



## Estimation by Intelligent Algorithm

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parameter for porosity calculation is the matrix porosity

(the log reading for a null porosity rock), which is, normally, determined in conventional core analysis. The formation water resistivity may be derived by the spontaneous potential log, or directly measure in laboratory on samples of formation water collected in the interest depths. The last possibility is the use of catalogs, with regional values of water resistivity. The Archie equation still involves parameters denominated as Archie's exponents (cementation exponent and saturation exponent), which only can be obtained by laboratory experiments with samples of reservoir rock. The cementation exponent (  $m$  ) is the slope of a straight line determined by linear regression in a plot involving the formation factor (ratio between water saturated rock resistivity and water resistivity) and porosity. The saturation exponent (  $n$  ) is the slope of linear regression produced in a graph involving the ratio between rock resistivity and water saturated rock resistivity in the ordinate, and water saturation in the abscissa.

In many practical situations, mainly in well site, a quick and realistic solution of Archie equation for water saturation may be a hard problem without core information. This implies in low confidence on porosity and the use of guessed values for cementation exponent and water resistivity. In this scenario, Hingle plot and Pickett plot are popular graphical solutions, which explore a linear relation between formation resistivity and porosity to solve Archie equation. Particularly, for water-bearing rocks, this linear relation is called as water line. Hingle plot is set up to the case where formation water resistivity and matrix parameter for porosity calculation are unknown. The slope of water line corresponds to formation water resistivity at formation temperature and the intersection with the abscissa gives the matrix porosity parameter. Pickett plot is indicated for the case where formation water resistivity and cementation exponent are unknown. The slope and intersection with 100% porosity of water line correspond to cementation exponent and formation water resistivity at formation temperature respectively. Thus, the association of these methods produces all parameters to solve Archie equation for water saturation.

Independent of the competence of Hingle plot and Pickett plot to solve Archie equation, as any graphical method, they are very sensible to visual misinterpretation and deeply dependent of interpreter expertise to locate water line correctly.

Intelligent algorithms are a large class of computing techniques, as artificial neural network, evolutionary computing, and fuzzy inference, mostly used for data analysis and interpretation. Intelligent algorithms are an increasingly powerful tool for making breakthroughs in the

science and engineering fields by transforming data into information and information into knowledge (Nikravesh, 2004).

Here, we interpret the water line as an angular pattern of all points from 100% water saturated zone in the Hingle plot or in the Pickett plot. To produce a computer-aid interpretation that may reduce the inherent imprecision in the visual location of water line, we present an intelligent algorithm that governs the operation of two angular competitive neural networks. The intelligent algorithm is able to recognize the angular pattern presented in water-bearing zones in Hingle plot and in Pickett plot and correctly locate the water line in both crossplots. The evaluation of this method is accomplished on synthetic data, that honor the Archie equation and actual well log data.

## Methodology

### Angular Competitive Neural Network

Angular competitive neural network was designed to find statistically relevant angular patterns in a Cartesian plane, where each point is associated to a position vector. Thus, an angular pattern can be understood as an angle that relates several vectors in the plane. This particular angle can be measured in relation to an orthogonal axis or in relation to a reference vector. Normally, the vectors chosen as reference integrates the training set.

The architecture of angular competitive network is composed by three layers: the input layer, the competitive layer and one intermediary layer, the selective layer, as shown in Figure 1.

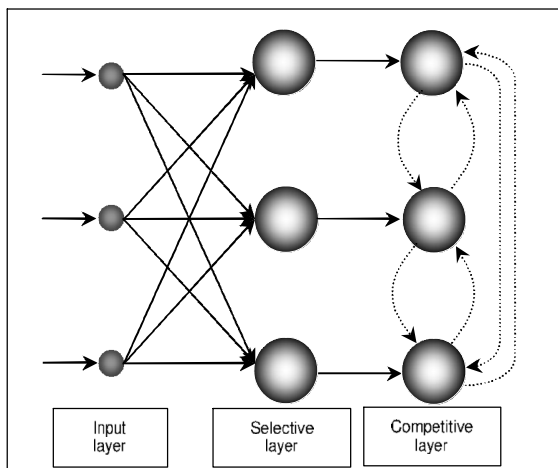


Figure 1 – Architecture of angular competitive neural network.

