Representation of the Earth’s Gravity Field in the Southern part of the African Plate

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Abstract

A new digital gravity field model (SAGM2005) is presented, representing the free air anomaly and the height anomaly in the southern part of the African plate.

A total of 12,203 gravity observations together with 5,486 free air GRACE-only data points were used in the process of Least-Squares Collocations (LSC) in order to carry the present representation out.

A new computer algorithm was written, as a Matlab application, to calculate the observed covariance function as a primary step of the collocation procedure.

The LSC method was applied as described by Moritz (1980) assuming a constant standard error of 3 mGal to the free air data points. The present work aimed to show the effect of introducing GRACE-only data with land observations on the precision of the calculated digital models, which represent the gravity field components in the studied area with spatial resolution of 0.1°, which is quite suitable to be used in regional studies.

Introduction

The calculation of the height anomaly (quasi-geoid) is one of the most discussed topics in the present days. The geotectonic applications for such quantity is getting wider and approaching a new level of importance, once the height anomaly does not lose power in long wavelengths as does happen with gravity anomaly. For that reason, together with the wide urban, military, and engineering applications, a precise height anomaly model is always looked for.

The Least-Squares Collocation method (LSC) is a mathematical tool used for the determination of the figure and the gravity field of the Earth combining data of various types. The same formulas could be interpreted as an inverse geophysical problem solution creating a statistical estimation. This solution is usually taken combining the Least-Squares Approximations and the prediction by Least-Squares Collocations. Such solution is considered as an analytical approximation for the geopotential using harmonic functions (Moritz, 1980).

One of the great challenges that faces the geodetic society is the lack of gravity and height observations over a great part of the world, which complicates the gravimetric quasi-geoid solutions. The Gravity Recovery and Climate Experiment (GRACE) gives the ideal solution for such problem with its high compatibility with the land surveys (Reigber, *et al.*, 2005).

The studied area occupies 6,325,000 km² of the southern Africa representing 10 countries (figure 1). The major topographic units of the area are the Drakensberg Mountains localized in the southeastern part of the region leaving a narrow coastal line to the Indian Ocean in the province of Orango near Durban (Republic of South Africa), the plateau of Namib, the Kalahari desert, which occupies more than two thirds of the continental part of the region and is connected to the plateau of Bié (Angola) and to the plateau of Katanga (Zambia).

The available data was first selected statistically, to get the best information with the minor errors (Vuolo, 2002). Then the gaps were filled using the EIGEN-GRACE02S geopotential model. The deterministic component was then removed of all the data set using the EGM96 model expanded to degree and order 360 calculated by the program GEOCOL16 (Tscherning, 1994).

The observed covariance function was calculated and then adjusted using the program COVFIT (Tscherning, 2001) according to the model suggested by Knudsen (1987). The LSC method was applied using the program COVAR V.1 written specially for this work to determine either the gravity anomaly (Δg) and the height anomaly (ζ).

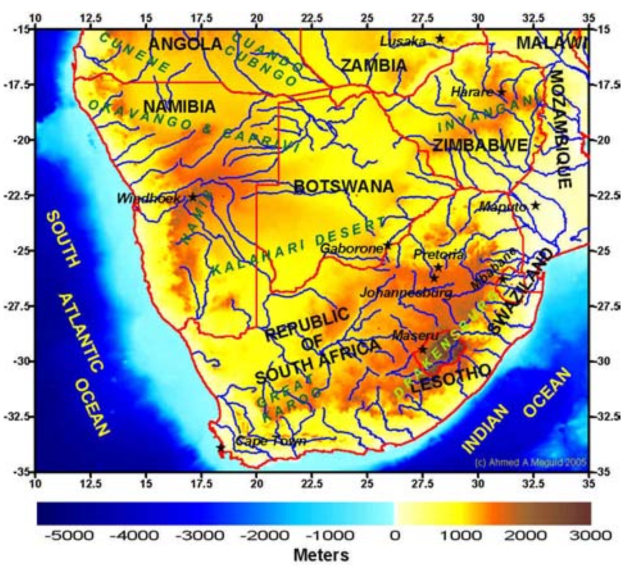

Figure 1 – Topographic map of the studied area.

