Use of reciprocity in a double plane wave Kirchhoff depth migration

R.K. Seifoullaev*, P.L. Stoffa, and M.K. Sen, Institute for Geophysics, The University of Texas at Austin

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Abstract

Slant stacking of typical marine and end-on land shot gathers generally produces high quality traces for positive ray-parameters while those for the negative ray-parameters are often noisy due to limited aperture. Use of such data in migration results in images with limited or missing negative dips. In this paper we employ the well-known reciprocity principle in an efficient manner such that a split-spread gather can be formed from existing one-sided offset data. This can be achieved by physically re-gathering the common shot gather data into common receiver gathers and collecting the data for a fixed surface reference position into the equivalent of a split spread gather. This approach encounters problems for irregular shot and receiver geometries. It can also cause processing difficulties since large volumes of intermediate data may need to be stored for purposes of collection into the common surface reference gathers. Thus, the intermediate volume of data to be used for pre stack depth imaging are significantly increased. We show that for plane wave pre stack imaging, reciprocity can be taken into account as the original (one sided offset) gathers are used in the construction of the plane wave transform. We demonstrate using a coupled ray-parameter Kirchhoff migration that the plane wave migrated data that include reciprocity are better imaged than the data that do not include reciprocity.

Introduction

We consider double plane wave Kirchhoff depth migration as developed by Stoffa et al. (2005) for p_s – p_r (absolute reference case) and p_s – p_o (relative reference case) plane wave domains. Here we report on imaging results using both nonreciprocal and reciprocal input data and provide a comparison and analysis of the results. Then we show how the original one sided shot gathers can be transformed to both plane wave domains without re-gathering the original data by using a simple additional phase shift in the plane wave transformations.

In Stoffa et al. (2005) we started with multi-coverage data, $P(s,r,t)$, recorded at the surface, where s is the source location and r is the receiver location. In the frequency domain we may generate source and receiver plane waves separately using the following forward slant-stacking formulas

$$P(p_s,r,\omega)=\int P(s,r,\omega)\exp(+i\omega p_s s)ds\,, \qquad (1)$$

$$P(s,p_r,\omega)=\int P(s,r,\omega)\exp(+i\omega p_r r)dr\,, \qquad (2)$$

or simultaneously using the formula

$$P(p_s,p_r,\omega)=\iint P(s,r,\omega)\exp(+i\omega[p_s s+p_r r])dsdr, \qquad (3)$$

where ω is angular frequency, $P(s,r,\omega)$ are the original data transformed into the frequency domain. All these transformations are reversible and the original data may be recovered from its plane wave components using the inverse slant-stacking formulas

$$P(s,r,\omega)=\omega^2\int P(p_s,r,\omega)\exp(-i\omega p_s s)dp_s\,, \qquad (4)$$

$$P(s,r,\omega)=\omega^2\int P(s,p_r,\omega)\exp(-i\omega p_r s)dp_r\,, \qquad (5)$$

$$P(s,r,\omega)=\omega^4\iint P(p_s,p_r,\omega)\exp(-i\omega[p_s s+p_r r])dp_s dp_r. \qquad (6)$$

Reciprocity

The typical marine acquisition geometry produces one-sided shot or receiver gathers where most of the reflections recorded correspond to the positive ray parameter values (Liu et al. 2004). This may be remedied by using the reciprocity principle to generate the apparently missing information that is actually present in the data available assuming reciprocity holds. Recall that the well-known representation theorem of deHoop (1958) can be used to show that if the source and receiver positions were interchanged, the recorded data would be the same provided the source and receiver directivities are also reciprocal (Aki and Richards 2002, Fokkema et al. 1993). For a marine streamer experiment that makes use of airgun sources and hydrophone receivers, application of reciprocity is straightforward. We can simply re-gather the data to recover the missing traces, which implies collecting common receiver gathers from the original common shot gathers.