The input layer receives the external input and the number of input neurons is problem dependent. The external data are position vectors in Cartesian plane and are stored as complex numbers, exploring the analogy between the Cartesian plane and complex plane. This

can be understood just as storage form of a vector as a simple element of a complex matrix. The input data in the angular competitive neural network will be formed by unitary vectors obtained from the vector resulting of the subtraction of each position vector in the input data by each one in the training set, as shown in Figure 2. The input data is stored in a complex matrix, the global difference matrix, which has number of rows equal to the number of neurons in the input layer and number of columns equal to the number of vectors in training set.

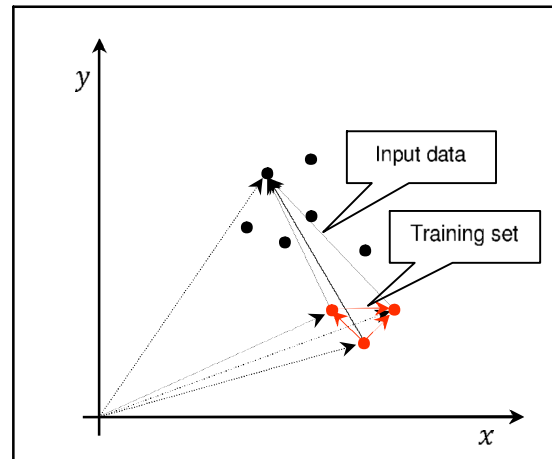


Figure 2 – Input data and training set of angular competitive network.

The selective layer operates to promote the selection of input data. A special activation function defines the selection criterion that acts in the sense of allowing or not the production of an effective output. Each selective neuron represents one vector of training set.

Training set is the subset of input data used to calculate the weight matrix linking input layer with selective layer. The criterion used to choose the components of training set is problem dependent. Each training set defines dynamically the number of neurons in the selective layer and in the competitive layer.

The weight matrix between the input and selective layers is a complex matrix constructed in a convenient form to storage the coordinate pair of each unitary difference vector as complex number, with the abscissa as real part and the ordinate as imaginary part. The unitary difference vectors are calculated from the vector resulting of the subtraction of each position vector in the training set by each other. This square complex matrix has order equal to the number of vectors in the training set and null diagonal.

For each time step, one row of global difference matrix is presented to the input layer. The operation accomplished in each selective neuron results in the input potential ( $u$ ) corresponding to the real part of the complex product of each element in the input vector (row of global difference matrix) and the complex conjugated of each element of the weight vector (column of the weight matrix). This operation is calculates the cosine of the

angle between this unitary difference vectors. The condition for two vectors have the same direction is the cosine to be equal to 1.0 or -1.0. The activation function for each selective neuron verifies the occurrence of input potential in the intervals [-1 -0.98] or [0.98 1] producing an output equal to the unit, as shown in Figure 3.

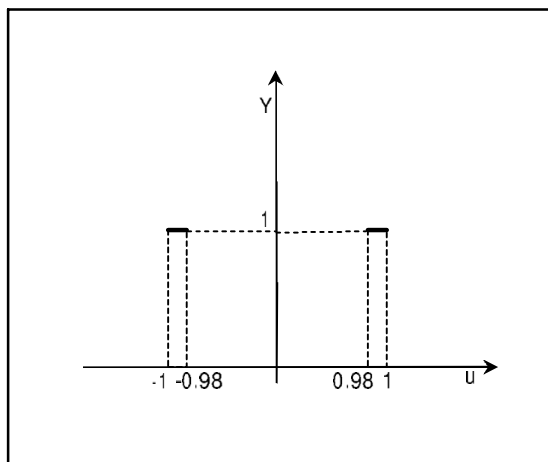


Figure 3 – Activation function. u is the input potential. Y is the neuron output.

The selective layer is activated by only one neuron of the input layer and it produces a binary vector as output that is passed to the competitive layer.

