# Using WCDP ("Wave Analogue Common Depth Point") to Generate Depth Interval Velocity for Migration

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**Abstract**

A successful prestack depth migration depends on a good depth interval velocity. The WCDP technique provides a reliable geological model after a composition based on interpretation of a series of time sections, which can be generated in a very short execution time when compared with the traditional prestack time migration. It will be discussed how this methodology works in order to obtain the geological model and the depth interval velocity field.

**Introduction**

The WCDP technique allows a better focalization of the reflectors in time domain disregarding a previous velocity model. These time sections allows the interpretation of a more reliable geological model, that will serve as input for the programs of generation of depth interval velocity models for PSDM (Prestack Depth Migration).

The WCDP technique outputs a series of time sections with a specific *background focalization* velocity, each one of them showing an improved image of a geological feature in a specific time. The time sections obtained with WCDP technique are interpreted taking into account the presence of geological features that can influence in the velocity model such as canyons, faults, pillows and domes of salt.

This geological time model will serve to get the interval velocity model in time as well in depth with coherent inversion, for the prestack time or depth migration.

**Mathematical Background**

The purpose of any survey is to obtain a specific parameter $a(\vec{r}\,')$ from the recorded data $d^{obs}(\vec{r})$:

$$d^{obs}(\vec{r}) = \int F(\vec{r},\vec{r}\,')a(\vec{r}\,')d\vec{r}\,'$$

The best equation will depend on the kind of survey, e.g., seismic, magnetometric, gravimetric, etc. The seismic survey uses the wave equation:

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} - \frac{1}{v^2}\frac{\partial^2 u}{\partial t^2} = 0 \text{, where}$$

$$u = u(x,y,z,t)$$

The Helmholtz acoustic wave equation is found by taking the temporal Fourier transform of the equation above:

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} + k^2(x,y,z,w)u = 0 \text{, where}$$

$$k(x,y,z,w) = {}^{w}\!/_{n(x,y,z)}$$

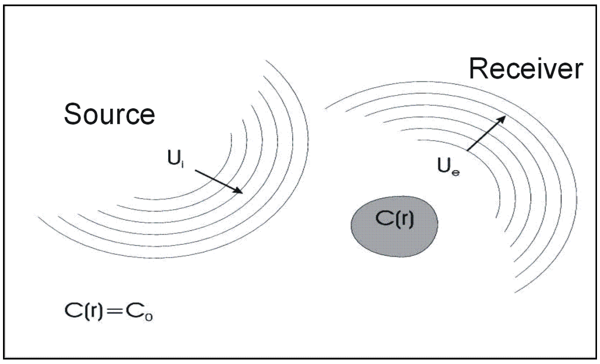


Figure 1 - The scattering ($u_e$) of an incident wave ($u_i$) from a source at a constant background velocity at an inhomogeneity, that acts as a second source.

Let $u(x,y,z,t)$ be the total wavefield:

$$u(x,y,z,t) = u_i(x,y,z,t) + u_e(x,y,z,t)\,,$$

so for the incident wave, we have:

$$\frac{\partial^2 u_i}{\partial x^2} + \frac{\partial^2 u_i}{\partial y^2} + \frac{\partial^2 u_i}{\partial z^2} - \frac{1}{v_o^2}\frac{\partial^2 u_i}{\partial t^2} = 0 \text{, or}$$

$$\frac{\partial^2 u_i}{\partial x^2} + \frac{\partial^2 u_i}{\partial y^2} + \frac{\partial^2 u_i}{\partial z^2} + k_0^2 u_i = 0$$

Now we reformulate $k(x,y,z,w)$ as a perturbation to a constant $k_0$ for the homogeneous background medium. Assuming that

$$k^2 = k_0^2 + [k^2 - k_0^2]$$

$$= k_0^2 + k_0^2[\frac{k^2}{k_0^2} - 1] = k_0^2 - k_0^2[1 - \frac{C_0^2}{C^2}]$$, where we can define a function $a(x,y,z) = 1 - \frac{C_0^2}{C^2}$

