Chaos and intermittency in interplanetary Alfvén waves

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Abstract

In this paper we present an investigation of intermittency in interplanetary Alfvén waves modeled by the DNLS equation. We show how the fluctuations of the magnetic field can evolve from periodic to chaotic dynamics, where two types of intermittency are identified: type-I Pomeau-Manneville and crisis-induced intermittency. The characteristic intermittency time follows a well defined power-law as a function of the plasma viscosity.

1. Introduction

The interplanetary space is a highly conducting, essentially collisionless plasma, with approximate equalities of energy between the thermal densities and magnetic field (Parker, 1991). The strong correlation between the fluctuations of the ambient magnetic field and plasma velocities suggest the presence of Alfvén waves in the interplanetary solar wind.

Alfvén waves are low-frequency electromagnetic plasma modes which propagate at the Alfvén velocity $\upsilon_A = B_0/(\mu_0\rho_0)^{1/2}$, where B_0 is the strength of the ambient magnetic field, μ_0 is the permeability of vacuum and ρ_0 is the average mass density of the plasma. From a linear analysis of MHD equations, the dispersion relation of the Alfvén wave is found as $\omega = k_{//}\upsilon_A$, where $k_{//}$ is the component of the wave vector $\mathbf{k}$ parallel to B_0. The wave electric field $\mathbf{E}$ is perpendicular to $\mathbf{B_0}$ and lies in the ($\mathbf{B_0}$, $\mathbf{k}$) plane. The perturbation of the fluid velocity $\mathbf{u}$ relates to the magnetic field's perturbation vector $\mathbf{b} = \delta\mathbf{B_0}$ by $\mathbf{u} = \pm\mathbf{b}/(\mu_0\rho_0)^{1/2}$, where the upper (lower) sign refers to the case $\mathbf{k}\cdot\mathbf{B_0} < 0$ ($\mathbf{k}\cdot\mathbf{B_0} > 0$). Thus, $\mathbf{u}$ and $\mathbf{b}$ are parallel and proportional to each other, and the plasma oscillates with the magnetic field lines (Benz, 1993; Bittencourt, 1995; Stasiewicz et al., 2000). Alfvén waves play an important role in space plasmas, since they can heat solar corona plasmas, and accelerate particles in auroral and solar plasmas (Cargill, 2000; Stasiewicz et al., 2000; Del Zanna and Valli, 2002).

Following Hada et al. (1990), Buti (1992), Chian et al. (1998, 2002a) and Rempel et al. (2004), we adopt here a low-dimensional model of Alfvén waves by seeking stationary solutions of the derivative nonlinear Schrödinger equation (DNLS) in the driver frame. This model enables us to acquire a clear insight of the nonlinear dynamical behavior of Alfvén systems which are difficult to obtain in the high-dimensional analysis of a partial differential equation. Stationary solutions can play a fundamental role in the full spatiotemporal solutions of the system, as demonstrated by He (1998) and He and Chian (2003) for a nonlinear drift wave equation.

Recent analyses of velocity and magnetic field fluctuations in solar wind data indicate that the solar wind plasma is strongly intermittent (Bruno et al., 2001; Bruno et al., 2003). Intermittent events are characterized by time series that display time intervals with low variability interrupted by bursts of very high variability (Bruno et al., 2001). We identify two types of intermittency in Alfvén waves modeled by the DNLS equation. In the Pomeau-Manneville type-I intermittency the fluctuations of the magnetic field display phases of approximately periodic behavior (laminar phases) interrupted by phases of chaotic behavior (bursty phases). In the crisis-induced intermittency, the magnetic field exhibits an interplay between phases of weakly chaotic oscillations ("laminar" phases) and strongly chaotic oscillations (bursty phases).

2. The derivative nonlinear Schrödinger equation

The nonlinear dynamics of a large-amplitude Alfvén wave propagating along an ambient magnetic field in the x-direction can be described by the derivative nonlinear Schrödinger equation:

$$\partial_t b + \alpha\partial_x\left(|b|^2 b\right) - i\left(\mu + i\eta\right)\partial_x^2 b = S(b,x,t), \qquad (1)$$

where η is the dissipative scale length, $b = b_y + ib_z$ is the complex transverse wave magnetic field normalized to the constant ambient magnetic field B_0, time t is normalized to the inverse of the ion cyclotron frequency $\omega_{ci} = eB_0/m_i$, position x is normalized to $c_A/\omega c_i$, c_A is the Alfvén velocity, $\alpha = 1/[4(1 - \beta)]$, $\beta = c_S^2/c_A^2$, $c_S = (P_0/\gamma\rho_0)^{1/2}$ is the acoustic velocity and μ is the dispersive parameter. The form of the driving force $S(b, x, t)$ can be chosen to model wave growth resulting from an external source or an instability. We assume $S(x, t) = A\ exp(ik\o)$ to be a monochromatic circularly polarized wave with a wave phase $\o = x - Vt$, where V is a constant wave velocity, A and k are real constants.

