Traveltime data profiles obtained using seismic ray tracing methods for the continental slope model parameterized by polynomials

Wilson M. Figueiró, Saulo P. Oliveira and Fábio C. Novaes, CPGG-UFBA, LAGEP, Federal University of Bahia, 40.170-290, Federação, Salvador, Brazil

Abstract

Geological structures known as continental shelf break, also called continental slope, are represented by seismic models characterized by two-dimensional, heterogeneous and isotropic velocity fields. By means of a polynomial parameterization of such fields, a smooth version of the model is generated with a strong variation near discontinuities that are, generally, associated to the seismic interfaces. In this kind of parameterization, all geometric and kinematics complications of the model are compacted in the polynomial coefficients, which are obtained by the least squares method. Traveltimes are calculated on the rays that represent paths for the seismic wave during its travel through the model. Seismic rays are connecting source positions to arrival points on the observation surface and they are traced by means of numerical approximations of ray equations (expansions of Taylor of second order). The calculated data tells us something about the geologic features of the considered seismic velocity field, such as: low lateral variation of velocity in one half of the model, predominance of low velocities in another half, break of model symmetry due to the increase of declivity of the layers in the accentuated slope region, increase of velocities with deep, etc. The same model is represented by different polynomial functions, it is observed that the increase of the polynomial degree produces an improvement on the representation when quantity and distribution of chosen points for polynomial adjustment are not altered. The used continental slope model comes from literature and, in this first approach, just its main features are considered. We believe that better results will be produced considering finer discrete models or polynomials of higher degree. Modeling traveltimes, such as it is done in this work, allows us the accomplishment of seismic inversions, in which the polynomials coefficients are the model parameters to be estimated.

Introduction

Geophysics has its main objective in the generation of images of structures of Earth interior that are not accessible to the direct observation. In seismology and seismics, whose distinction is mostly in the scale of the object of study, it is not different. They aim to produce images of distribution of subsurface seismic properties, such as seismic velocities. However, in the major part of approaches used to estimate subsurface parameters, the synthetic modeling of data (or the resolution of the direct problem) appears as one of their central elements. In this work, traveltimes of first arrival of no direct compressional waves are modeled. Such waves are generated in different source points located on the observation surface and they arrive on this same surface. The traveltimes are calculated on the ray trajectories, whose tracing theory and technique are presented in the next section.

Seismic ray tracing and traveltime calculation

The seismic ray tracing system (Červený, 1987) is given by:

$$\begin{cases} \dfrac{d\vec{X}(\tau)}{d\tau} = \vec{P}(\tau) \\ \dfrac{d\vec{P}(\tau)}{d\tau} = \dfrac{1}{2}\vec{\nabla}\left[\dfrac{1}{v^2(x,z)}\right], \end{cases} \tag{1}$$

where: $\vec{X}(\tau)$ is the position vector of the points of the curve that represents the ray trajectory at τ; $\vec{P}(\tau)$ is the tangent vector to the ray trajectory at τ (known as the slowness vector); $v(x,z)$ is the wave propagation velocity at the point (x,z) of the model and τ is a parameter, with international system dimension L^2T^{-1} and without a direct physical interpretation, it is defined by $\int_0^t v^2 dt$ with the time t measured along the ray trajectory.

The rays are traced by means of expansions of vectors position and slowness until the second term of the Taylor's series using system (1). It means:

$$\begin{cases} \vec{X}(\tau+\Delta\tau) = \vec{X}(\tau) + \vec{P}(\tau)\Delta\tau \\ \vec{P}(\tau+\Delta\tau) = \vec{P}(\tau) + \dfrac{1}{2}\vec{\nabla}\left[\dfrac{1}{v^2(x,z)}\right]\Delta\tau. \end{cases} \tag{2}$$

The traveltimes are numerically calculate by the following expression:

$$t = \sum_{i=0}^{J} \frac{1}{v_i}\left\|\vec{X}_{i+1} - \vec{X}_i\right\|_2, \tag{3}$$

where $\vec{X}_0$ is the source position, $\vec{X}_i = \vec{X}(i.\Delta\tau)$, v_i is the velocity at $\vec{X}_i$, $\|\cdots\|_2$ is the Euclidean norm and J is the number of lines on the polygonal line that represents the ray trajectory inside the space where the model is delimited.