Pre stack depth imaging can be accomplished in several ways: shot gathers can be rearranged into receiver gathers and after each are individually migrated both images can be combined (averaged) into a single image; or, equivalently, shot and receiver gathers may be combined into split spread gathers and then migrated; finally, in accordance with the reciprocity principle, the roles of sources and receivers in (3) may be inter-changed, which is the approach we present now:

In (3) it is assumed that the data were recorded everywhere. For single-sided offset marine data recorded, say, for $r \ge s$ we define the observed data for $r \ge s$ as $P_D(s,r,\omega)=P(s,r,\omega)$ and for $r<s$ as $P_D(s,r,\omega)=0$. Then, the reciprocal data may be defined as $P_R(r,s,\omega)=P_D(s,r,\omega)$. If we now apply (3) to P_D and P_R, we get

$$P_D(p_s,p_r,\omega)=\iint P_D(s,r,\omega)\exp(+i\omega[p_s s+p_r r])dsdr, \qquad (D)$$

$$P_R(p_r, p_s, \omega) = \iint P_R(r, s, \omega)\exp(+i\omega[p_r r + p_s s])dsdr =$$
$$\iint P_D(s, r, \omega)\exp(+i\omega[p_r r + p_s s])dsdr.$$

Interchanging p_s and p_r in the last formula, we get

$$P_R(p_s, p_r, \omega) = \iint P_D(s, r, \omega)\exp(+i\omega[p_s r + p_r s])dsdr. \qquad \text{(R)}$$

Combining and normalizing equations (D) and (R) we obtain

$$P(p_s, p_r, \omega) = \tfrac{1}{2}\iint P_D(s, r, \omega)(\exp(+i\omega[p_s s + p_r r] + \exp(+i\omega[p_r s + p_s r])dsdr, \qquad (7)$$

which represents the plane wave data corresponding implicitly to the split spread data that would have been gathered using reciprocity. If we change to source–offset coordinates *s*, *o*, p_s, p_o and assume field invariance under the change of variables, then (7) becomes

$$P(p_s, p_o, \omega) = \tfrac{1}{2}\iint P_D(s, o, \omega)(\exp(+i\omega[p_s s + p_o o] + \exp(+i\omega[p_s s + (p_s - p_o)o])dsdo. \qquad (8)$$

There are several advantages of this approach. First, we don't physically have to collect the reciprocal data from the original data; we simply include an additional phase delay operator in the plane wave transforms. Second, we avoid all the difficulties associated with source or receiver locations not being on a regular grid or being of different or even random spacing. The ability to construct the plane wave response properly will of course still depend on adequate spatial samplings but no regularity is required.

Double Plane Wave Kirchhoff Depth Imaging

Following from (7) a p_s–p_r volume can be migrated using the equation

$$P(x) = -K(x)\iiint P(p_s, p_r, \omega) \exp(i\omega[\tau(x, p_s) + \tau(x, p_r) + (p_s + p_r)\xi])d\omega dp_s dp_r, \qquad (9)$$

or

$$P(x) = -K(x)\iint P(p_s, p_r, \tau(x, p_s) + \tau(x, p_r) + (p_s + p_r)\xi)dp_s dp_r, \qquad (10)$$

(see Stoffa et al., 2005) where *x* is the image point at depth, ξ the projection of *x* onto the recording surface, $P(x)$ is the migrated image, and $\tau(x, p_s)$, $\tau(x, p_r)$ are source and receiver plane wave vertical delay times computed from the origin to the isochron of x, respectively (see Figure 1 in Stoffa et al., 2005).

Resulting from (8) a p_s–p_o volume can be migrated using the equation

$$P(x) = -K(x)\iiint P(p_s, p_o, \omega) \exp(i\omega[\tau(x, p_s - p_o) + \tau(x, p_o) + p_s\xi])d\omega dp_s dp_o, \qquad (11)$$

or

$$P(x) = -K(x)\iint P(p_s, p_o, \tau(x, p_s - p_o) + \tau(x, p_o) + p_s\xi)dp_s dp_o. \qquad (12)$$

Stoffa et al. (2005) show examples of using equation 12 to migrate singe sided simulated marine data shot over the EAEG salt model.

Migration Examples

The examples we use here are based on the same 2D staggered grid elastic finite difference simulation (Levander 1988) (see Figure 2 in Stoffa et al. 2005). The data were acquired every 20m along the top of the model for 675 shot positions. The acquisition proceeded from the left (X=0.0km) to the right (X=13.48km). We simulated a marine survey with the array towed behind the ship. 240 channels were acquired with the first complete shot gather occurring at shot point 240 (X=4.78 km). The receiver spacing was 20m. For example shot records from the middle of the survey and over the salt are shown in Figure 1.