Seeking stationary solutions with $b = b(\o)$, and setting $\partial_t b = 0$, the first integral of Eq. (1) reduces to a low-dimensional system of coupled ordinary differential equations:

$$\dot{b}_y - \nu \dot{b}_z = \partial H/\partial b_z + a\cos\theta, \quad (2)$$

$$\dot{b}_z + \nu \dot{b}_y = -\partial H/\partial b_y + a\sin\theta, \quad (3)$$

$$\dot{\theta} = \Omega, \quad (4)$$

where $H = (\mathbf{b}^2 - 1)^2/4 - (\lambda/2)(\mathbf{b} - \hat{\mathbf{e}}_y)^2$, the upper dot denotes derivative with respect to the phase variable $\tau = \alpha b_0^2 \o \mu$, $\nu = \eta/\mu$ is the normalized dissipation parameter, $b \rightarrow b/b_0$ (where b_0 is an integration constant), $\mathbf{b} = (b_y, b_z)$, $\tau = \Omega\phi$, $\Omega = \mu k/(\alpha b_0^2)$, $a = A/(\alpha b_0^2 k)$, $\lambda = -1 + V/(\alpha b_0^2)$. We assume $\beta < 1$, hence $\alpha > 0$.

In order to study the nonlinear dynamics of the system (2)-(4), we define a stroboscopic Poincaré map:

$$P : [b_y(\tau), b_z(\tau)] \rightarrow [b_y(\tau + T), b_z(\tau + T)], \quad (5)$$

where $T = 2\pi/\Omega$ is the driver period.

Thus, one iteration of the Poincaré map, P(b(t)), corresponds to integrating Eqs. (2)-(4) from time τ to time $\tau + T$. This type of projection defined in fixed time intervals is called stroboscopic projection, or time-T map. In the following sections we use $\tau = 0$ for the initial phase and generate trajectories by plotting one Poincaré point at each value of $\tau + nT$, $n = 1, 2, \ldots$

3. Nonlinear analysis of the DNLS equation

The results discussed in this section were obtained by numerically integrating Eqs. (2)-(4) and varying the dissipation parameter ν while keeping the other parameters constant, with $\lambda = 1/4$, $\mu = 1/2$, $a = 0.3$ and $\Omega = -1$, thus the external driver is left-hand circularly polarized. The choice of parameter values is based on Chian et al. (1998, 2002a).

Figure (1.a) shows the bifurcation diagram of the Alfvén attractor as a function of ν (Rempel and Chian, 2004). For this range of ν, the attracting set can be either chaotic or periodic. A periodic attractor is seen in the Poincaré map as a finite set of points that are periodically revisited by the flow of b. In a chaotic attractor the magnetic field oscillates irregularly in a bounded region of the phase space, never repeating its behavior. SNB denotes the saddle-node bifurcation that takes place at $\nu_{SNB} \approx 0.07738$ and IC denotes the interior crisis that occurs at $\nu_{IC} \approx 0.06212$. The saddle-node bifurcation at ν_{SNB} marks the beginning of a *periodic window* in the bifurcation diagram. In the saddle-node bifurcation the simultaneous creation of a period-2 (p-2) attractor and a p-2 unstable periodic orbit occurs. The dashed lines in Fig. (1.a) represent the unstable periodic orbit (UPO) created at ν_{SNB}. As the value of ν is decreased, the p-2 attractor undergoes a cascade of period-doubling bifurcations, in each of which the period of the attractor is doubled. As the period tends towards infinity, a chaotic attractor is formed in two separate bands in the bifurcation diagram, called *band region* (B), following references (Szabó et al., 1996) and (Szabó et al., 2000). At $\nu = \nu_{IC}$ the chaotic attractor collides with the p-2 UPO created at SNB, called the *mediating unstable periodic orbit* (MPO). The collision is responsible for an interior crisis, which is a sudden enlargement in the size of a chaotic attractor (Grebogi et al. 1983).