Polynomial fitting

As some motivations for representation of velocity fields by polynomial functions, we have: economy of computational memory space, facility for mathematical manipulation (a polynomial function can be infinitely and easily differentiated and integrated), simplicity of the parameters set to be estimated by an inversion procedure, and every continuous function defined in a compact set is the limit of a sequence of polynomial functions, this is the Stone-Weierstrass approximation theorem (Bartle, 1983).

The velocity field discontinuities can be thought as points at which the polynomial function varies strongly.

The geophysical model comes from a geological model of the continental shelf break. In fact, such geophysical model is a seismic model that becomes a discrete seismic compressional velocities field. Here, it is represented by the following matrix:

$$M = \begin{pmatrix} x_1 & z_1 & v_{1,1} \\ \vdots & \vdots & \vdots \\ x_k & z_l & v_{k,l} \\ \vdots & \vdots & \vdots \\ x_n & z_m & v_{n,m} \end{pmatrix}, \tag{4}$$

where $k \in \{1,2,3,\cdots,n\}$, $l \in \{1,2,3,\cdots,m\}$ and $v_{k,l}$ is the velocity at the point (x_k, z_l). We fit to the points of M a polynomial in the form:

$$v(x,z) = \sum_{i+j=0}^{N} c_{ij}.x^i.z^j, \tag{5}$$

where i and $j \in \{0,1,2,\cdots,N\}$.

Combining (4) with (5), when i and j vary, we have the following system of equations:

$$\sum_{i+j=0}^{N} c_{ij}.x_k^i.z_l^j = v_{k,l}. \tag{6}$$

In a more synthetic way, (6) can be expressed as:

$$A.C = V, \tag{7}$$

where

$$A = \begin{pmatrix} \cdots & \cdots & \cdots \\ \vdots & \vdots & \vdots \\ \cdots & x_k^i.z_l^j & \cdots \\ \vdots & \vdots & \vdots \\ \cdots & \cdots & \cdots \end{pmatrix} \tag{8}$$

is the matrix that has the polynomial forms, with $x_k^i.z_l^j$ being the entry of the line

$$r = m(k-1) + l \tag{9}$$

and column

$$s = \frac{(i+j)(i+j+1)}{2} + j + 1 \tag{10}$$

of the matrix A;

$$C = \begin{pmatrix} \vdots \\ c_{i,j} \\ \vdots \end{pmatrix} \tag{11}$$

is the coefficient vector with $c_{i,j}$ its s-th component being s given by (10) and

$$V = \begin{pmatrix} \vdots \\ v_{k,l} \\ \vdots \end{pmatrix} \tag{12}$$

is the vector of velocities at the knots of the discrete model, with $v_{k,l}$ its r-th component being r given by (9). The solution of (7) given by the least squares method (Menke,1989), is as follows:

$$C = (A^T A)^{-1} A^T V. \tag{13}$$

Continental shelf break model

The geologic model of the continental shelf break or continental slope used as a start point, Figure 1, comes from geologic literature (Morelock, 2004).

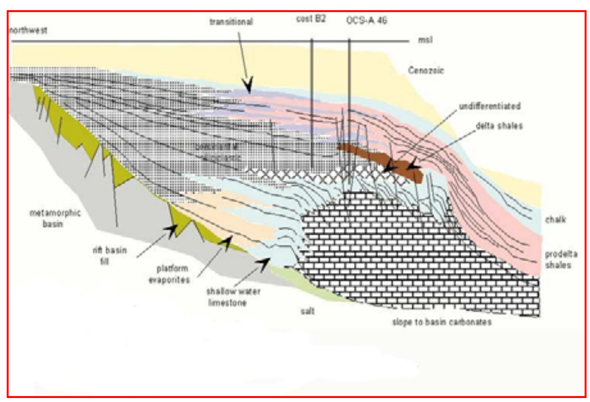

Figure 1 – Shelf break continental geologic model took as a starting point. For each different kind of rock in the model is attributed a typical compressional velocity. This picture can be found in the internet in the following address: http://cima.uprm.edu/~morelock/6_image/17divmarg.jpg.

The delimited region of the considered model is shown in Figure 2.

