Velocity-independent quality factor inversion based on the frequency centroid shift: basic theory

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Abstract

We present an approach for *Q*-quality factor inversion which is based on the frequency centroid temporal shift and uses the S-transform to monitor this evolution. This method relies solely on the pulse deformation and can be applied directly on the trace. It does not require amplitude modeling and as consequence it does not requires either the knowledge of the field velocity as far as the following condition is met: for a given layer, both pulses reflected at the base and top of this layer have approximately the same ray path along the upper layers. To implement this inversion approach, it is required a reference pulse as, for example, a pulse reflected at a known reference interface which can be extracted from the trace itself. The proposed inversion methodology can be applied either to GPR or seismic data.

Introduction

Obtaining quality factor (*Q*) estimates is of paramount importance to improve the quality of seismic and GPR images because these estimates can be used to perform inverse propagation operators which restore the original spectra of the signals. In addition, *Q* estimates obtained from seismic data may also have importance for petrophysical characterization of hydrocarbon reservoirs because they can be used for fluid diagnostic. However, obtaining *Q* estimates directly from the trace is a difficult task.

In this research we present a method to obtain *Q* estimates directly from the trace. Nonetheless the simplicity of the presented approach, the used hypothesis can serve as a basis upon which more complex situation can be modeled.

The method we developed to estimate *Q* is based on the temporal monitoring of the frequency centroid shift along the trace. Regarding using the frequency centroid shift, we followed Quan and Harris (1997) approach and the temporal monitoring was done using the S-transform (Stockwell *et al.*, 1996).

Methodology

<u>Dependence of the frequency centroid shift with Q</u>

Quan and Harris (1997) showed that the wave propagation can be described by a linear system in the frequency domain:

$$R(f) = G(f) \cdot H(f) \cdot S(f).$$

S(f) and *R(f)* are the amplitude spectrum of the incident and received waves, respectively. The instrument/medium response is *G(f).(H(f)*, where *G(f)* includes geometrical spreading, instrument response, source/receiver coupling, radiation patterns, and reflection/transmission coefficients, and the phase accumulation caused by propagation, and *H(f)* describes the attenuation effect on the amplitude

Assuming that attenuation is proportional to frequency (Ward and Toksöz, 1971), *H(f)* can be written as

$$H(f) = \exp\left(-f \int_{ray} \alpha_0 dl \right)$$

where α_0 is the intrinsic attenuation coefficient ($\alpha = f.\alpha_0$) in a constant-*Q* model and the integral is evaluated along the ray path. Here we are following Quan and Harris (1997)´s definition of attenuation parameters. Thus, in the case that *G* is frequency independent, one can estimate the integrated attenuation along the ray path simply using the spectral ratio *S(f)/R(f)* in a given frequency range. In this case,

$$f \int_{ray} \alpha_0 dl - \ln G = \ln\left[\frac{S(f)}{R(f)} \right]$$

In the case one has a Gaussian spectrum, Quan and Harris (1997) showed that the integrated attenuation along the ray path can be given by:

$$\int_{ray} \alpha_0 dl = (f_S - f_R) / \sigma_S^2$$

where f_S and f_R are, respectively, the source and receiver frequency centroids and σ_S^2 is the variance of the source signal spectrum.

For a constant-*Q* homogeneous media, the above equation provides an expression relating the frequency centroid shift with *Q* and the travel-time τ_D:

$$f_S - f_R = \frac{\pi \tau_D}{Q} \sigma_S^2 \quad (1)$$

which is exact if the wave spectrum is Gaussian. Quan and Harris (1997) showed that for a triangular input pulse spectrum Equation (1) is the dominant term of a Taylor series expansion of the integral defining the centroid frequency. However, the triangular case may not be a good approximation of a seismic or GPR real pulse spectrum; the Ricker pulse is a better approximation. For an input Ricker pulse spectrum with a travel time given by τ_D in a constant-*Q* homogeneous isotropic media, we show (using also a Taylor series expansion) that the centroid shift is now given by:

$$f_S - f_R = \frac{\pi \tau_D}{Q} \sigma_S^2 + \sqrt{2} \frac{(16 - \ \pi)}{(8 - 3\pi)^{\ /2}} \left(\frac{\pi \tau_D}{Q} \right)^2 \sigma_S \quad \left(\left(\frac{\pi \tau_D}{Q} \right)^3 \sigma_S^4 \right)$$

Note that in the seismic or GPR cases, usually τ_D << 1 and Q > 10, so that second and higher order terms in the above equation are negligible when compared with the first term. Thus, Equation (1) might be a good approximation even for a real seismic or GPR pulse.

