Numerical analysis of electromagnetoelastic waves propagation in heterogeneous media

Lucas A. P. Gomes*, Viatcheslav Priimenko, Mikhail Vishnevskii, LENEP/UENF, Brazil

Abstract

Interaction of electromagnetic fields with elastic bodies is the subject of many theoretical investigations in mechanics of continua for the last decades. Some variants of direct and inverse problems have been studied leading into the determination of some characteristics of a medium. It is very interesting to study this phenomenon due to the possibility of applying this theory to geophysical prospecting and study of earthquake sources. It is well known that when an electrical-conducting elastic body oscillates in an electromagnetic field, variations of the electrical and magnetic fields are observed as a result of this motion. Similar processes are also observed when seismic waves propagate in the Earth's crust. Variations of the seismic and electromagnetic fields arising in this case are called **electromagnetoelastic waves**. Our work is dedicated to investigation of such interaction. The model considered here is based on a simple variant of combination of the Lamé and Maxwell equations. We form the basic equations for the description of some variant of this coupling and consider mathematical and numerical models of this process.

Introduction

Study of electromagnetoelastic interactions is interesting from the viewpoint of application of this effect to continuum physics and geophysical exploration. There are many works in which direct and inverse problems arising in investigation of such phenomena are studied, see Eringen and Maugen, 1991; Avdeev, Priimenko and Vishnevskii, 2004. Basically, we may divide the electro-magnetoelastic interactions into interactions based on electrokinetic effect, piezoelectric effect, and seismo-magnetic effect. We consider the last one. The electro-magnetoelastic field arising as result of such interaction is defined as local geomagnetic variations propagating simultaneously with the seismic field. In this case the dominant frequency and the velocity of such electro-magnetoelastic waves are equal to the frequency and the velocity of seismic waves. The electromagnetic waves produced by this effect are transversal as any electromagnetic wave is, but they propagate with the same velocity of the longitudinal seismic waves that produce quasi-stationary state of the seismomagnetic effect. Such waves contain information about the electromagnetic and seismic properties of the medium.

Mathematical model

The phenomena of seismomagnetic induction in the constant magnetic field of the Earth are described mathematically by the solution of the coupled electromagnetic and elastic equations considered in the quasi-stationary electromagnetic state. The interaction of electromagnetic fields with deformable media is considered with point of view of linear elasticity connected with electrodynamics of elastic moving media by means of motion of particles in the electromagnetic field. We do not consider any effects of interactions, which could arise as a result of some kind of relations in the constitutive equations besides velocity. The basic equations governing the electrodynamics process are the following ones, see Dunkin and Eringen, 1963

$$\begin{aligned} &\nabla\times\vec{H} = \sigma\vec{E} + \sigma\mu_e\vec{U}_t\times\vec{H} + \vec{J}, \\ &\nabla\times\vec{E} = -\mu_e\vec{H}_t,\ \nabla\bullet(\mu_e\vec{H}) = 0. \end{aligned} \tag{1}$$

For describing of the elastic waves propagation we use the following system

$$\rho\vec{U}_{tt} = \nabla\bullet T(\vec{U}) + \mu\nabla\times\vec{H}\times\vec{H} + \vec{F}, \tag{2}$$

where $T(\vec{U})$ is the tensor with components $T_{i,j}, 1 \leqslant i,j \leqslant 3,$ defined by the formulas

$$T_{i,j} = \lambda\nabla\bullet\vec{U}\delta_{i,j} + \mu(U_{i,x_j} + U_{j,x_l}). \tag{3}$$