The discrete version of seismic aspects (in the case, velocities of compressional waves) of the delimited region is shown in Figure 3.

The discrete geophysical model generated is a two-dimensional, heterogeneous and isotropic compressional velocity field with total horizontal length of $32.0\ km$ and depth of $4.0\ km$. Such discrete model is constituted by $17 \times 33 = 561$ blocs. The velocity is homogeneous and isotropic inside each block.

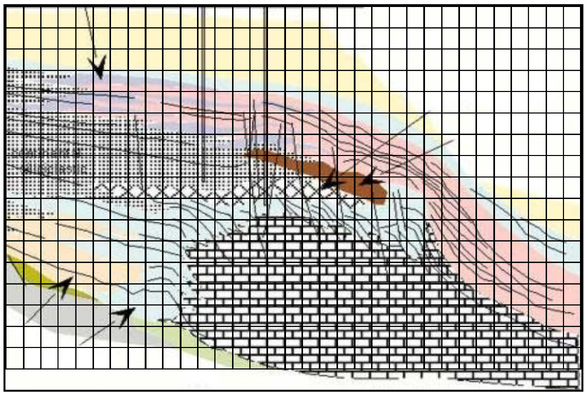

Figure 2 – The knots of the mesh put on the delimited region of the model are points used to produce a discrete model using typical velocities of rocks present in the geologic model.

	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32
0	1.5	1.5	1.5	1.5	1.5	1.5	1.5	1.5	1.5	1.5	1.5	1.5	1.5	1,5	1.5	1.5	1.5	1.5	1.5	1.5	1.5	1.5	1.5	1.5	1.5	1.5	1.5	1.5	1.5	1.5	1.5	1.5	1.5
0.25	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	1.5	1.5	1.5	1.5	1.5	1.5	1.5	1.5	1.5	1.5	1.5	1.5	1.5	1.5
0.5	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	1.5	1.5	1.5	1.5	1.5	1.5	1.5	1.5	1.5	1.5	1.5	1.5	1.5
0.75	3	2.7	2.7	2.7	2.7	2.7	2.7	2.7	2.7	2.7	2.3	2.3	2.3	2.3	2.3	2.3	2	2	2	2	2	1.5	1.5	1.5	1.5	1.5	1.5	1.5	1.5	1.5	1.5	1.5	1.5
1	3	2.7	2.7	2.7	2.7	2.7	2.6	2.6	2.6	2.6	2.6	2.7	2.7	2.7	2.7	2.7	2.6	2.6	2.6	2.3	2.3	2	2	1.5	1.5	1.5	1.5	1.5	1.5	1.5	1.5	1.5	1.5
1.25	3	3	3	3	3	3	3	3	3	3	2.7	2.7	2.7	2.6	2.6	2.6	2.6	2.6	2.6	2.6	2.6	2.3	2.3	2.3	2	1.5	1.5	1.5	1.5	1.5	1.5	1.5	1.5
1.5	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3.6	3.6	3.6	3	2.6	2.6	2.6	2.6	2.6	2.6	2.3	2.3	1.5	1.5	1.5	1.5	1.5	1.5	1.5
1.75	3	3	3	3	3	3.9	3.9	3.9	3	3	3	3	3	3.9	3.9	3.9	3.9	3.9	3.6	3.6	3.6	2.6	2.6	2.6	2.6	2.6	2.3	1.5	1.5	1.5	1.5	1.5	1.5
2	3	4.4	4.4	4.4	4.4	4.4	4.4	3	4.4	4.4	4.4	4.4	4.4	4.4	4.4	4.4	4.4	4.4	4.4	3.9	4.4	4.4	4.4	4.4	2.6	2.6	2.6	2.3	2.3	2	2	2	2
2.25	3	4.2	4.2	4.2	4.2	4.2	4.2	4.4	4.4	4.4	4.4	4.4	4.4	4.9	4.9	4.9	4.9	4.9	4.4	4.4	4.4	4.4	4.4	2.6	2.6	2.6	2.6	2.6	2.6	2.3	2.3	2	2
2.5	5.1	4.2	4.2	4.2	4.2	4.2	4.4	4.4	4.4	4.4	4.4	4.9	4.9	4.9	4.9	4.9	4.9	4.9	4.9	4.9	4.9	4.9	4.9	4.9	4.9	2.6	2.6	2.6	2.6	2.6	2.6	2.3	2.3
2.75	5.2	5.1	5.1	5.1	4.2	4.2	4.2	4.2	4.4	4.4	4.4	4.9	4.9	4.9	4.9	4.9	4.9	4.9	4.9	4.9	4.9	4.9	4.9	4.9	4.9	4.9	2.6	2.6	2.6	2.6	2.6	2.6	2.6
3	5.2	5.2	5.2	5.2	5.2	5.2	5.2	4.4	4.4	4.4	4.4	4.9	4.9	4.9	4.9	4.9	4.9	4.9	4.9	4.9	4.9	4.9	4.9	4.9	4.9	4.9	4.9	4.9	4.9	2.6	2.6	2.6	2.6
3.25	6.2	6.2	6.2	5.2	5.2	5.2	5.2	5.2	5	5	5	4.9	4.9	4.9	4.9	4.9	4.9	4.9	4.9	4.9	4.9	4.9	4.9	4.9	4.9	4.9	4.9	4.9	4.9	4.9	4.9	4.9	4.9
3.5	6.2	6.2	6.2	6.2	6.2	6.2	6.2	6.2	6.2	6.2	6.2	5	5	5	4.9	4.9	4.9	4.9	4.9	4.9	4.9	4.9	4.9	4.9	4.9	4.9	4.9	4.9	4.9	4.9	4.9	4.9	4.9
3.75	6.2	6.2	6.2	6.2	6.2	6.2	6.2	6.2	6.2	6.2	6.2	6.2	6.2	6.2	6.2	6.2	6.2	6.2	6.2	6.2	6.2	6.2	6.2	6.2	6.2	6.2	6.2	4.9	4.9	4.9	4.9	4.9	4.9
4	6.2	6.2	6.2	6.2	6.2	6.2	6.2	6.2	6.2	6.2	6.2	6.2	6.2	6.2	6.2	6.2	6.2	6.2	6.2	6.2	6.2	6.2	6.2	6.2	6.2	6.2	6.2	6.2	6.2	6.2	6.2	6.2	6.2