<u>Inverse problem formulation for a layered medium</u>

Equation (1) was already used for obtaining *Q* estimates from transmission tomography data. Below we show that this equation can be used for obtaining *Q* estimates from usual seismic or GPR reflection data.

We assume that the Earth is flat and composed of sub horizontal layers in which the laws of geometrical optics for the wave are valid. The conditions a seismic or GPR data and the associated earth model should satisfy in order to better assure the validity of the inversion approach to be described are: (i) *Q* and layer thickness values may vary spatially but there is good lateral continuity (at least by parts); (ii) the layers are sufficiently thick so that reflections at base and top are resolved; (iii) the data should be suitably processed so that zero-offset condition is met and multiples are removed; (iv) reflection coefficients are frequency-independent.

As can be noticed from Figure 1, for a stratified earth composed of *N* layers and at the zero-offset condition (*S≡R*), ray paths obey the equalities

$$P_{1i}^D = P_{2i}^D = P_{3i}^D = \ldots = P_{(n-1)i}^D$$
$$P_{1i}^U = P_{2i}^U = P_{3i}^U = \ldots = P_{(n-1)i}^U$$

for all layers *i* and all (*N-1*) pulse arrivals recorded in the trace; superscripts D and U stand for Downgoing and Upgoing wave fronts respectively.

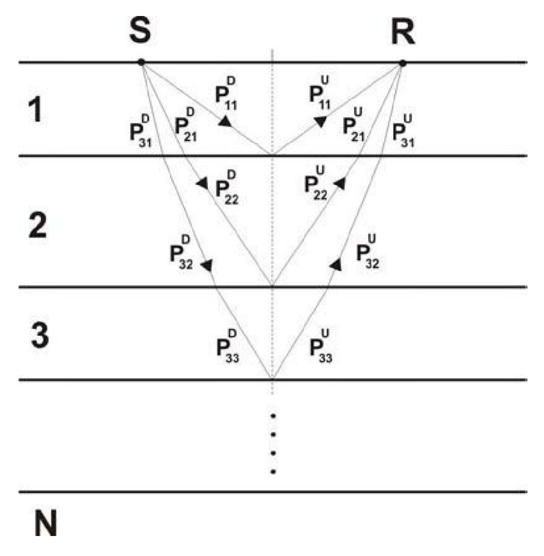

Figure 1: stratified earth composed of N layers showing ray paths from source S to receiver R.

For a given layer *j*, let t_j and t_{j-1} be the arrival times for pulses reflected at top and base of this layer respectively. So the *j-th* and (*j-1*)-*th* arrivals have dispersion associated with source-to-receiver paths respectively given by:

$$SR_{jj} = \underbrace{P_{11}^D + P_{22}^D + \ldots + P_{jj}^D}_{\text{Downgoing}} + \underbrace{P_{jj}^U + \ldots + P_{22}^U + P_{11}^U}_{\text{Upgoing}}$$

$$SR_{j-1j-1} = \underbrace{P_{11}^D + P_{22}^D + \ldots + P_{j-1j-1}^D}_{\text{Downgoing}} + \underbrace{P_{j-1j-1}^U + \ldots + P_{22}^U + P_{11}^U}_{\text{Upgoing}}$$

Note that:

$$SR_{jj} = SR_{j-1j-1} + \underbrace{P_{jj}^D}_{\text{Downgoing}} + \underbrace{P_{jj}^U}_{\text{Upgoing}} \quad (2)$$