The competitive layer acts exactly as a classical competitive layer, promoting a competition among their neurons and allowing that only one of them wins the competition and produce the network output.

Each neuron in the competitive layer produces a summation of the number of times that its linked neuron in selective layer was activated. After the presentation of all the columns of global difference matrix, the neurons in the output layer compete among them. The winner neuron is that one linked to the selective neuron with the largest number of activations.

Pickett plot

In Pickett plot (Pickett, 1966), the Archie equation is rewrite considering the saturation exponent (n) equal to 2.0 and takes the logarithm on both sides of Archie equation, in the form.

$$\log R_t = -m \log \phi + \log \left( \frac{R_w}{S_w^n} \right) \tag{1}$$

represents porosity.  $S_w$  is the water saturation.  $m$  is the cementation exponent and  $R_w$  is the formation water resistivity. In a log-log paper, equation 1 describes a family of lines with slope equal to cementation exponent.

All points with unit water saturation describe a straight line called as water line. The slope of water line is formation temperature, is determined by the intersection

of water line with the vertical by 100% porosity, as shown in Figure 4.

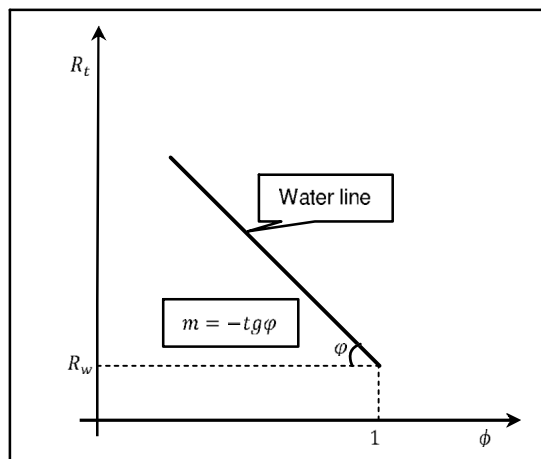


Figure 4 – Pickett plot.

Hingle plot

The Hingle plot assumes the saturation exponent and cementation exponent, both equal to 2.0 and rewrites the Archie equation in the form

$$\frac{1}{\sqrt{R_t}} = S_w \frac{1}{\sqrt{R_w}} \phi \tag{2}$$

Clean formations, 100% water saturated will produce points in Hingle plot that will fall in a line of maxim inclination, the water line, as shown in Figure 5.

Water resistivity can be calculated by the slope of water line.

When porosity cannot be calculated, the horizontal axis can be assigned, directly, in log readings. The intersection of water line with porosity log axis results in the matrix porosity parameter.

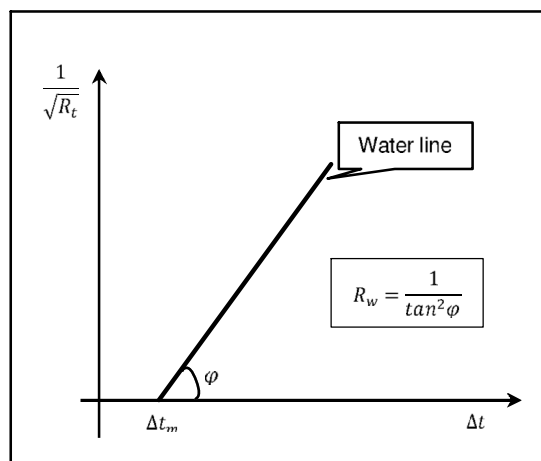


Figure 5 – Hingle plot.

### Association of Hingle and Pickett Plots

In many practical situations, mainly in well site, a quick and realistic solution of Archie equation for water saturation may be a hard problem without core information. Considering a sonde composed by one deep resistivity tool and sonic tool, for instance. The water saturation can be accomplished associating the Hingle and Pickett methods.

In the first step, Hingle plot is built with deep resistivity log readings and sonic log readings. The intersection of determined water line with horizontal axis gives a first approach for the matrix transit time, which permits the porosity calculation, considering water as interstitial fluid.