Least-Squares Collocations (LSC)

Considering (I) to be a vector of observations associated with a generic element of the gravity field and is described by the form

$$I = LT \quad (1)$$

where L is a linear functional applied on the anomalous potential T, assuming a random error (n) contaminating all the data set, then the vector of observations becomes

$$I = s + n \quad (2)$$

where (s) represents the pure values of the elements after removing the error, therefore

$$s = AX + BT \quad (3)$$

Where (AX) represents the deterministic component and (BT) represents the stochastic component (Moritz, 1980) of the observations' vector, hence

$$I = AX + BT + n \quad (4)$$

The representation of the Earth's gravity field using heterogeneous data with a uniqueness condition is given by

$$\hat{T} = \left(\beta BK\right)^T \left(\beta C + \alpha D\right)^{-1} \left(I - AX\right) \quad (5)$$

Where α and β are numerical parameters, C and D are positive defined symmetric matrices and K is the nucleus of the Hilbert space.

The solution of such least squares problem depends on the estimation of an empirical covariance function calculated for the observations. That covariance is a function only of the spherical distance between each two elements of the observations' vector and, considering gravity anomaly only to form the vector of observations, is given by

$$C\left(\Psi\right) = \mathrm{cov}_{\Psi}\left\{\Delta g\right\} \quad (6)$$

Where ψ is the spherical distance and Δg is the gravity anomaly.

The covariance in some distance interval is the mean value of all the elementary products localized in that interval, hence

$$\mathrm{cov}_s\left\{\Delta g\right\} = M\left\{\Delta g_P \bullet \Delta g_Q\right\} \quad (7)$$

Where M{.} is the mean value calculated using all the point pairs P and Q separated by distance *s*.

After the modelling and the propagation of the covariance function using the method described by Knudsen (1987), the LSC is applied to the data set representing the gravity field as follows

$$S = C_{sl} C_{ll}^{-1} \left(I - AX\right) \quad (8)$$

where C_{sl} is the vector of the covariance between the desired signal to be calculated and each element of the observation, C_{ll} is the matrix of the covariance among the elements of the observations. The root-mean square (rms) associated to that signal is given by

$$\sigma_s = \sqrt{\left|C_{ss} - C_{sl} C_{ll}^{-1} C_{ls}\right|} \quad (9)$$

Where C_{ss} is the adjusted variance of the signal.

Model Calculations

The covariance function of the free air anomaly was calculated using the stochastic component of the data set. Later, such function was adjusted using the procedure suggested by Knudsen (1987).

There is an important relationship that permits the propagation of the associated covariance of any element of the gravity field to another using linear transformations of the anomalous potential function. Using these transformations, one can calculate the covariance function associated with the height anomaly propagating the covariance function associated with the free air anomaly. This fact forms the base of the calculations of the gravimetric quasi-geoid by means of the LSC method (Moritz, 1980; Knudsen, 1987).

The covariance functions for the free air anomaly are shown in figure (2). The agreement between the observed and the adjusted covariance functions has been detected to continue until reaching distance of 0.5°. That observation was taken as a guiding factor to choose the data search radius, which was taken to be 0.5° as well, which agrees also with the removed deterministic component of the model EGM96 expanded to degree and order 360. The propagation of the covariance associated with the free air anomaly resulted in the cross covariance function and the covariance function associated with the height anomaly.

Using these covariance functions, both the free air anomaly model and the height anomaly model were calculated for the region using formula (8). The rms was calculated for both quantities using formula (9).

Model Structure

The new digital model SAGM2005 was calculated, either for free air anomaly (figure 3) and height anomaly (figure 4), in 50,000 points with spatial resolution of 0.1°. The rms was not homogeneously distributed over the area and varied between 11 mGal (continental areas without data) and 3 mGal (well-covered continental areas) for free air anomaly (figure 5) and between 0.25 m (continental areas without data) and 0.1 m (well-covered continental areas) for height anomaly (figure 7).

Results and Conclusions

While comparing the new digital model SAGM2005 with other existing models that represent the same quantities, it shows a high agreement level, either in case of free air anomaly (with the data set used to calculate the model and with the model EGM96 expanded to degree and order 360) and in case of height anomaly (with the height anomaly of the model AGP2003 (Merry, 2003)).

Some important features were observed over the rms distribution map for both the calculated quantities:

- The greatest rms values were localized where there had no gravimetric data available specially over the mountain ranges of Drakensberg (Republic of South Africa and Lesotho) and the Inyangani Heights (Zimbabwe).