The original shot gather data were then transformed into the plane wave domain by simple slant stacking (Stoffa et al. 1981). 121 plane wave seismograms for ray parameters +0.6 to -0.6 sec/km every 0.01 sec/km were recovered from the input shot gathers. The origin was taken relative to each shot's position to display the data of Figure 2. The plane wave gathers of Figure 2 correspond to the common shot gathers of Figure 1. The migrated and stacked plane wave data are shown in Figure 5.

The original shot data were then gathered into common receiver gathers, Figure 3. We note that now the first receiver has full coverage from the 240 shots that it recorded. But, on the right hand side, because the shooting stopped at the end of the model the number of traces continuously decrease from X=8.68 km to the end of the line. The common receiver data were transformed to the plane wave domain again by simple slant-stacking and , 121 plane wave seismograms for ray parameters +0.6 to -0.6 sec/km every 0.01 sec/km were recovered from the input common receiver gathers. Figure 4 shows the transformed data corresponding to the receiver gathers of Figure 3 with the origin of each plane wave gather shifted to be relative to the receiver position for display purposes.

We note that Figures 2 and 4 are partial transforms and do not include the transforms over the source or receiver position. They are intended to show that different events are recorded based on whether the original or reciprocal data are inspected. Comparing Figures 1 and 3, and Figures 2 and 4, we note that different events are stronger in each gather and that their moveouts are different. The plane wave gathers, see Figures 2 and 4, show that each gather has recorded plane waves from predominantly opposite directions. This is as we expect from the acquisition geometry and gathering process. These plane wave data were further transformed over source and receiver position respectively, to complete equation (3). Here, we keep the original shot and receiver gathers separate and migrated each data set individually.

In Figure 5 the image is derived from the plane wave transformed shot gathers. Figure 6 is the corresponding result for the receiver gathers. For the shot gathers, Figure 5, the image on the left side of the deep structure is not recovered as well as in Figure 6 because of the acquisition geometry. The same is true for the right hand side of the section for the migrated plane wave common receiver gathers, Figure 6. The final result is formed by stacking the two images, Figure 7a. The reflectivity is shown in Figure 7b for comparison.

Reciprocity Examples

The shot gathers and the receiver gathers generated assuming reciprocity were also transformed to construct source and receiver plane waves both in $p_s - p_r$ and $p_s - p_o$ domains. We show the effect of adding reciprocity by physically including the receiver gathers into the plane wave transforms, equation (3) or equation (1) of Stoffa et al. (2005). Figures 8 and 9 show plane wave transforms with and without reciprocity. Each panel shows all p_r for the cases where p_s = -0.5, p_s = 0.0 and p_s = 0.5 sec/km from left to right. The plane wave transforms without reciprocity are shown in Figure 8 and with reciprocity in Figure 9. Figures 10 and 11 show similar cross sections from the $p_s - p_o$ volumes without reciprocity (Figure 10) and with reciprocity (Figure 11). It is clear that by assuming reciprocity additional information is included in the plane wave transforms. These additional data will contribute to the image directly rather than by summing the individually migrated plane wave data as shown above.

Conclusions

We have shown that reciprocity should be taken into account during pre stack depth imaging. Our examples are for plane wave Kirchhoff migrations which show that missing plane wave components in the original marine shot gathers can be recovered by using reciprocity. Separate images formed using both the original and the receiver gathers constructed using reciprocity each illuminate different parts of the target better. By combining these partial images the final image is significantly improved. We have shown examples in the $p_s - p_r$ and $p_s - p_o$ domain of how, by using reciprocity, the missing plane wave components can be recovered, so that these can be used directly in the imaging without the need to construct separate partial images. Finally, we suggest a new way to take into account reciprocity which does not require physically re gathering the data. Instead, reciprocity is taken into account directly as part of the plane wave transform.

References

Aki, K., and P. G. Richards, 2002, Quantitative Seismology, second edition, University Science Books.

deHoop, A. T., 1958, Representation theorems for the displacement in an elastic solid and their applications to elastodynamic diffraction theory, D. Sc. Thesis, Technische Hogeschool, Delft.