Here $\vec{E} = (E_1, E_2, E_3)$ is intensity of the electrical field and $\vec{H} = (H_1, H_2, H_3)$ is intensity of the magnetic field, $\vec{U} = (U_1, U_2, U_3)$ is displacement of the points of the medium from the reference configuration, $\sigma, \mu_e, \rho, \lambda, \mu$ denote the electroconductivity, the magnetic permeability, the density of medium and the parameters of Lamé, respectively, $\vec{J} = (J_1, J_2, J_3), \vec{F} = (F_1, F_2, F_3)$ are sources of the electromagnetic and elastic oscillations, and $\delta_{i,j}$ is the Kronecker symbol. Constant vector $\vec{H}_0$ characterizes the constant magnetic field of the Earth. It is possible to show that a 1-D non-dimensional variant of system (1)-(3) can be formulated in the following form, see Priimenko and Vishnevskii, 2005

$$
\begin{aligned}
&h_t = (rh_z - hu_t - rj)_z,\\
&u_{tt} = (v^2 u_z)_z - phh_z + f, \qquad (4)\\
&e = rh_z - hu_t,
\end{aligned}
$$

where h, e, u, f, j are dimensionless analogues of the functions H_1, E_2, U_3, F_3, J_3, respectively; $r^{-1} = \mu_e L V_0 \sigma$ is the magnetic Reynolds's number, $p = \mu_e H_0^2 \rho^{-1} V_0^{-2}$, $v = v_p V_0^{-1}$ and $v_p = \sqrt{(\lambda + \mu)/\rho}$ is the velocity of longitudinal waves propagation. Here L, V_0, H_0 are characteristic values of length, seismic velocity and the magnetic field of the Earth, respectively.

The problem statement

Now we are able to formulate the basic problem, which will be studied. Consider in the cylinder $Q_T = \Omega \times (0,T)$, $\Omega = (-1,1)$, equations

$$
\begin{aligned}
&h_t = (rh_z - hu_t - rj)_z,\\
&u_{tt} = (v^2 u_z)_z - phh_z + f, \qquad (5)
\end{aligned}
$$

where $r(z), v(z)$ are positive piecewise smooth functions, and p is a positive number. The following initial boundary-value problem is considered for equations (5) with initial data

$$
\begin{aligned}
&h(z,0) = h_0(z), z \in \Omega,\\
&u(z,0) = u_0(z), u_t(z,0) = u_1(z), z \in \Omega, \qquad (6)
\end{aligned}
$$

and boundary conditions

$$
\begin{aligned}
&h(-1,t) = h(1,t) = 0, t \in (0,T),\\
&u(-1,t) = u(1,t) = 0, t \in (0,T). \qquad (7)
\end{aligned}
$$

This initial boundary-value problem can be considered as a diffraction problem for parabolic-hyperbolic system (5). The following transmission conditions are assumed in points of discontinuity $z = z_k$: *physical absence of discontinuities of a medium and equilibrium of effective forces*. Mathematically it means fulfillment of the following conditions in the points of discontinuity

$$
\begin{aligned}
&[h(z,t)] = [u(z,t)] = 0,\\
&[r(z)h_z(z,t)] = [v^2(z)u_z(z,t)] = 0. \qquad (8)
\end{aligned}
$$

Here $[\cdot]$ means the jump of corresponding function across the discontinuity point. Priimenko and Vishnevskii, 2005, proved the existence and uniqueness theorems of the solution of problem (5)-(8).

Numerical simulation

Knopoff, 1955, has studied the influence of electro-magnetic fields on the propagation of elastic waves and arrived at the conclusion that in the class of geophysical problems the effect of electromagnetic phenomena on the process of elastic waves propagation is negligible, at least in the case of not too large external electromagnetic fields. It means that in equations (5) we can put $p = 0$.

Consider a simple variant of the model when electromagnetoelastic waves, propagated in the constant magnetic field of the Earth, are generated by the seismic source only, i.e., the initial data of the problem and the source of electromagnetic oscillations are taken equal to zero

$$h_0 = u_0 = u_1 = j \equiv 0.$$

These conditions and the assumption that the electro-magnetoelastic waves are propagated in the constant magnetic field make possible to reformulate problem (5)-(8) in the following form

$$
\begin{aligned}
&h_t = (rh_z - hu_t - u_t)_z, (z,t) \in Q_T,\\
&u_{tt} = (v^2 u_z)_z + f, (z,t) \in Q_T,\\
&h(z,0) = u(z,0) = u_t(z,0) = 0, z \in \Omega,\\
&h(\pm 1,t) = u(\pm 1,t) = 0, t \in (0,T),\\
&z = z_1 : [h] = [u] = [rh_z] = [v^2 u_z] = 0, t \in (0,T).
\end{aligned}
$$