- Also, the rms approached its maximum value over the desert areas (Kalahari Desert and Bié plateau).

- The rms of the oceanic part of the studied area, where the GRACE-only data were introduced homogeneously, was homogeneously distributed in case of height anomaly, whereas, in case of free air anomaly, there were some deviations of the homogeneity.

We suggest, in order to get better results in similar future representations, the usage of highly homogeneous land observations over the target area and, if it was necessary, filling over the gaps with an adequated geopotencial model expanded to high degree and order to guarantee the existence of the anomalies of short wavelengths.

It is strongly recommended, also, the careful choice of the search radius used to calculate the model, which forms, together with the model used for such search process, one of the most important factors that could, once chosen correctly, reduce better results.

Acknowledgments

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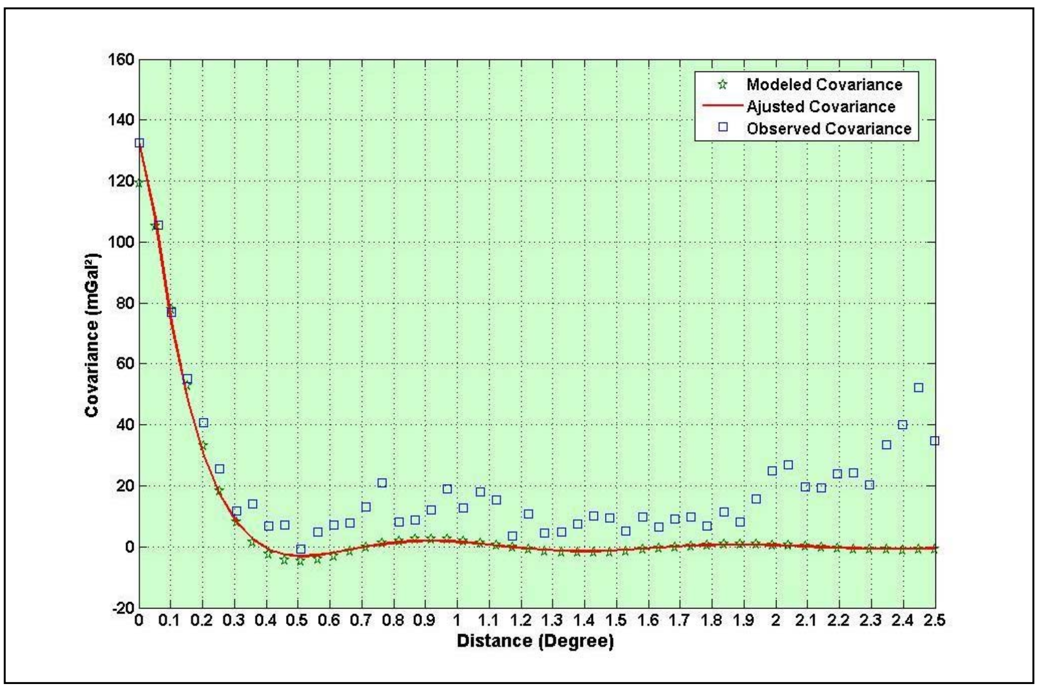

Figure 2 – The observed covariance function – in blue squares, together with the adjusted covariance function – in red solid line – and the modeled covariance function – in green pentagrams, of the free air anomaly.