Fokkema, J.T., and P.M. van den Berg, 1993, Seismic applications of acoustic reciprocity, Elsevier Science Publishers B.V.

Liu, Fagi, Whitmore D.N., Hanson D.W., Day R.S., Mosher C.C, 2004, The Impact of Reciprocity on Prestack Source Plane Wave Migration: SEG Expanded abstracts **23**, p1045-1048.

Levander, A.R., 1988, Forth-order finite-difference *P-SV* seismograms: Geophysics, Vol. 53, p1425-1436.

Stoffa, P.L., Buhl, P., Diebold, J.B., and Friedemann, W., 1981, Direct mapping of seismic data to the domain of intercept time and ray parameter – A plane wave decomposition: Geophysics, Vol. 46, p255-267.

Stoffa, P.L., Sen, M.K., Seifoullaev, R.K., Pestana R., Fokkema, J.T., 2005, Double Plane Wave Kirchhoff Migration: Ninth International Congress of the Brazilian Geophysical Society, this volume.

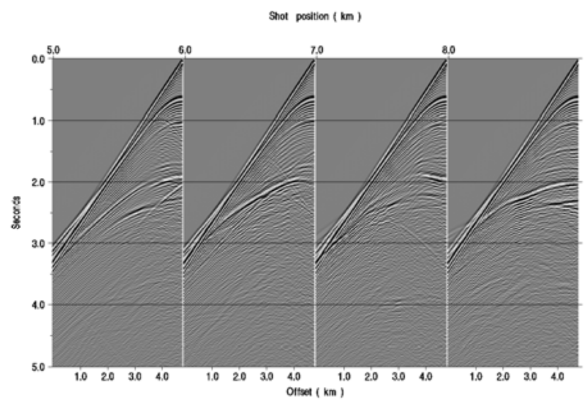

Figure 1. Finite difference common shot gathers at source positions 5, 6, 7 and 8 km simulating a marine survey with the array towed behind the ship. 240 channels were acquired with a receiver spacing 0.02 km. The maximum offset is 4.78 km}

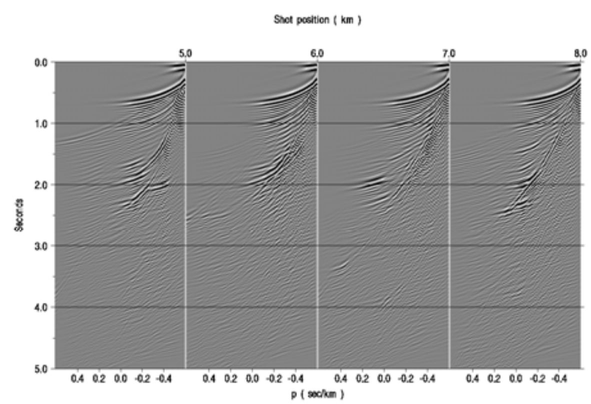

Figure 2. $\tau - p$ transformed shot point gather at source positions 5, 6, 7 and 8 km. 121 traces in each panel correspond to ray parameters from +.6 to -.6 sec/km every .01 sec/km

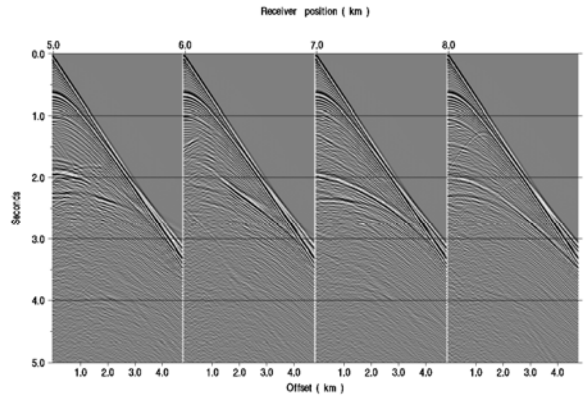

Figure 3. Finite difference common receiver gathers at receiver positions 5, 6, 7 and 8 km collected from the original shot point gathers. The maximum offset is 4.78 km