For such case there were done some numerical simulations with aim to understand the level of influence of the parameters of the model on the magnetic response. As the elastic source there was used the second derivative of the Gaussian wavelet:

$$f(t) = \left[1 - 2\pi(\pi f_c t)^2\right] e^{-\pi(\pi f_c t)^2},$$

where $f_c = \omega_s / 3\sqrt{\pi}$ is the dominant frequency.

Numerical modeling was made with the following characteristic values

$$
\begin{aligned}
&\omega_s = 50Hz, \mu_e = 1.26 \times 10^{6} H/m, H_0 = 40A/m,\\
&L = 20m, \rho = 2650kg/m^3, V_0 = 2500m/s.
\end{aligned}
$$

Examples and Results

With aim to analyze the variations of magnetic field h caused by propagation of the elastic wave u, there were made some tests divided in several stages. On the first stage we calculated the magnetic field, changing position of the discontinuity point z_1 and considering the electrical conductivity σ and the seismic velocity v as fixed ones. On the second stage the magnetic field was calculated when we changed the values of the electrical conductivity and the seismic velocity, considering the discontinuity point as fixed one. And finally, an analysis of the magnetic field was done in the frequency domain. The non-dimensional values of the electrical conductivity and the seismic velocity are given in Table 1.

Table 1 - First experiment, case of the discontinuity point variation only.

	First Case	Second Case
σ	1.0, for $0 \le z \le 0.3$ 2.0, for $0.3 < z \le 1.0$	1.0, for $0 \le z \le 0.7$ 2.0, for $0.7 < z \le 1.0$
v	1.0, for $0 \le z \le 0.8$ 1.4, for $0.8 < z \le 1.0$	1.0, for $0 \le z \le 0.3$ 1.4, for $0.3 < z \le 1.0$

The results obtained in the first experiment are given in Figure 1.

The non-dimensional values of the electrical conductivity and the seismic velocity used in the second experiment are represented in Table 2.

Table 2 - Second experiment, case of variation of σ , v only.

	First Case	Second Case
σ	1.0, for $0 \le z \le 0.3$ 2.0, for $0.3 < z \le 1.0$	1.5, for $0 \le z \le 0.3$ 1.0, for $0.3 < z \le 1.0$
v	1.0, for $0 \le z \le 0.8$ 1.4, for $0.8 < z \le 1.0$	1.2, for $0 \le z \le 0.8$ 1.0, for $0.8 < z \le 1.0$

The results obtained are given in Figure 2. On the final stage we analyzed the amplitude spectrum of magnetic field, calculated with the values, used on previous stages. Figure 3 represents the results for the first case and Figure 4 represents the results for the second case.

Conclusions

Numerical modeling of the electromagnetoelastic effect pointed at the possibility to record the variations of magnetic field, generated by the propagation of seismic waves. Comparison of the amplitude variations of magnetic field showed that the influence of the coefficients variations becomes apparent stronger than the influence of the variations of discontinuity point position. The same situation we observe analyzing the amplitude spectrum of magnetic response. It makes possible to consider some inverse problems of the determination of the electromagnetic and elastic properties of the medium using registered magnetic field only.

Acknowledgments

The first author is grateful to PIBIC/UENF for financial support.

References

Avdeev, A.V., Priimenko, V.I., and Vishnevskii, M.P., 2004. Mathematical problems of electromagnetoelastic interactions. Series: Mathematical Modeling in Geo-physics, Special issue of NCC Bulletin: 9 (2004), 1-90.

Dunkin, J.W., and Eringen, A.C., 1963. On the propagation of waves in an electromagnetic elastic solid, Intern. J. Eng. Sci., 1., 461-495.

Eringen,A.C., and Maugen, G.A., 1991. Electrodynamics of continua. Vol. I, II. Springer Verlag, Berlin, Germany.

