Numerical and analytical amplitude comparison on acoustic modeling

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This paper was prepared for presentation at the 9th International Congress of the Brazilian Geophysical Society held in Salvador, Brazil, 11-14 September 2005.

Abstract

This paper presents the comparison of amplitudes response of P wave calculated by a finite difference algorithm for 2D acoustic modeling, 2D AM, and analytically calculated amplitudes. For the comparison a simple model containing 3 horizontal layers was built. It was given a single shot in the middle part of the model and direct and reflected P wave behavior were analyzed. It was observed that geometric spreading amplitudes calculated by 2D acoustic finite difference algorithm exhibited smaller values than the ones calculated with cylindrical divergence formula. The same behavior was observed on reflected amplitudes when compared with Zoepprittz equation results. Both, direct and reflected waves calculated with finite difference routine are linearly related to analytical amplitudes. Such linear relation permits us to improve numerical results in studies that ask for more accuracy.

Introduction

Finite difference algorithms are wide used in exploration and reservoir seismic studies. Specially on 2D simulations, it permits to test different geological hypothesis relatively fast enough to support decision making. In finite difference algorithm we usually need to inform rock parameters in a grid based field, condition which permits modeling of highly complex geologic structures. In some studies, mainly in exploration approach, it is useful to suppose rocks behave as an acoustic medium. On such approach we only need to create a P velocity grid to perform the model.

Acoustic modeling using second order solver is widely used for structural check of subsurface geology. With that, we usually pay little attention on amplitude behavior of numerical results of acoustic algorithms.

In this work an acoustic modeling routine, 2D AM (2D Acoustic Modeling), which solves 2D wave equation by finite difference method, is tested by comparisons of analytical results. The direct and reflected waves are analyzed. The first one is compared with geometric spreading prediction which in a 2D model is calculated through a cylindrical divergence. For reflected wave analysis, numerical model amplitudes are compared to Zoepprittz equation results.

To perform such analysis it was used three routines: the 2D AM used to perform the model, described with more detail on the next topic; the MODBUILD used to build the P velocity model and; AVOY, used to calculate multiple interfaces amplitude results by Zoepprittz equation also better described below.

2D Acoustic Model routine

The 2D AM routine, built in C++ language, simulates a real seismic survey outputting snapshots and seismograms containing pressure, velocity or displacement fields. The routine solves the 2D wave equation by a second order approximation.

To run the 2D AM routine it is necessary to inform survey parameters (spread length, receiver (geophone or hydrophone) interval, initial and final shot position, shot interval and record time), number of grid points in vertical and horizontal direction, grid spacing and P velocity field. The source used is a Ricker wavelet whose dominant frequency is a function of grid spacing and velocity range. Despite we are not working with a staggered grid in an elastic simulation, we have used the same 10 grid points/wavelength used by Levander (1988). Such ratio also avoids numerical dispersion in this acoustic routine.

Analytical amplitude calculation

Analytical calculation of transmitted and reflected P wave was made by AVOY subroutine which contains the implementation of Zoeppritz equation extracted from Yilmaz (1987). This routine, applied only for horizontal and parallel layers model, simulates a shot on the position (x, z) = (0, 0) and outputs a three column file with the arrival position on the surface (horizontal distance from the source), time (lasted time from the shot until arrival on surface) and amplitude of reflected P wave. The amplitude calculation is made through a Gauss Jordan method applied for each interface downward and upward.

Despite the cited subroutine permits the calculation of P reflected wave responses of multiple horizontal interfaces, in this work, just one interface was used.

To run AVOY routine it is necessary to fill properties file and a parameter file. The first one contains layer properties as follow: layer thickness (m), P velocity (m/s), density (kg/m3) and Vp/Vs ratio. The parameter file contains the initial and final incidence angle on the first interface, angle increment and the input and output file names. The obtained amplitude results are exact and do not take account geometrical spreading.