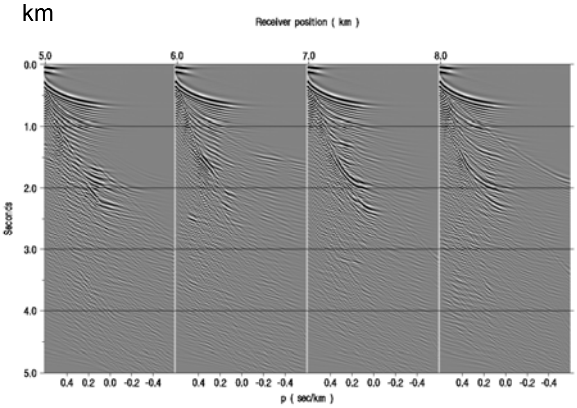

Figure 4. $\tau - p$ transformed receiver gathers at receiver positions 5, 6, 7 and 8 km. 121 traces in each panel correspond to ray parameters from +.6 to -.6 sec/km every .01 sec/km

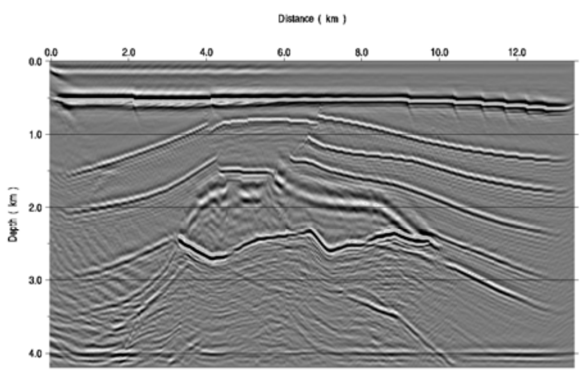

Figure 5. Migrated common shot receiver plane wave gathers. 121 receiver plane wave were migrated and stacked to produce the image

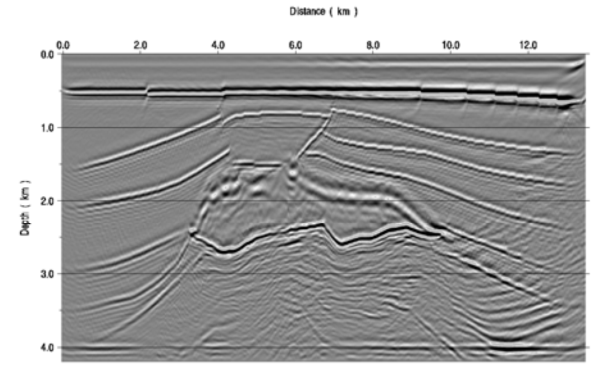

Figure 6. Migrated common receiver source plane wave gathers. 121 source plane waves were migrated and stacked to produce the image

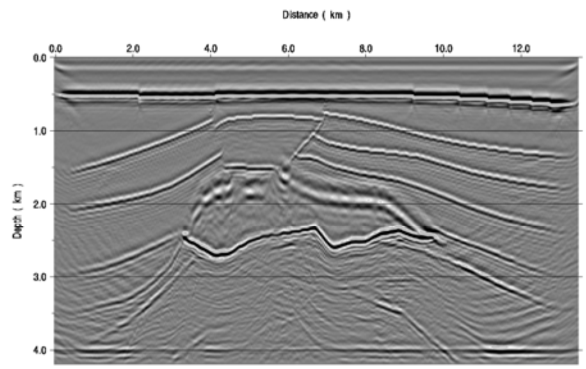

Figure 7a. Migrated shot and receiver gathers combined together (stacked)