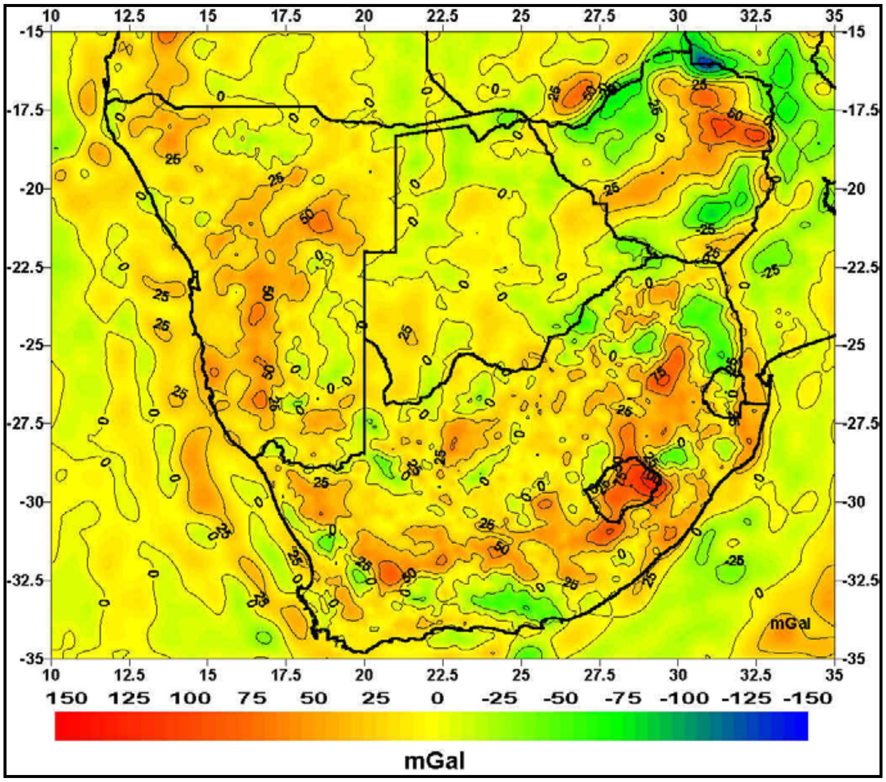

Figure 3 – The free air anomaly of the model SAGM2005 (contour interval = 25 mGal).

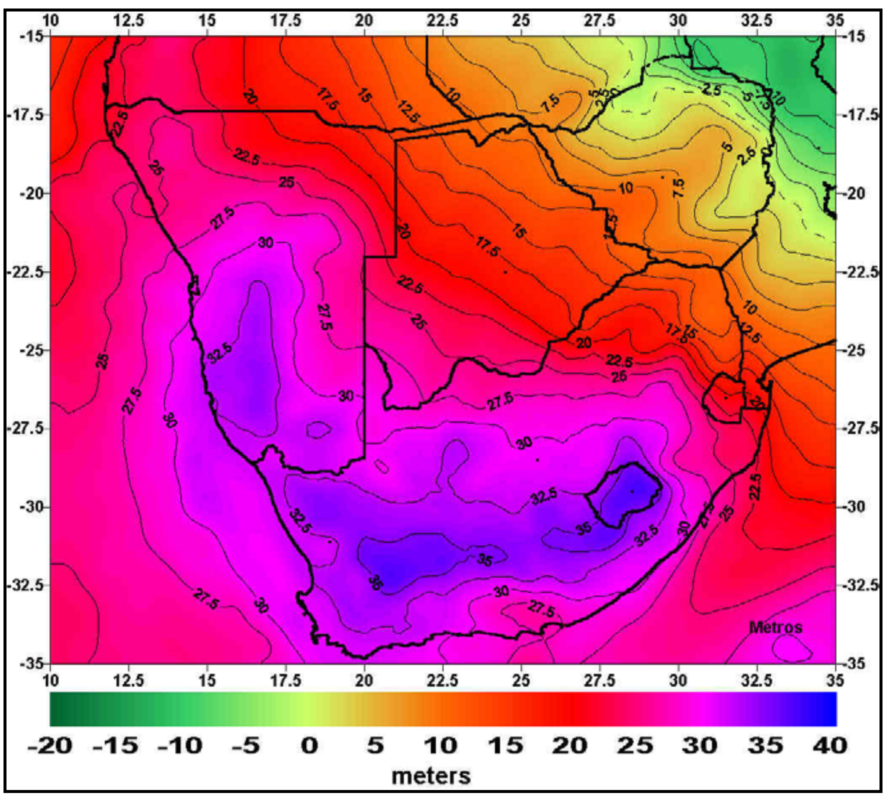

Figure 4 – The height anomaly of the model SAGM2005 (contour interval = 2.5 m).