Nonetheless the simplicity of the above equation, it is very worth mentioning that it opens the possibility of estimating Q_j (*j=1,2,…, N-1*) using data from the trace only and the equation:

$$Q_j = - \ \pi \ \ldots \quad (3)$$

where C_{j-1} is the variance of the amplitude spectrum of the pulse reflected at the top of the jth-layer and f_{j-1}^R and f_j^R are the frequency centroids for pulses reflected at top and base of *j-th* layer. Estimating *Q* in layers 2 until (*j-1*) is possible if we assume that the first pulse recorded in the trace is the reference pulse. So Q_2 can be estimated because all terms in the right-hand side of Equation (3) can be directly obtained from quantities associated with the first arrival recorded in the trace (C_{j-1}, f_{j-1}^R, f_j^R)or from the second one (f_j^R,t_j). Using identical approach Q_3 can be estimated from the second and third arrivals recorded in trace and so forth. Let us stress that Equation (3) is equivalent to Equation (1) but that Equation (3) implicitly violates causality due to the rearrangement used to express SR_{ij} as function of SR_{i-1j-1} in Equation (2); this rearrangement groups rays paths without obeying temporal sequence. Despite this implicit non-causality, the above-proposed Q inversion approach is valid because while estimating Q_j (at zero-offset condition), we are using the final overall accumulated dispersion effect recorded in time arrival j that, besides the dispersion associated with the jth layer, incorporates also the final overall accumulated dispersion effect recorded in time arrival *j-1*.

Because we are estimating *Q* using the trace itself, our inversion approach does not demand the knowledge of the velocity field.

<u>Frequency centroid shift temporal evolution monitoring using the S-transform</u>

The S-transform *ts[h]* of a function *h(t)* is defined as (Stockwell et al., 1996):

$$ts[h](\tau,f_0)=\int_{-\infty}^{\infty} h(t)w(\tau - t, f_0)e^{-i2\pi f_0 t}dt$$

where $w(\tau\text{-}t,f_0)$ is a Gaussian window centered at time τ given by:

$$w(\tau - t, f_0)=\frac{|f_0|}{\sqrt{2\pi}}e^{-\frac{(\tau-t)^2 f_0^2}{2}}$$

The S-transform can thus be understood as an extension of the Fourier transform for a short time Gaussian window with width

$$\sigma^2 = 1/f_0^2$$

Being f_0 the frequency of interest (Stockwell et al., 1996).

Results

In sequence, we show that the S-transform provides a suitable framework for time evolution monitoring of the frequency centroid shift.

Figure 2 show a GPR trace composed of six arrivals. This trace was generated by using a model with seven parallel layers having the deepest layer infinite thickness. By applying the S-transform at regular steps along the trace (upper figures) it is possible to monitor the evolution of the frequency content along the trace. This monitoring can be improved if each arrival is isolated from the composite trace, as shown in the middle panel of figure 2. In particular, it is possible to detect when the frequency content of any particular arrival is already well defined because a clear maximum is reached for the frequency centroid (observe that these frequency centroid maxima are visible even when the S-transform is applied to the original trace is shown in the upper figure of the right panel of figure 2).

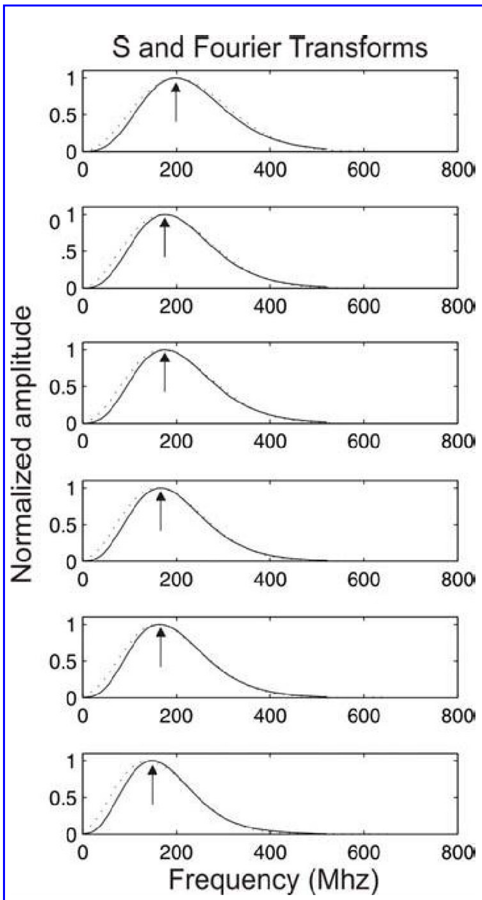

Figure 3: comparaion of S-transform (continuos line) with Fourier (dotted line) amplitutide spectra for the six arrivals of trace shown in Figure 2. The arrows identifies the centroid`peaks.