Knopoff, L., 1955. The interaction between elastic waves motions and a magnetic field in electrical conductors. J. Geophys. Res., 60, 441-456.

Priimenko, V.I., and Vishnevskii, M.P., 2005. An inverse problem of electromagnetoelasticity in the case of complete nonlinear interaction. J. of Inverse and Ill-posed Problems, vol.13, No.3,1-26.

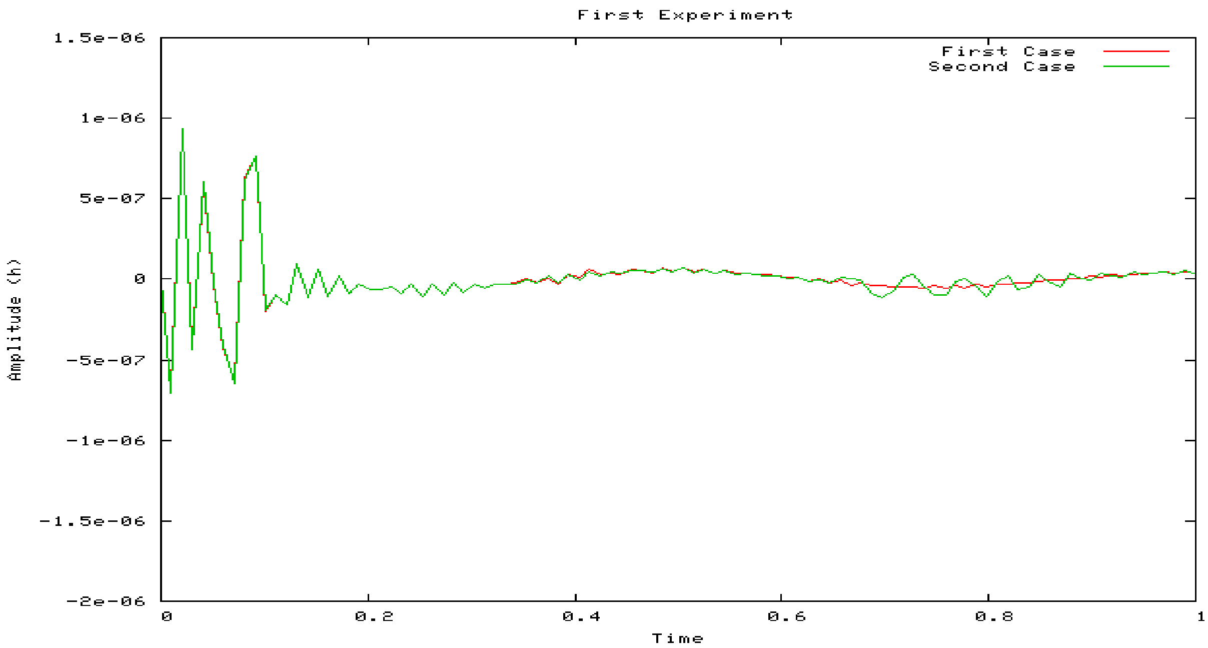

Figure 1 - Comparison of the amplitude variations of magnetic field for different positions of the discontinuous point.