The equation that describes the propagation is

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} + (k_0^2 - k_0^2[1 - \frac{C_0^2}{C^2}])u = 0,$$

or

$$\left(\nabla^2 + (k_0^2 - k_0^2[1 - \frac{C_0^2}{C^2}])\right)u = 0$$, where

$$u = u_i + u_s.$$

Rearranging the last equation:

$$(\nabla^2 + k_0^2)u_i + (\nabla^2 + k_0^2)u_s = k_0^2 a[u_i + u_s]$$

The right-hand side term is the source of scattered wavefield. The first term on the left side represents the Helmholtz acoustic wave equation, so, $(\nabla^2 + k_0^2)u_i = 0$ and $(\nabla^2 + k_0^2)u_s = k_0^2 a[u_i + u_s]$, which describes the propagation of scattered wavefield. Using some properties of Green's function to solve the last equation we get the Lippmann-Schwinger equation:

$$u_s(\vec{r}) = -k_0^2 \int G(\vec{r} \mid \vec{r}\,')a(\vec{r}\,')[u_i(\vec{r}) + u_s(\vec{r}\,')]d\vec{r}\,'.$$

Considering a source at a vector position $\vec{r}_s$ and a receiver at $\vec{r}_p$, using the Born approximation, we can write:

$$u_s(\vec{r}_s, \vec{r}_p) \approx -k_0^2 \int a(\vec{r}\,)G(\vec{r} \mid \vec{r}_s)G(\vec{r}_p \mid \vec{r}\,)d\vec{r}\ .$$

The last equation can be rewritten in a simpler way like:

$$d_{obs}(\vec{r}) = \int F(\vec{r}, \vec{r}\,')a(\vec{r}\,')d\vec{r}\,',$$

we are trying to obtain the parameter $a(\vec{r}\,')$ from the recorded (observed) data $d_{obs}(\vec{r})$.

The main aim is to find $F$. After inversion of the equation of $u_s$, the WCDP method generates sections of derivative of $a$ function.

**Input Data – Pre Processing**

The input data should have:

a) Deconvolution

b) Multiples attenuation

c) No geometrical spreading correction

d) Band pass filter in the target zone

e) Source order from the nearer to the farther offset and no missing shot or trace

**What will the technique provide?**

The technique will provide a series of time sections that will be interpreted and will tend to provide the best possible geological time model (Figure 2) that will be used for the attainment of a RMS velocity model for prestack time migration or the interval velocity model with coherent inversion for the prestack depth migration. Moreover, they can also be used for the demigration, if necessary, for the coherent inversion modeling in order to attain an interval velocity model in depth for the prestack depth migration.

These sections are provided in a processing speed approximately ten times faster than the traditional methods of prestack time migration.

**How will the velocity model be generated?**

The seismic sections provided by WCDP technique do not require a previous velocity model. But one same horizon can be tracked in sections with different background velocity (Figure 3). Each WCDP section helps a better imaging of some geological features on some regions of the model. In such case to achieve a final geological time model, is required to put together the interpretation of several WCDP sections with distinct background velocity. It can be said that one single WCDP section is not enough to interpret a single horizon. This geological time model will serve for the attainment of the best RMS velocity field in time or best interval velocity in depth by layer stripping in coherent inversion.

The time RMS velocity or, the depth interval velocity will be the input for a prestack migration, on whichever the methodology employed.

In short the following strategy must be employed to find a depth interval velocity model:

1. Interpretation of the different time sections generated by the WCDP to build the model.
2. Velocity interpretation, on the geological model based approach.
3. Interpret RMS time velocity model, horizon to horizon from top to bottom of section, if necessary.
4. Demigrate model for input in a coherent inversion modeling of depth interval velocity, if recessary.

5. Interval depth velocity modeling, horizon to horizon from top to bottom of section.
6. Residual velocity analysis and anisotropy analysis.
7. Generate interval depth velocity model.
8. Migrate prestack in depth with any methodology.
9. Repeat 6 to 8 until the best flattening of events.

## Conclusions

The strategy of using WCDP imaging technique in order to obtain a velocity model becomes attractive as a consequence of well focused time section offered by this technique. Using a series of background velocity fields, we are able to get a more reliable geological time model. This model is generated from sections obtained with a short machine time and is the input of whichever inversion application employed.

