. Note that the projection depends on the desired configuration, in this case, given by the common-angle geometry. For a given fixed reflection angle, γ, the points $M$ and $M_n$ at the reflector define, on the measurement surface, the corresponding trace points x and $x_n$, namely the spatial CMP coordinates of the center and one-sided edge of the projected Fresnel Zone. This leads to following expression of the n-th projected Fresnel Zone

$$PFZ_{(n)} = \frac{v}{2}\frac{\sqrt{n\mathrm{T}_w t_m \cos(q)\cos(g)}}{\cos^2(q) - \sin^2(g)}. \quad (7)$$

Equation (7) shows that, for an expected geological planar dipping reflector within a homogeneous model of velocity, *v*, and an estimate of the dominant source-wavelet period, one can compute the n-th projected Fresnel Zone for the common-angle configuration.

We point out that the true Fresnel Zone for a dipping plane reflector is not symmetrical with respect to the reflection point, being larger in the downdip direction. The (symmetrical) Fresnel and projected Fresnel Zones radii expressed by Equations (6) and (7), respectively, are a consequence of the second-order Taylor polynomial employed to approximate the diffraction traveltime $t_D$.

The next step is to determine the reflector dip at the reflection point in depth (see point M(y,z) in Figure 1), since, according to Equation (7), the knowledge of that quantity is sufficient to determine the n-th projected Fresnel Zone, so that a true-amplitude common-angle Kirchhoff migration in the least aperture migration approach can be performed.

Following our assumption of constant velocity above the reflector, the reflection point M(y,z) in depth lies vertically below to the given time-migrated point N($x_m$,$t_m$), namely, y=$x_m$ and z=v$t_m$/2, where v is the velocity. Moreover, the reflector dip at M coincides with the emergence angle the zero-offset ray (the normal to the reflector from M) makes with the vertical.

**Estimation of the reflector dip**

Kirchhoff processes can be used to extract information from stationary rays from seismic data by the so-called multiple-weighted diffraction stack (Bleistein, 1987; Tygel et al., 1993). This is done by performing migration twice with slightly different weighting functions and dividing both results. This quotient represents an estimation of the parameter to be extracted. Following Bleistein (1987), we apply the procedure to zero-offset Kirchhoff migration and design the weights so as to obtain the emergence angle of zero-offset primary rays.

To describe the multiple-weighted diffraction stack procedure, we briefly recall the amplitude behavior of the migration integral. For that, we consider the 2.5D zero-offset time migration integral in the frequency domain

$$\hat{V}(M,w) = \sqrt{iw}\int_A dx.W(x,M)\hat{U}(x,w)e^{iw(t_D - t_R)}. \quad (8)$$

In this case, ξ represents the location of the coincident source-receiver pair (compare with Equation (1) for the analogous common-angle situation).

Due to the oscillating behavior of the complex exponential in Equation (8), and assuming the high-frequency situation that is typically valid for seismic Kirchhoff migration, the integrand goes to zero except in the region where $\tau_D$ approaches to $\tau_R$. This limit is governed by Equation (4). Recall that, in the present case, $\tau_D$ and $\tau_R$ represent the diffraction and reflection curves for the zero-offset configuration. Samples which effectively contribute to the migrated image are the ones related to straight rays that fulfill Equation (4). During zero-offset migration, each sample along the migration operator has its own migration dip angle. Therefore, modifying the original weighting function (in this case it could be a constant, say 1) by attaching the factor (migration dip angle), one obtains a new weighting function. Its use in the migration operator produces an image whose amplitudes ($V_1$(*M*)) can be considered as the original amplitudes *V*(*M*) multiplied by a weighting mean of the weighted samples that come from the region around the stationary point. Finally, dividing amplitudes $V_1$(*M*) by *V*(*M*) one obtains a section containing, for each sample, an estimative of the reflector dip angle, *q*. Note that some kind of smoothing have to be applied on this estimative of reflection angle, *q* section, because of values near zero amplitude in the input sections, *V(M)* and $V_1(M)$. The reader is referred to the works of Sun (1998; 2000) for more details on the actual implementation these procedures.