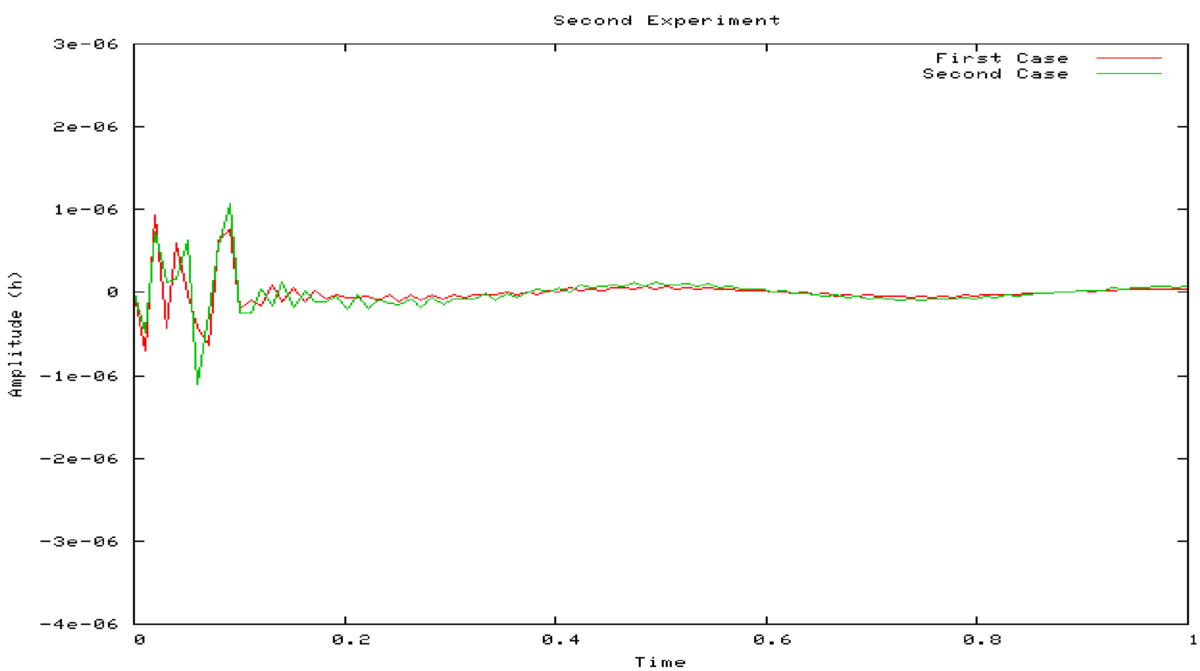

Figure 2 - Comparison of the amplitude variations of magnetic field for different values of σ , V .