Model Building

To perform the 2D AM routine it was chosen a three horizontal layer model. The upper, mid and lower layers have respectively 1500m/s, 2200m/s and 2300m/s P velocities. The two interfaces which apart each layer are 800m and 1000 m deep. The grid is 1000x1000 with 2m cell size. As we are working with a plan-parallel model, the velocity field for 2D AM routine was easily generated by a routine in which number of layers, thickness and velocity of each one is informed, the MODBUILD. Just one shot was performed in x=1000m and z=600m position. It was used a Ricker source which, according to employed parameters, has a dominant frequency of 75 hz.

To build the correspondent model to perform AVOY subroutine we fill the parameter file with P velocities and layer thickness in the same way already described in 2D AM. The same density was used for all layers and, to simulate an acoustic medium, Vp/Vs ratio was filled with an extremely high value (over 1E+20), a value that approaches Poisson ratio very close to 0,5.

Geometric spreading

As 2D AM works with 2D wave equation, the geometric spreading do not obey the spherical divergence, but a cylindrical one which is given by equation 1.

$$A(x) = Ao / (x)^{1/2} \quad \text{equation 1}$$

A(x) - amplitude at a distance x from the source

Ao - amplitude near the source (~ 0)

Direct wave amplitudes was sampled on different model positions and exhibited good but not exact approximation of cylindrical amplitude decay (Figure 1).

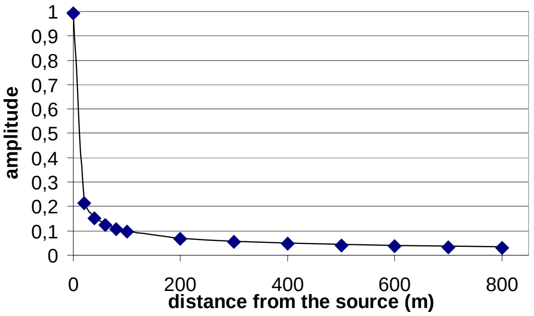

Figure 1: Predicted cylindrical divergence amplitudes (line) and 2D AM amplitudes (dots).

The amplitudes calculated by 2D AM routine was sampled on stations 20, 40, 60, 80, 100, 200, 300, 400, 500, 600, 700 and 800m (distance from the source). They follow the predicted values but they are systematically a little bit smaller than analytical ones. What most surprises us on such comparison was the increasing error as we get farther from the source. The error calculated has varied from 3%, in station 100m for instance, to 16%, in station 800m. This increasing error can be explained by the asymptotical behavior of cylindrical divergence leading to zero in infinite. In this sense we hope that error really do increase with distance.

Cross-plotting distance from the source and the difference between predicted amplitude and 2D AM one (Figure 2) show a linear behavior which regression gives equation 2:

$$y = 3E\text{-}06x + 0{,}0028 \quad \text{equation 2}$$

y - difference between analytical and numerical amplitude for direct wave;

x – distance from the source.

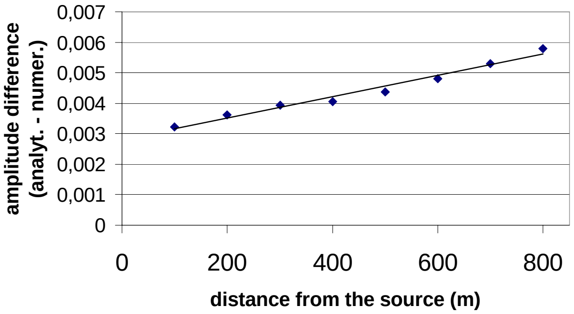

Figure 2: Cross-plot of distance from the source (horizontal) and the difference between predicted cylindrical amplitude and 2D AM one (vertical).

Equation 2 is then used to correct calculated amplitudes of 2D AM furnishing better amplitudes for direct P wave.