In the second step, Pickett plot is constructed using deep resistivity log readings and calculated porosity. The interpretation of Pickett plot allows the determination of the cementation exponent and water resistivity. This process is interactive being looked for a better approach for water line in both graphs.

### Methodology

To produce a computer-aid interpretation that may reduce the inherent imprecision in the visual location of water line in Hingle plot and in Pickett plot, we present an intelligent algorithm that governs the operation of two angular competitive neural networks.

We assume the particular resistivity-porosity dependence showed by well log data in the Hingle plot and Pickett plot as angular pattern. An angular pattern can be understood as a sub set of input data with the largest number of vectors with the same direction.

The input data for each one angular competitive network will be formed by unity vectors calculated with the result of the subtraction of each position vector in the Hingle plot and Pickett plot for each position vector in the training set.

The training set used by both angular competitive networks is formed based on deep resistivity log readings with small resistivity values. These points are natural candidates to represent water-bearing formation.

The weight matrix between the input and selective layers for each angular competitive neural network will be a complex matrix constructed in a convenient form to storage the coordinate pair of each unity difference vector as complex number.

The location of water line in Hingle plot and Pickett plot performed by angular competitive neural network assumes two premises. The first one supposes the presence of at least two water-bearing points in the training set. The second premise considers that water-bearing interval has a sufficient number of sampled points.

In each plot, the location of water line is associated to the winner neuron in the competitive layer. Let be  $k$  the position of the winner neuron, this indicates the direction of the difference vector  $k$  corresponding to the  $k$  element of the training set is the one with the largest number of

align points, this defines the direction of water line by the  $k$  point in the Hingle plot and Pickett plot, respectively

### Results

The competence of intelligent algorithm to control the operation of angular competitive neural networks to interpret the association of Hingle plot and Pickett plot is presented with synthetic logging data. Resistivity log readings are generated using Archie equation. Sonic log readings are produced using Wyllie equation considering quartz matrix ( $\Delta t_m = 55.5 \mu s.ft^{-1}$ ). True porosity is generated as random values in the closed interval [0.05, 0.3]. Water saturation are random values in the interval

$0 < S_w < 1$  and cementation exponent is taken equal to 2.0.

Figure 6 shows a set of synthetic points represented by red circles in Hingle plot (Figure 6-A) and in Pickett plot (Figure 6-B). In both graphics, the true water line is drawn as red line.

Figure 7 shows the first result of this intelligent algorithm in the interpretation of Hingle plot. The blue circles show the training set. The interpreted water line (blue line) is coincident with the true water line and determines the quartz transit time. It can be observed that even in the presence of noise data, the angular competitive network was robust to obtain the correct location of the water line.

Figure 8 shows the final result of this intelligent algorithm, used the matrix transit time determined early to porosity calculation and interpret the Pickett plot. The blue circles show the training set. The interpreted water line (blue line) is coincident with the true water line and determines the cementation exponent and water resistivity.

In Figure 9, we compare the results of present intelligent algorithm (blue lines) with the straight line located by linear regression (black line) with the points in training set. It can be observed that linear regression is unable to locate the water line.

We show an evaluation of intelligent algorithm with actual wireline data. We use a published data (Darling, 2005). In Figure 10 we show the correspondent Hingle plot, with wireline points as red circles. The circles in blue represent the training set. To produce this Hingle plot, we use raw deep resistivity and density log readings.

Figure 11 shows the interpreted water line (blue line) in Pickett plot. Porosity is calculated using the density matrix parameter determined early. The black line represents the water line defined by linear regression with same points of the training set (blue circles). It is observed that the linear regression is unable to produce the correct location of water line.

The Table 1 allows the comparison among the values of water resistivity, matrix density and cementation exponent obtained by three methods: core analysis (Darling, 2005), intelligent algorithm, and linear regression.

**Conclusions**

This work presents an intelligent algorithm that controls two sequential angular competitive neural networks to interpret Hingle plot determining the matrix parameter need to porosity calculation, which permits the construction of Pickett plot generating the water resistivity and cementation exponent. Thus, all quantities involved in Archie equation are defined and the water saturation is straightforward.