It is to be observed that, in the case of 3D pre-stack depth migration the multiple-weighted diffraction stack may be economically unfeasible because several common-offset sections must be at least twice migrated. Our time migration scheme, however, considers straight rays, so the problem of finding the stationary point location is dramatically simplified.

Consider that there is a stacked section that satisfactorily represents a zero-offset section. After preprocessing, this stacked section is supposed to contain mostly primary reflections. Let us choose the reflector dip angle, *a=q*, as the attribute to be extracted from the multiple-weighted diffraction stack approach.

By means of Equations (2), this *q* section can directly provide the center the n-th projected Fresnel Zone for any common-reflection angle, as computed by Equation (7).

**Conclusions**

In this work, we have reviewed and discussed some important aspects of 2.5D true-amplitude Kirchhoff migration in the common-angle domain. We have emphasized the definition and construction of projected Fresnel Zones, which can be seen as optimal apertures to perform the migration. The construction of the projected Fresnel Zone depends on the estimation of the reflector dip. For that estimation, we proposed the multiple-weighted diffraction stack procedure applied in the post-stacked domain. Restriction the migration aperture to a projected Fresnel Zone is expected to provide migration results with better quality and less computational effort.

At the present stage, we are working to derive a more stable way to obtain the reflector dip using the multiple-weighted diffraction stack. We expect to show soon good results of this technique. Finally, we mention that an analogous approach of this paper can be also used for the common-offset configuration.

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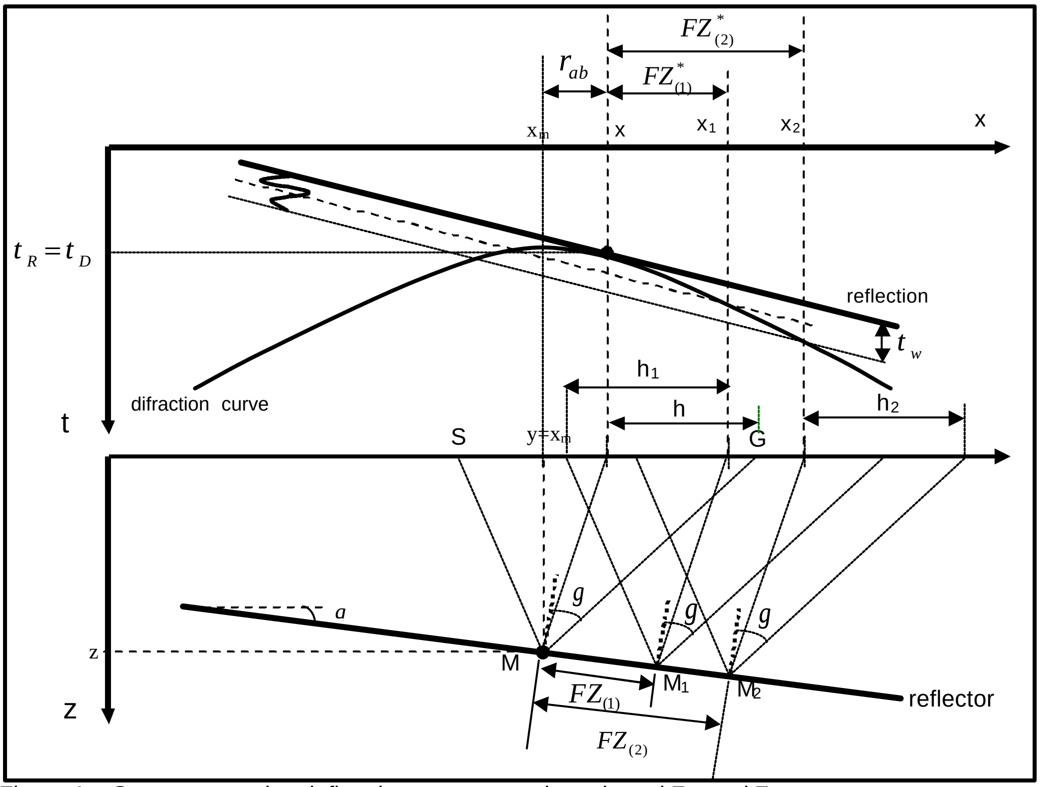

Figure 1 – Geometry used to define the common angle projected Fresnel Zone.