Reflected amplitudes

The AVOY subroutine was run and reflected amplitudes from the first interface - the one which apart 1500m/s upper layer from the 2200 m/s layer - was taken and subjected to cylindrical divergence correction. That was necessary because, as it was mentioned, AVOY subroutine do not calculate geometric spreading.

Numerical results from 2D AM were taken from the seismograms on stations 0, 20, 40, 60, 80, 100, 200 and 300m. All the used station was affected by reflected waves with reflections below the critical angle - 42,99º). 2D AM and AVOY were compared and again numerical results, 2D AM, follow analytical amplitudes with a little bit smaller values (Figure 3).

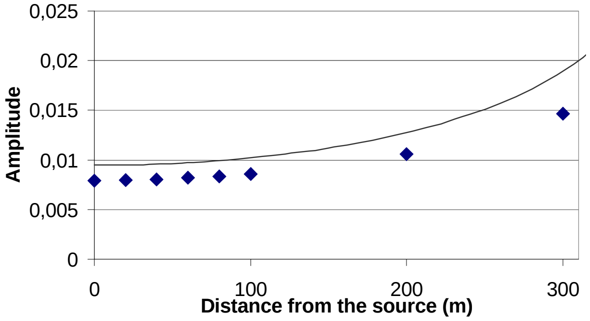

Figure 3: Reflected amplitudes calculated by Zoepprittz equation (line) and 2D AM (dots).

The cross-plot of 2D AM reflected amplitudes vs Zoepprittz corrected amplitudes exhibit linear relation which motivated us to apply equation 2 on numerical results. It was made some tests and it was verified that the best correction for reflected P wave amplitude is reached when we apply equation 2 without linear coefficient 0,0028. Nevertheless such correction still gives smaller amplitudes than predicted. It suggests that numerical algorithm overestimates transmitted amplitude and underestimates reflected one.

It was not done yet, but additional tests should be made with multiple layers to verify if there is any kind of trade off or compensation between number of layers, and a consequent increase in number of transmitted waves from the deeper portion of the model, such that propagation error get smaller with increase in model complexities.

Discussion

Numerical results show very good amplitude results on seismic modeling as it was pointed out on Figure 1. It is very clear that amplitudes far from the source follow the shape of predicted cylindrical divergence. The same behavior was observed for reflected P wave on 2D AM. As we are working with an acoustic model, differing from geological environment, amplitudes increase when offset increases (Figure 3). For relative results or comparison of different model hypothesis of subsurface on exploration or reservoir studies, it is not necessary a great accuracy. Acoustic modeling applied on structural analysis does not ask for a high level of accuracy on amplitude results.

Nevertheless, when the control on amplitude is greater, some corrections should be made. As we can realize with the simple example of direct and reflected P wave, the second order finite difference algorithm for 2D wave propagation must incorporate a correction described by equation 2, or higher (4 th) order solvers should be used.

Similar care should be taken on elastic modeling, as numerical approximation sometimes are far from desired accuracy necessary on detailed studies of reservoir behavior and fluid migration.

Conclusions

It was realized that, despite the excellent results achieved with finite difference routines, there are differences between analytical and numerical results reaching relative error of 16% or more. Such differences are not important when we are working with relative amplitude results as finite difference algorithm follow analytical prediction and show systematic smaller values. Nevertheless, if we wish higher accuracy or work with absolute results, corrections must be imposed on the routines or higher order solvers, may be 4th, must be used. In this work, up to the moment, it was realized that equation 2 correct the amplitudes obtained with 2D AM routine.

It was also realized that the use of equation 2 do not correct the 2D AM reflected amplitudes. We obtain better results when we apply that equation without its linear coefficient. Nevertheless, such correction still gives smaller results than the ones predicted by Zoepprittz equation.

For homogeneous medium the suggested correction – equation 2 – works. Further studies should be made to improve reflected amplitudes result.

Acknowledgments

We would like to thanks the colleague Andre Bulcão for discussions and suggestions and PETROBRAS S.A. for permission to publish this work.

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