For many geologists and petroleum engineers Hingle plot and Pickett plot are considered as a quick look method. This kind of sense is not due to naive simplifications of the rock model or theoretical contradictions in the relationship defined in the Archie equation, but mainly by the visual interpretation needed for the location of water line based only in interpreter expertise.

The evaluation of this method showed estimate of the water resistivity matrix porosity parameter and cementation exponent values very close to that one obtained by direct measure in laboratory by core analysis.

Table 1 – Water resistivity, matrix density and cementation exponent.

Method	$R_w$	$\rho_b$	$m$
Core analysis (Darling, 2005)	0.025	2.66	1.9
Intelligent algorithm	0.028	2.65	1.85
Linear regression	0.27	12.1	0.74

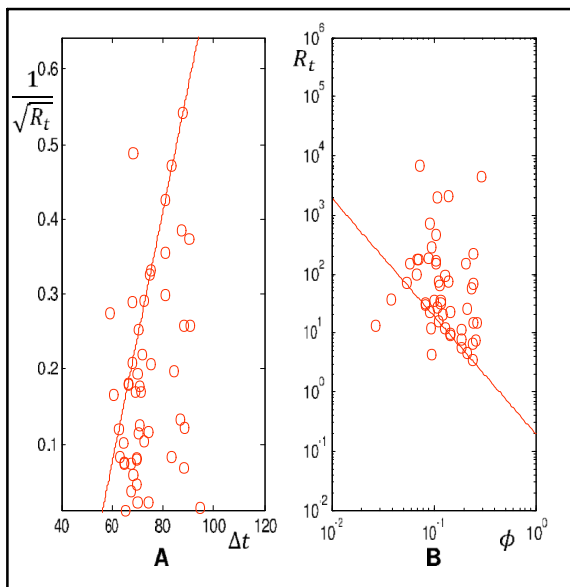


Figure 6 – Synthetic data. A- Hingle plot. B- Pickett plot. True water line is in red.

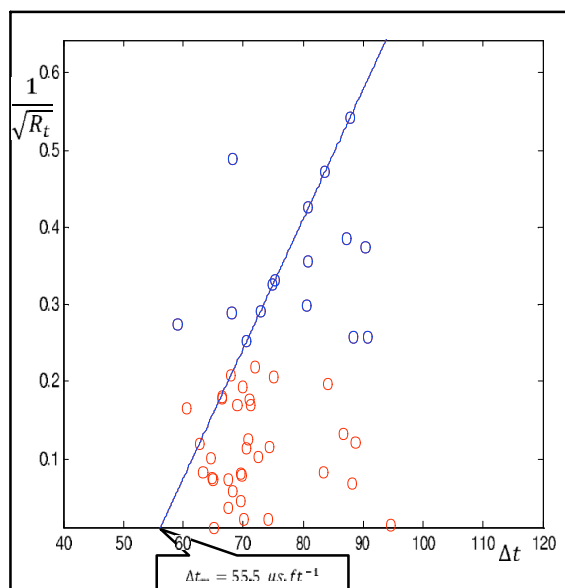


Figure 7 – Hingle plot. Synthetic data are red circles. Training set are blue circles. Interpreted water line is in blue.

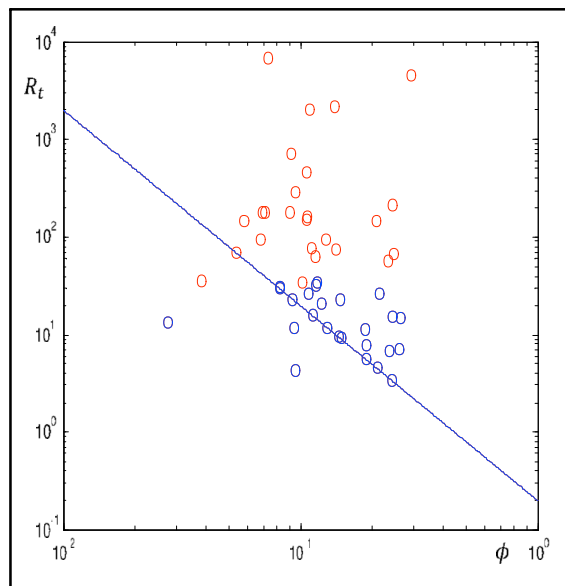


Figure 8 – Pickett plot. Synthetic data are red circles. Training set are blue circles. Interpreted water line is in blue.