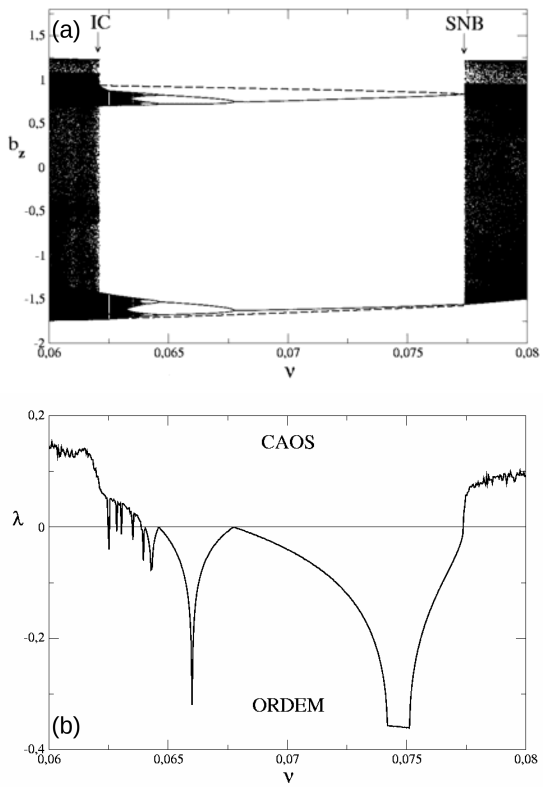

Figure 1 (a) Bifurcation diagram of the attracting set in a period-2 window. IC denotes interior crisis and SNB denotes saddle-node bifurcation. The dashed lines denote the p-2 unstable periodic orbit. (b) Variation of the maximum Lyapunov exponent (λ_{max}) of the attracting set as a function on ν.

In Fig. (1.b) we plot the variation of the maximum Lyapunov exponent (λ) of the Alfvén attractor, as a function of ν. Positive values indicate the presence of a chaotic attractor, and negative values indicate that the attractor is periodic. Note that λ jumps abruptly at ν_{SNB} and ν_{IC}, indicating a sudden increase in the attractor's chaoticity.

Figure (2) shows the transition from order to chaos at the SNB. Figures (2.a), (2.c) and (2.e) display, respectively, the time series, stroboscopic time series and power spectrum at $\nu = 0.07738$, where we can see a laminar behavior due to the p-2 Alfvén periodic attractor. The power spectrum of this periodic time series is characterized by discrete peaks. However, Figs. (2.b) and (2.d), at $\nu = 0.0780$, show the occurrence of intermittency just to the right of SNB. This intermittency is know as type-I Pomeau-Manneville intermittency (Pomeau and Manneville, 1980), and it can be clearly seen in Fig. (2.b) that the Poincaré points show bursts interrupting laminar periods that resemble the p-2 attractor of Fig. (2.c). Figure

(2.f) displays the power spectrum of the chaotic time series in log-log scale. This broadband spectrum reflects the excitation of new wave frequencies at the onset of chaos.

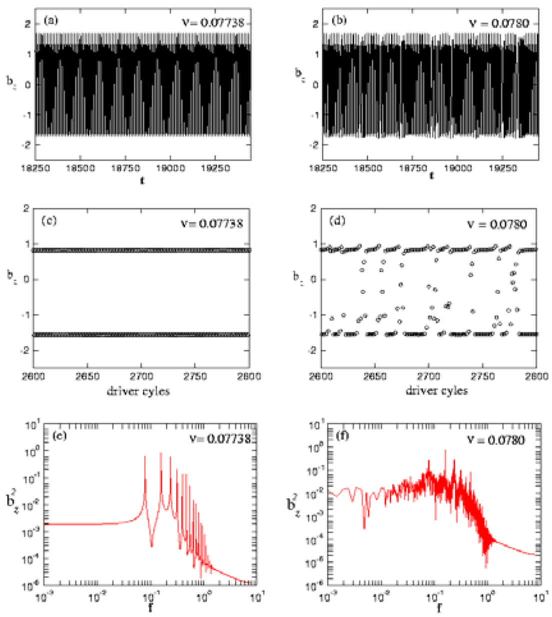

Figure 2 Transition from order to chaos in SNB: (a) and (b) time series, (c) and (d) stroboscopic time series and (e) and (f) power spectra of the time series in log-log scale.

The Alfvén crisis-induced intermittency is characterized by time series containing weakly chaotic laminar phases that are randomly interrupted by strongly chaotic bursts. Figure (3) shows the transition from strong chaos to weak chaos at IC. In Figs. (3.a) and (3.b) we have the times series at $\nu = 0.0620$ ($\nu < \nu_{IC}$) and $\nu = 0.06213$ ($\nu > \nu_{IC}$), respectively. Figures (3.c) and (3.d) show the time series with driver cycles (Poincaré points). At $\nu = 0.0620$ there are energy bursts which can be seen in both Figs. (3.a) and (3.c). In Fig. (3.d) the values of b_z lie in the bands of the chaotic attractor shown in Fig. (1) and the bursty phase is not present. Thus, the interior is responsible for the onset of strong intermittent chaos. Finally, Figs. (3.e) and (3.f) show the power spectra for both cases. Both spectra are broadband, but the main peaks are more pronounced in Fig. (3.f) than in Fig. (3.e), since for $\nu = 0.06213$ the time series reflect a behavior which is closer to the periodic behavior of Fig. (2.a).