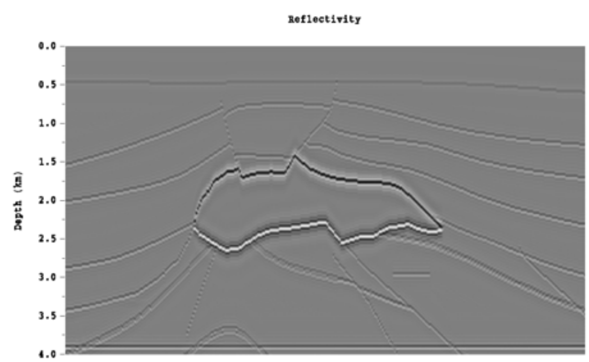

Figure 7b. Reflectivity for EAEG salt data

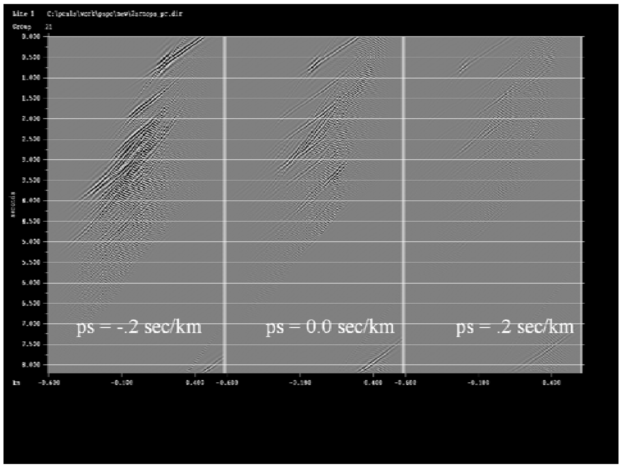

Figure 8. p_s cross sections from p_s – p_r volume without data reciprocity

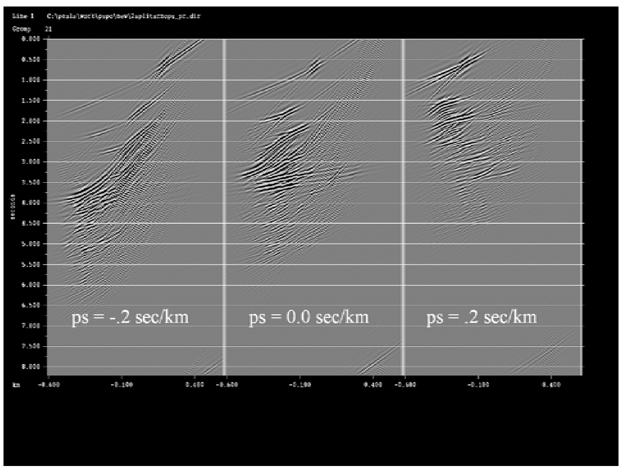

Figure 9. p_s cross sections from p_s – p_r volume with data reciprocity

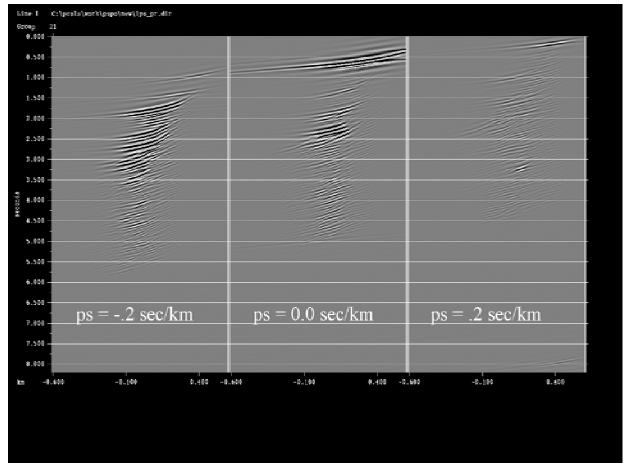

Figure 10. p_s cross sections from p_s – p_0 volume without data reciprocity

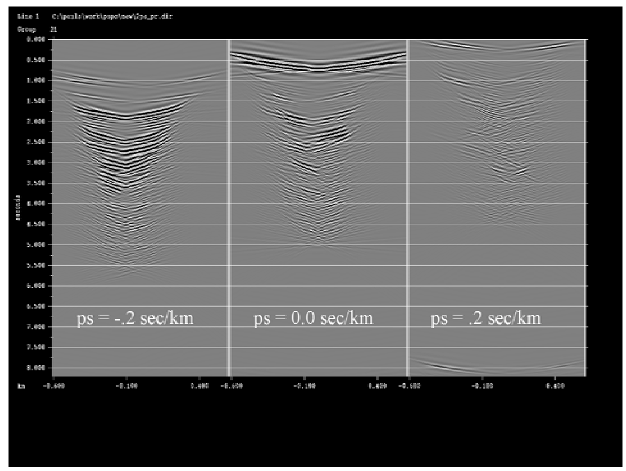

Figure 11. p_s cross sections from p_s – p_0 volume with data reciprocity