If the S-transform is computed at the time position where the frequency centroid maximum is attained, its amplitude spectrum is very similar to the usual Fourier spectrum, as shown in Figure 3. In this figure, the evolution of centroid peak can be evaluated. The evolution of the frequency centroid is similar. Note that it is possible to identify if the layer has a significant *Q* value by monitoring the centroid frequency shifts. By using the first arrival as the reference pulse, the *Q* values of layers 2 to 6 can be estimated by solving an inverse problem where all centroid shifts are matched jointly. We implemented this inverse problem through and optimization procedure based on a sequential search technique, which has proven effective in solving problems with nonlinear objective functions subject to nonlinear inequality constraints (Richardson and Kuester, 1973).

Conclusions

We presented the basic ideas of a method to estimate *Q* quality factor by using the frequency centroid temporal shift as monitored by the S-transform. This method combines the advantages of the current two-folded approaches to obtain *Q* estimates done separately in time or in frequency domains (Tonn, 1991).

The *Q* estimation approach presented here relies solely on the pulse deformation and is applied directly on the trace, after a generalization of the Quan and Harris (1997) approach to deal with non-causal situations. Because the inversion approach does not demand the knowledge of pulse amplitude, it does not either requires the knowledge of the field velocity as far as the following condition is met: for a given layer, both pulses reflected at the base and top of this layer have the same ray path along the upper layers.

It is important to stress that the hypothesis of cumulative dispersion for all arrivals registered in a single trace although rigorously valid only if the layers are horizontal and the zero-offset condition are met, is still approximately valid even for gently non-parallel dipping layers exhibiting dips of up to 20 degrees. In addition, it is robust to the violation of the zero-offset condition and to noise content.

Acknowledgments

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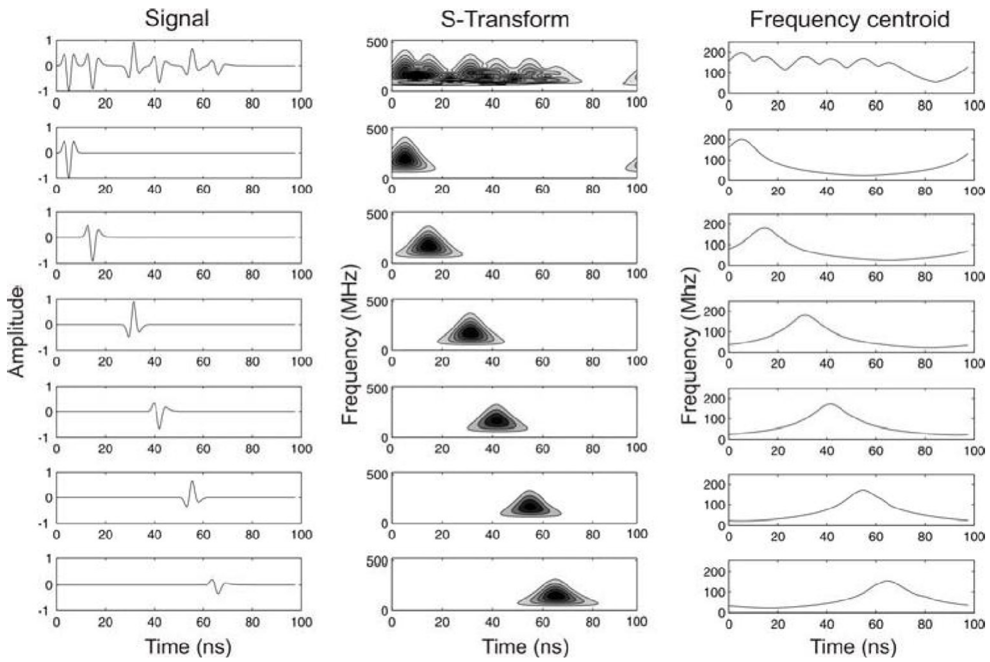

Figure 2: Composite panels showing the application of the S transform to monitor the frequency centroid evolution. Left panel: in the upper figure, a GPR trace which is composed by six arrivals; the remaining figures shows isolately each arrival. Middle panel: amplitude of the corresponding S transform. Right panel: evolution of the corresponding frequency centroid revealing that for each arrival a maximum is identified to the frequency centroid. Observe that separating the arrivals by windows increases the resolution in the frequency centroid determination. Because no amplitude modeling is necessary, all amplitudes are normalized.