The average transient time between bursts in an intermittent time series, obtained from a series of Poincaré points, tend the decreases as decreases ν in the IC and increases ν in the SNB. Grebogi et at. (1987b) have shown that for a wide class of dynamical system the dependence of the average duration of the laminar phases with the distance of the control parameter from its critical value follows a power-law.

Intermittent time series can be characterized by the average time between bursts (average duration of laminar phases). Close to the critical values, ν_{IC} and ν_{SNB}, the laminar phases are long and their duration decreases as the dissipation is varied away from the critical value, and

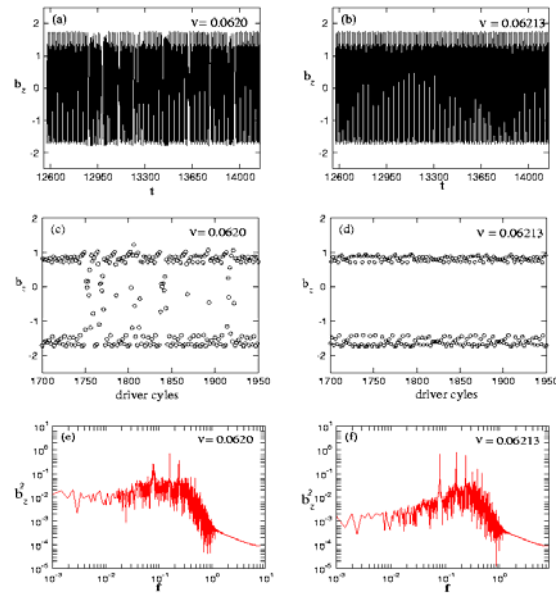

Figure 3 Transition from strong chaos to weak chaos in IC: (a) and (b) time series, (c) and (d) stroboscopic time series and (e) and (f) power spectra of the time series in log-log scale.

in the direction of the strongly chaotic regime. In Fig. (4.a) we show the characteristic time between bursts as a function of the distance of ν from the critical value ν_{IC} in loglog scale. The dots are the values computed from time series and the line with slope $\gamma \approx -0.78$ is a linear fit. In Fig. (4.b) the characteristic intermittency time for the type-I Pomeau-Manneville intermittency is shown to follow a power-law with characteristic exponent $\gamma \approx -0.62$.

4. Discussion and concluding remarks

In this paper we showed the onset of intermittent chaos in nonlinear Alfvén waves modeled by the DNLS equation. We presented the occurrence of two types of intermittency: type-I Pomeau-Manneville and crisis-induced intermittency. Also, we reported changes in the power spectra obtained from the times series during the transition from order to chaos (in the saddle-node bifurcation) and from weak chaos to strong chaos (in the interior crisis). The average time between bursts in the intermittent time series follows a well defined power-law as a function of the plasma viscosity.

Although we have exemplified our analysis using the stationary solutions of the DNLS equation, the bifurcation reported in this paper (saddle-node bifurcation, period-doubling and interior crisis) are ubiquitous nonlinear dynamical phenomena in low and high-dimensional systems (Chian et al., 2002b; Rempel and Chian, 2003).

5. Acknowledgments

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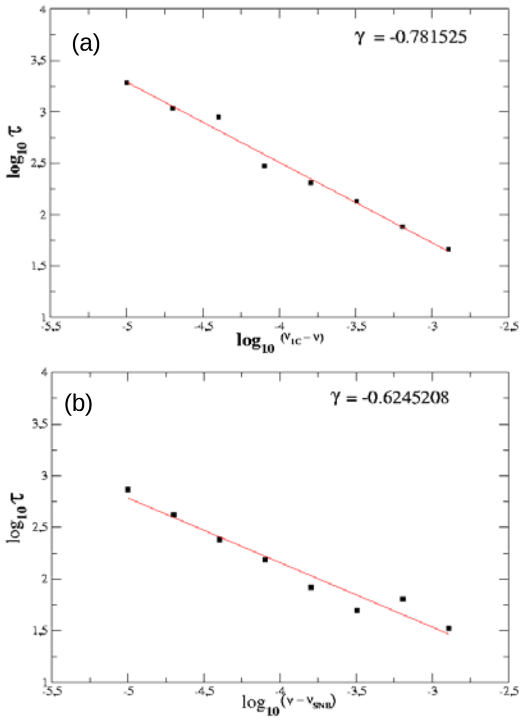

Figure 4 Characteristic intermittency time for the time series (a) crisis-induced intermittency and (b) type-I Pomeau-Manneville intermittency.

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