Figure 3 – Discrete geophysical model constituted by $33 \times 17 = 561$ blocks of homogeneous and isotropic compressional velocities.

Numerical experiments

The used geometry of acquisition is constituted by three sources located on observation surface at the following positions: $8.0\ km;\ 16.0\ km$ and $24.0\ km.$ The arrival position $\vec{X}$ of the ray is, also, position where traveltime is recorded. It is determined by the direction of slowness vector in the source.

Firstly, the model is parameterized by a polynomial function of the form:

$$v_1(x,z) = c_{0,0} + c_{1,0}.x + c_{0,1}.z + c_{2,0}.x^2 + c_{1,1}.x.z + c_{0,2}.z^2, \qquad (12)$$

it is called M_I and can be seen in the Figure 4. For it, the found coefficients have the following values: $c_{0,0} = 1.6822\ km.s^{-1}$, $c_{1,0} = 0.0430\ s^{-1}$, $c_{0,1} = 0.7319\ s^{-1}$, $c_{2,0} = -0.0026\ km^{-1}.s^{-1}$, $c_{1,1} = -0.0018\ km^{-1}s^{-1}$, and $c_{0,2} = 0.1232\ km^{-1}.s^{-1}$.

The original geological model is far from M_I, but some important structural features can already be observed, such as: the increasing velocity with depth, the dip layers after the shelf break, and the delimitation of velocities to the established range. For this model and chosen source positions, Figure 5 shows the ray field obtained. For the ray tracing, the points of ray trajectories are spaced of $\Delta\tau = 0.015\ km^2/s$, in terms of τ.

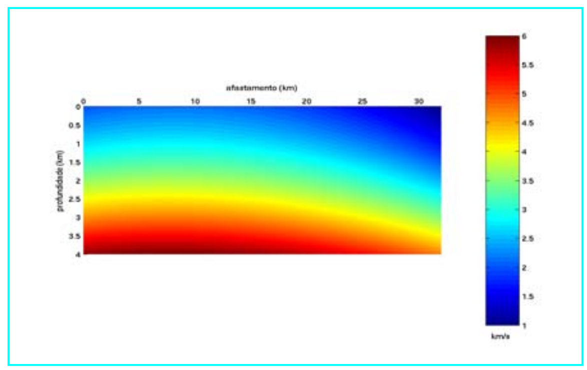

Figure 4 – Seismic velocity field correspondent to the continental slope model parameterized by the polynomial $v_1(x,z)$.