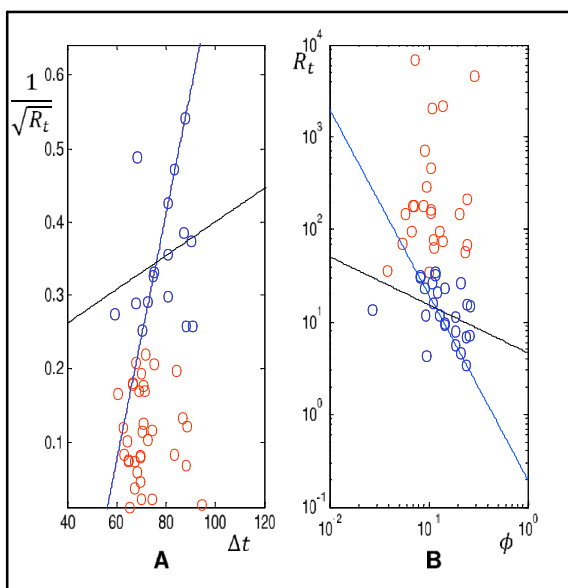


Figure 9 – Synthetic data. A- Hingle plot. B- Pickett Intelligent algorithm (blue line) compared with linear regression (black line).

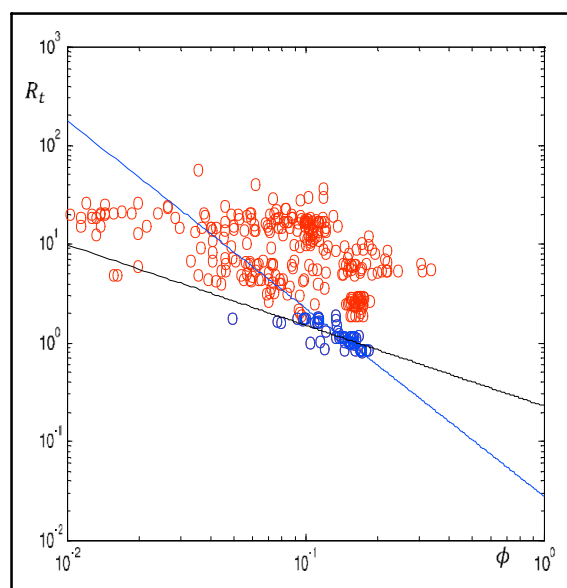


Figure 11 – Pickett plot. Actual wireline data (red circles). Training set (blue circles). Interpreted water line is in blue. Black line is generated by linear regression.

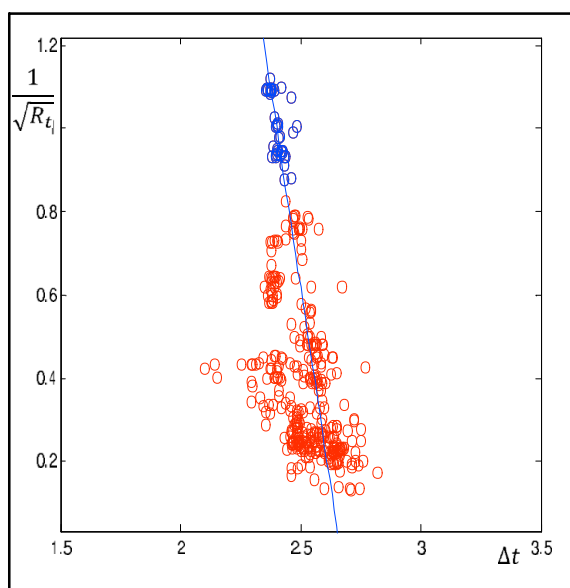


Figure 10 – Hingle plot. Actual wireline data (red circles). Training set (blue circles). Interpreted water line is in blue.

### Acknowledgments

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