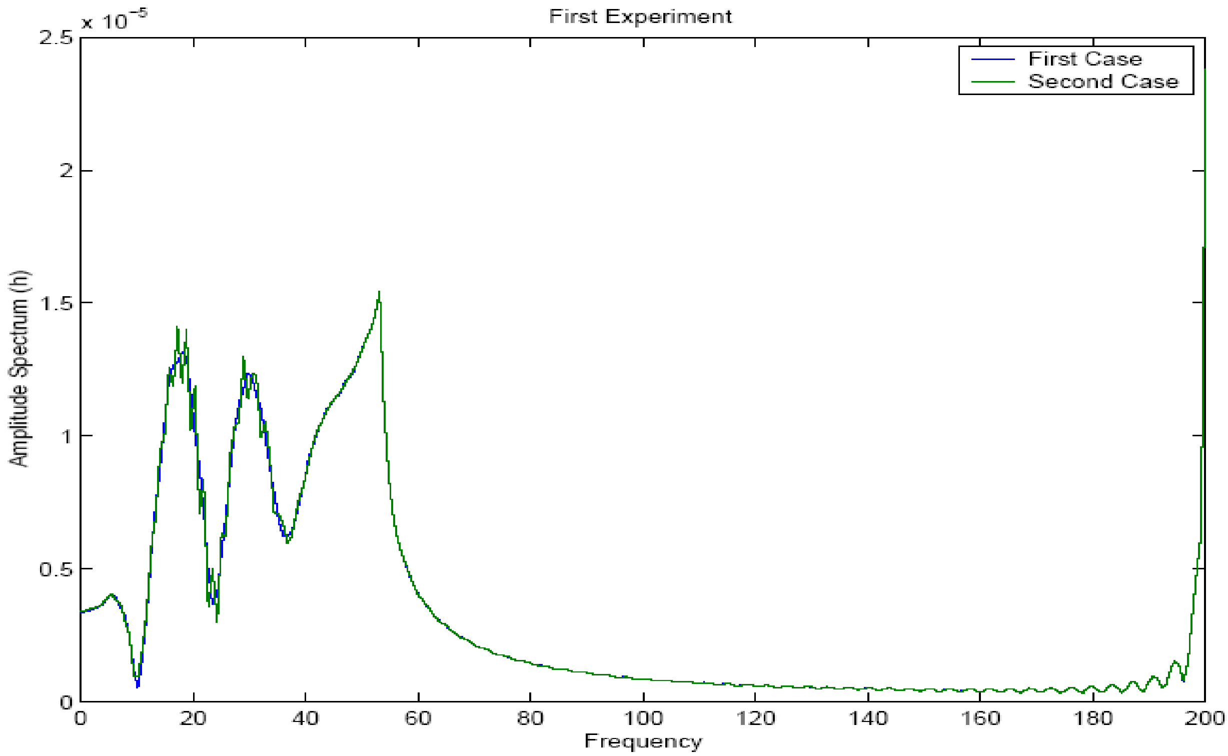

Figure 3 - Amplitude spectrum of the magnetic field for different positions of the discontinuous point.

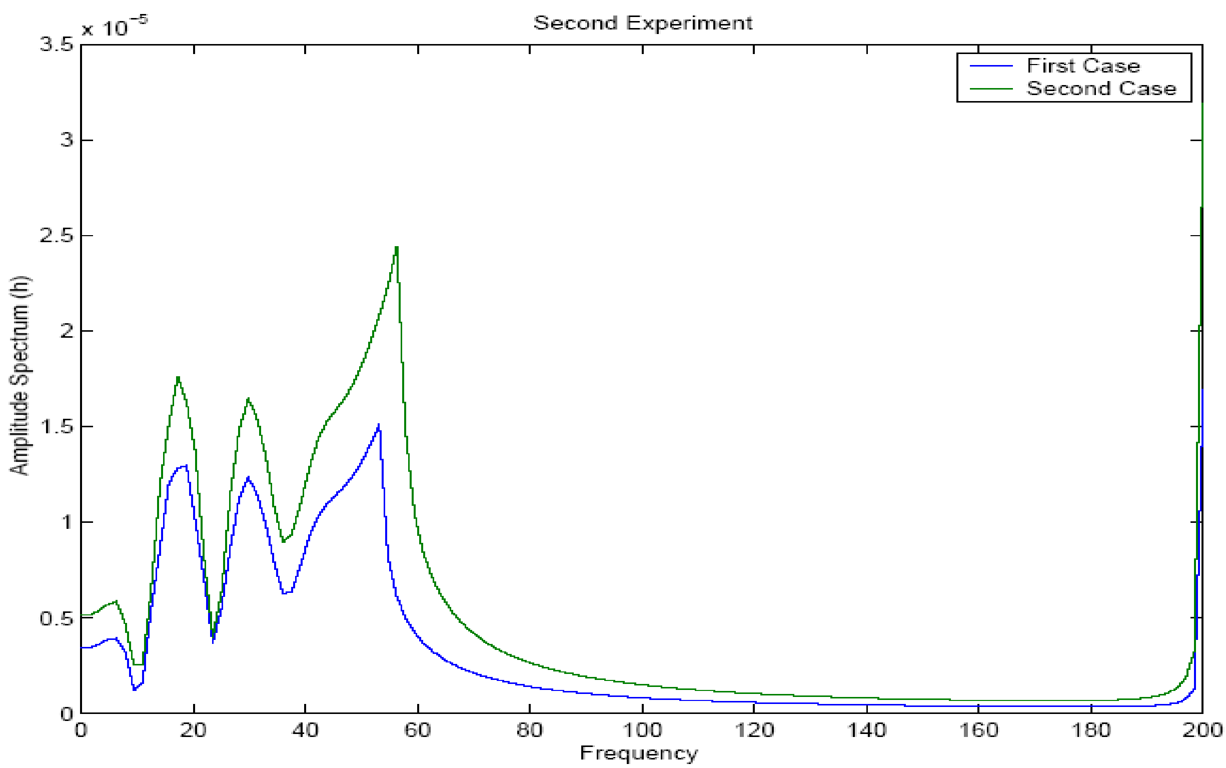

Figure 4 - Amplitude spectrum of the magnetic field for different values of σ , V .