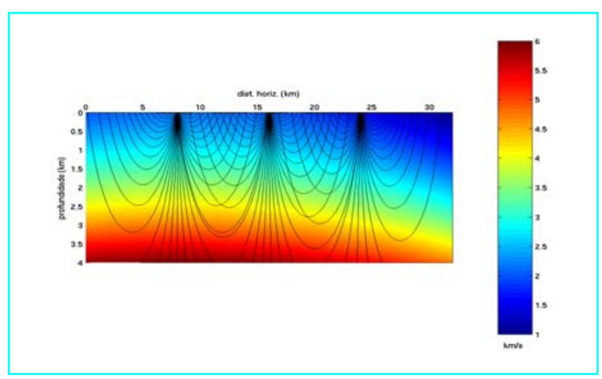

Figure 5 – Ray field in M_I.

Figure 6 shows the traveltime profiles for the proposed acquisition arrangements used in the case of model M_I.

The traveltime graphics are in concordance with the lateral monotony of the region close to the position

$x = 0.0\ km$ and have an anomalous behavior corresponding to the dip of layers on the shelf break region.

The same geologic model is also parameterized by a polynomial function of the form:

$$v_2(x,z) = c_{0,0} + c_{1,0}.x + c_{0,1}.z + c_{2,0}.x^2 + c_{1,1}.x.z + c_{0,2}.z^2 + c_{3,0}.x^3 + c_{0,3}z^3, \tag{13}$$

being called M_{II} and is shown in the Figure 7. The values of coefficients of v_2 obtained by least squares method are the following:

$c_{0,0} = 1.8377\ km.s^{-1}$, $c_{1,0} = 0.0075\ s^{-1}$, $c_{0,1} = 0.4902\ s^{-1}$, $c_{2,0} = 0.0002\ km^{-1}.s^{-1}$, $c_{1,1} = -0.0018\ km^{-1}.s^{-1}$, $c_{0,2} = 0.2789\ km^{-1}.s^{-1}$, $c_{3,0} = -0.0001\ km^{-2}.s^{-1}$, and $c_{0,3} = -0.0260\ km^{-2}.s^{-1}$.

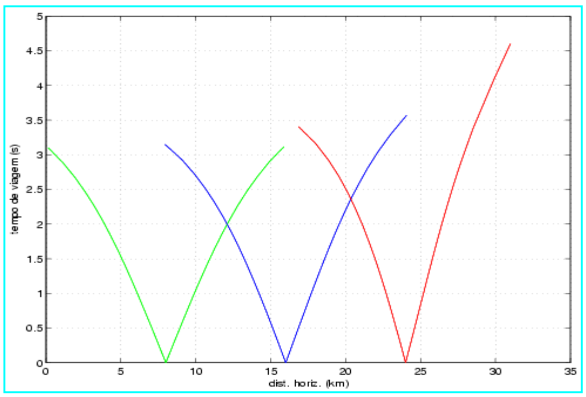

Figure 6 – Traveltime profiles recorded on the observation surface for model M_I.

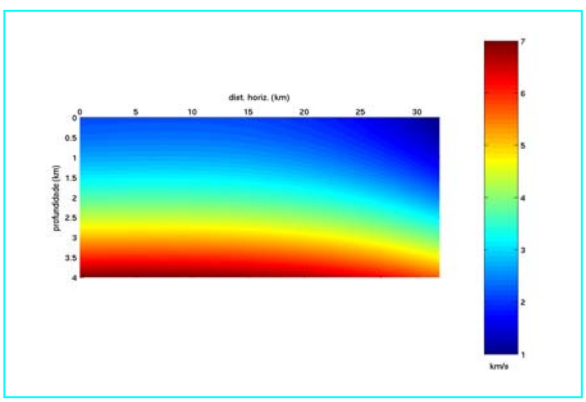

Figure 7 - Seismic velocity field correspondent to the continental slope model parameterized by the polynomial v_2.