This model enhances the quality of the velocity field obtained using any horizon based methodology for RMS or interval velocity generation.

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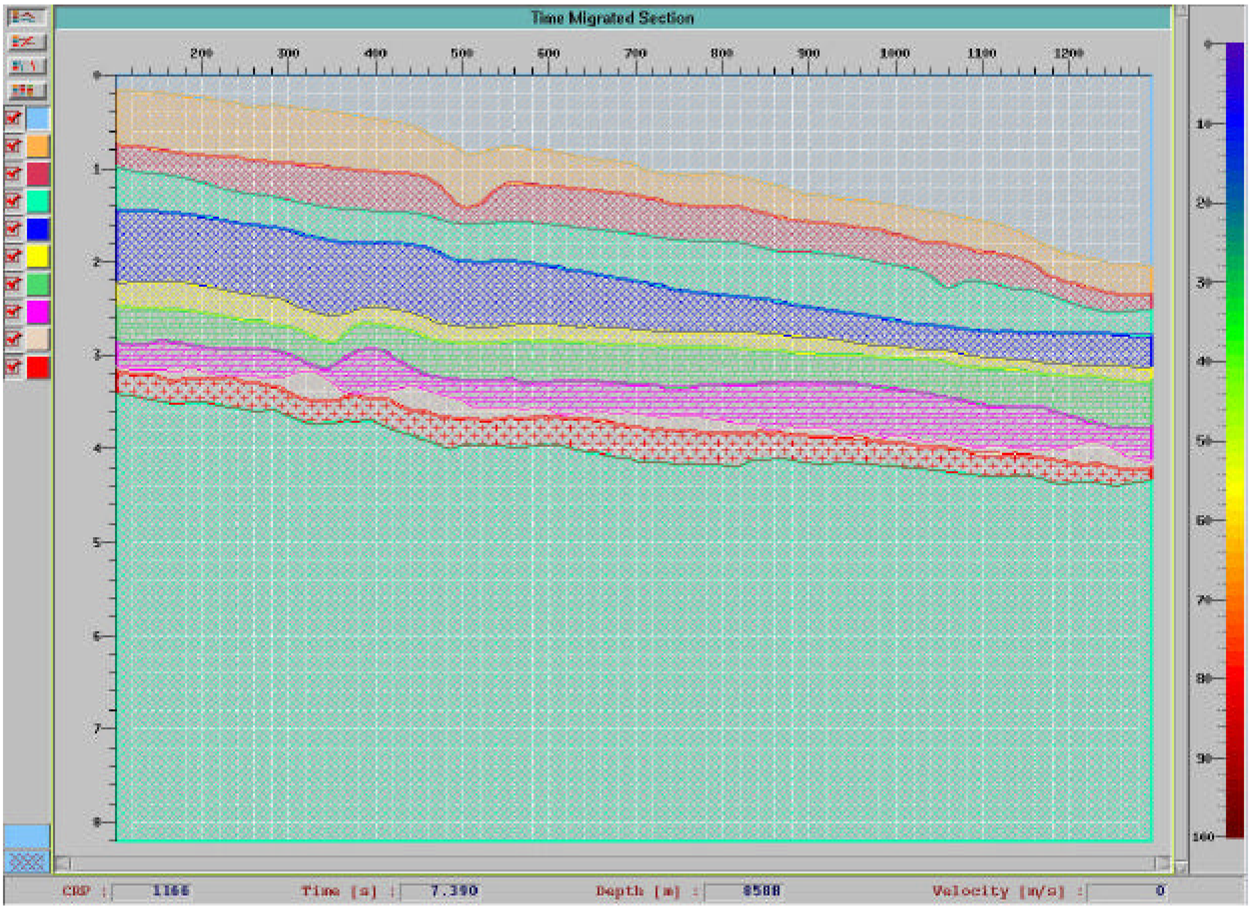


Figure 2 - Geological Time Migrate Model, obtained after interpretation of horizons in several time migrated sections provided by WCDP Technique.