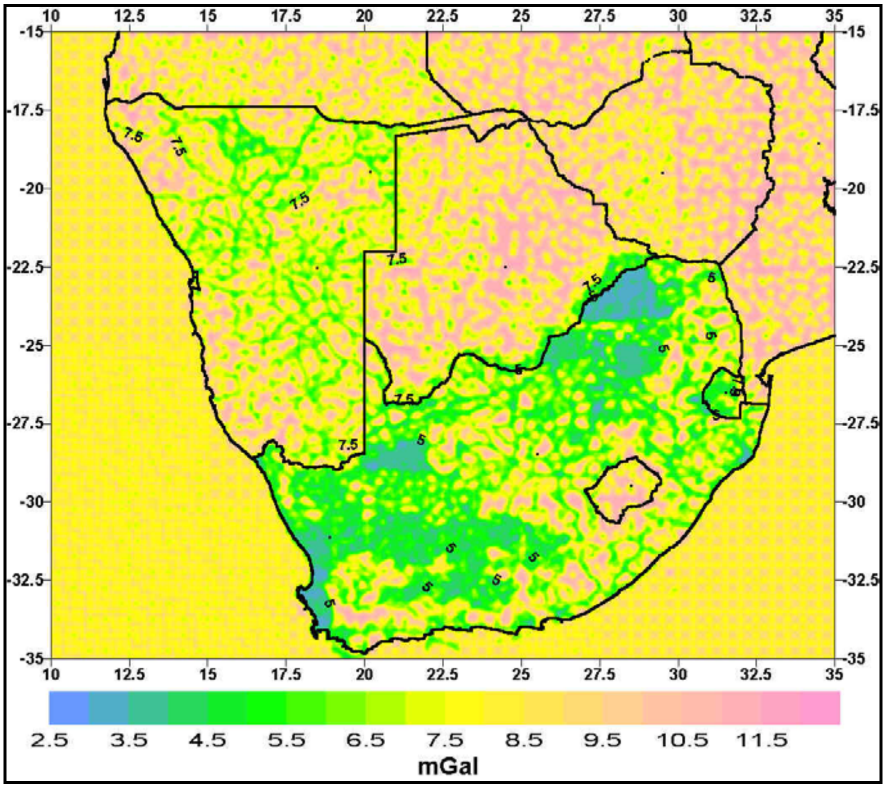

Figure 5 – The rms associated with the free air anomaly of the model SAGM2005.

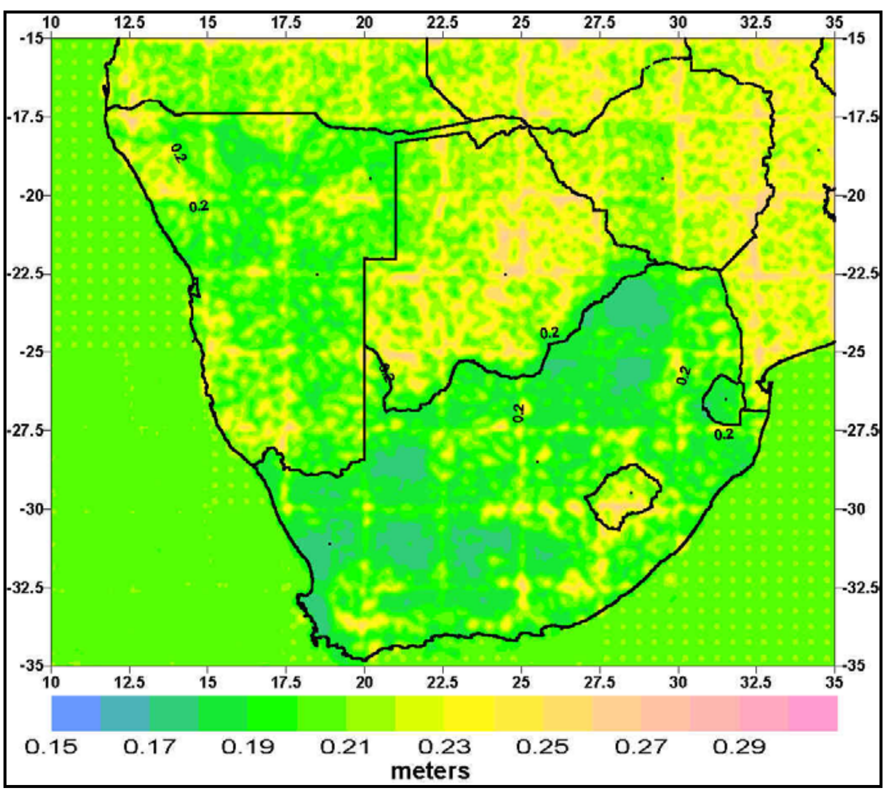

Figure 6 – The rms associated with the height anomaly of the model SAGM2005.