The same comments presented for M_I, with respect to the representation of the geological model, also apply to M_{II}, with the addition that all aspects of the representation were improved. Figure 8 shows the ray field in M_{II}.

To the model M_{II}, the traveltime profiles, shown in Figure 9, are not significantly different of those observed for model M_I, this occurrence is due to the use of the same discrete model for fitting with the two different polynomial functions and, besides, there was not an abrupt change of function used in the parameterization.

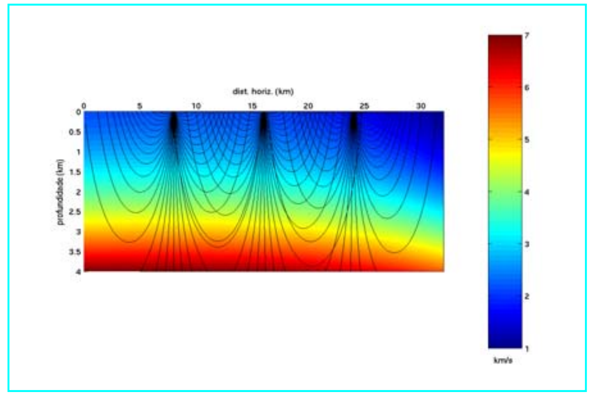

Figure 8 – Ray field for the model M_{II}.

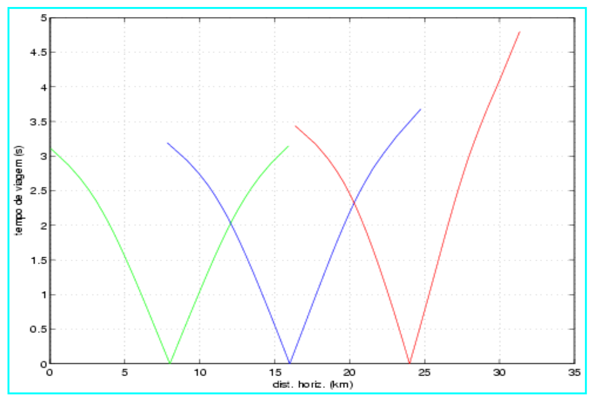

Figure 9 – Traveltime profiles recorded for the model M_{II}.

The third polynomial parameterization of the model is given by:

$$v_3(x,z) = c_{0,0} + c_{1,0}.x + c_{0,1}.z + c_{2,0}.x^2 + c_{1,1}.x.z + c_{0,2}.z^2 + c_{3,0}.x^3 + c_{2,1}.x^2.z + c_{1,2}.x.z^2 + c_{0,3}.z^3, \tag{14}$$

it is called M_{III} and is shown in Figure 10, with the following coefficients values obtained by fitting:

$c_{0,0} = 1.3338\ km.s^{-1}$, $c_{1,0} = 0.0309\ s^{-1}$, $c_{0,1} = 1.3637\ s^{-1}$, $c_{2,0} = 0.0006\ km^{-1}.s^{-1}$, $c_{1,1} = -0.0523\ km^{-1}.s^{-1}$, $c_{0,2} = 0.0529\ km^{-1}.s^{-1}$, $c_{3,0} = -0.0001\ km^{-2}.s^{-1}$, $c_{2,1} = -0.0002\ km^{-2}.s^{-1}$, $c_{1,2} = 0.0141\ km^{-2}.s^{-1}$, and $c_{0,3} = -0.0260\ km^{-2}.s^{-1}$.

The inclusion of new terms to the polynomial function, relatively to M_{II}, improved the general representation of the discrete model. Figure 11 shows the ray field in the velocity field $v_3(x,z)$ obtained by the least squares fitting, it means: in M_{III}.

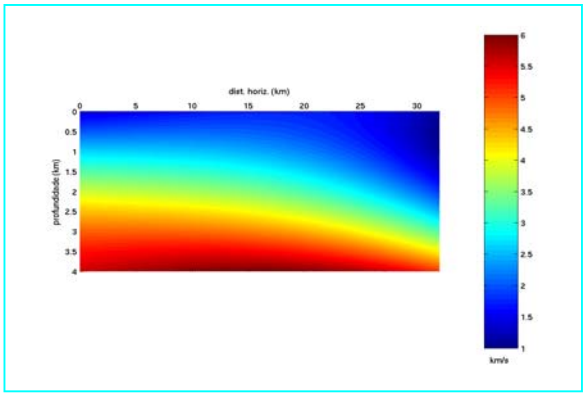

Figure 10 - Seismic velocity field for the polynomial function $v_3(x,z)$ obtained by fitting to the discrete model.