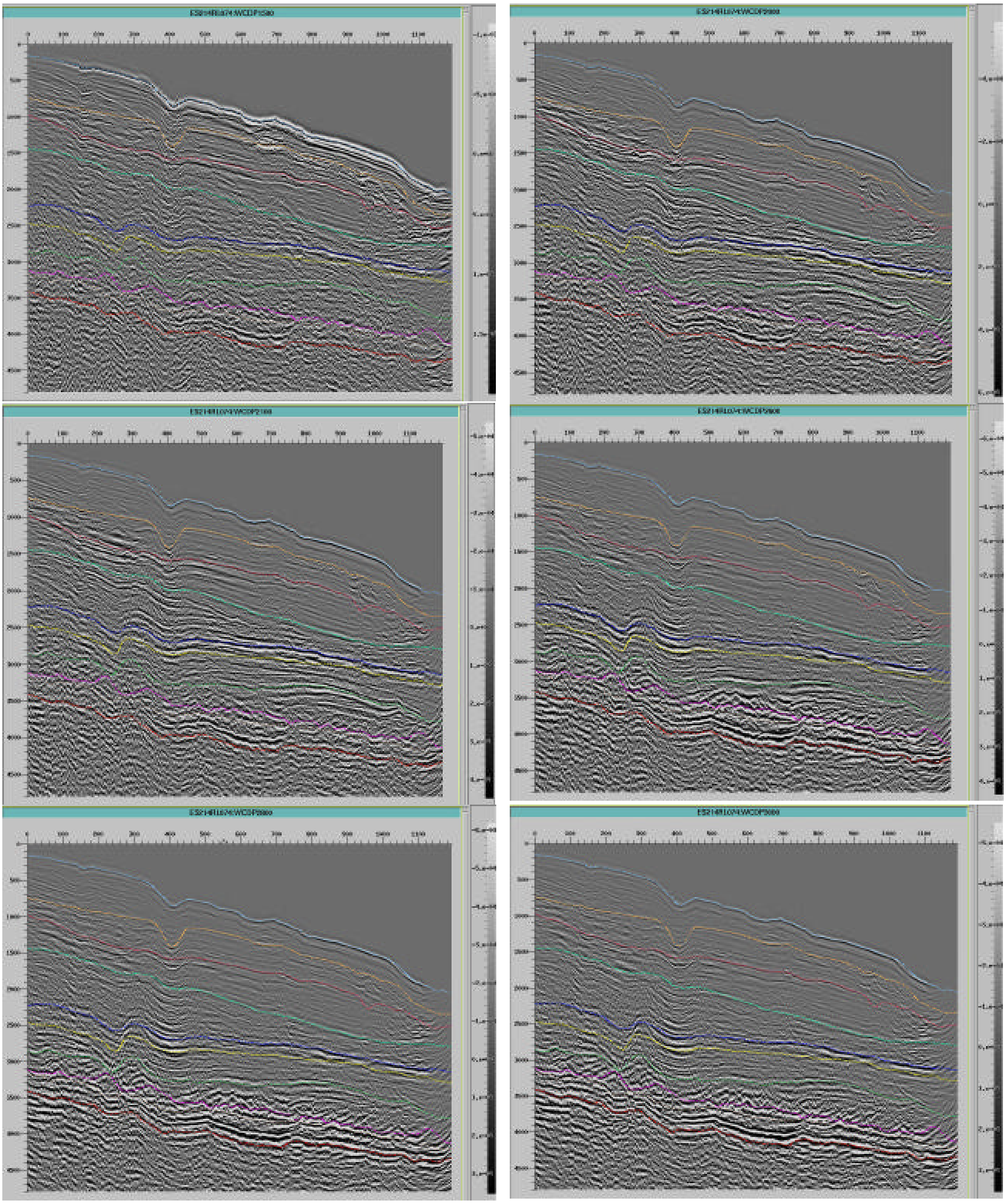

Figure 3 – Sections showing the improving image of specific geological feature with different background velocity. The Sea Bottom (Sky Blue) has a good image with a velocity of 1500 m/s and the Base of Salt (Pink) has a good image with a velocity of 2800 m/s.