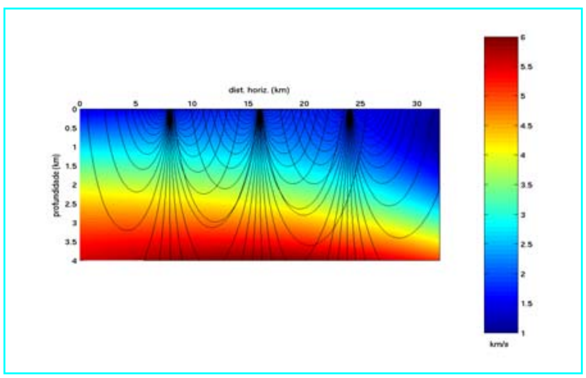

Figure 11 – Ray field in the model M_{III}, for the used source distribution.

With respect to the traveltime profile, Figure 12, there is not significant changes relatively to the profiles already presented in this work, for the same reasons previously exposed.

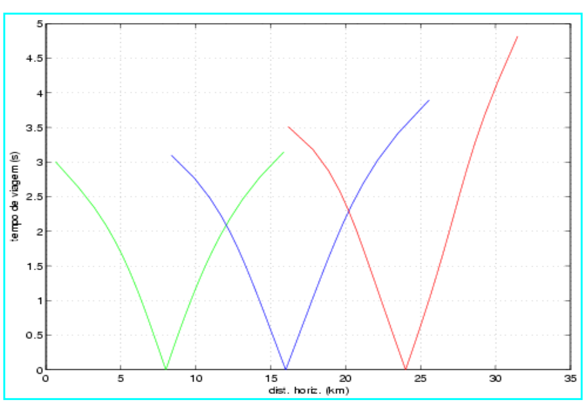

Figure 12 – Traveltime profiles obtained for the model M_{III}, for the proposed source distribution.

The triplication phenomena of traveltime does not happen in the presented experiments, because the considered velocity field is always increasing with depth, not allowing the appearance of low velocity zones.

A finer discrete model and higher polynomial functions degree are considered in another work (Figueiró et al, 2005).

Discussions and Conclusions

With this work is possible to have the following conclusions: the polynomial parameterization with term of degree 3 is able to represent the main structural features of the seismic velocity field extracted from a geologic model of the continental shelf break; we believe that polynomial functions with degree higher than three will be able to represent more detailed aspects of the continental slope model; in addition the increase of the number of points used to construct de discrete model will improve the adjustment; the obtained ray fields are very symmetric, independently of the degree of the used polynomial and of the source positions; the increase of the polynomial degree, for the three cases considered in this work, does not significantly affect the traveltime profiles; and the profile relative to the third source has an anomalous behavior due to the proximity to the region where begins the break shelf, it means: in the region where the slope begins to have an accentuated change.

Acknowledgments

We present our Thanks to the CPGG-UFBA, FINEP, PETROBRAS and to the “Rede Cooperativa de Pesquisa em Risco Exploratório em Petróleo e Gás”.

References

Bartle, R. G., 1983, Elementos de Análise Real. Editora Campus Ltda., Rio de Janeiro.

Červený, V., 1987, Ray Method for Three-Dimensional Seismic Modeling. Petroleum Industry Course, The Norwegian Institute of Technology.

Figueiró, W. M., Novaes, F. C., and Oliveira, S. P., 2005, Obtenção de tempos de trânsito usando raios sísmicos em modelos de talude continental parametrizado por polinômios. II Workshop da Rede Cooperativa de Pesquisa em Risco Exploratório em Petróleo e Gás, CDROM, Belém, Pará.

Menke, W., 1989, Geophysical Data Analysis: Discrete Inverse Theory. International Geophysics Series, Academic Press, Volume 45.

Morelock, J., 2004, Marine Geology. Department of Marine Sciences, University of Puerto Rico, http://cima.uprm.edu/.