Traveltime approximations for qP-wave in VTI and orthorhombic media

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ABSTRACT

As exploration targets have gotten deeper, cable lengths have increased accordingly, making the conventional two term hyperbolic traveltime approximation produce increasingly erroneous traveltimes. To overcome this problem, many traveltime formulas were proposed in the literature that provide approximations of different quality. In this paper, we give an overview over a number of those approximations and compare their quality. Moreover, we propose some new traveltime approximations based on the approximations found in the literature. The main advantage of our approximations is that some of them are have rather simple analytic expressions that makes them easy to use, while achieving the same quality as the better of the established formulas. Moreover, we derive another traveltime approximation based on a more recent approximation for the phase velocity.

INTRODUCTION

Traveltime approximations play a key hole in the processing of reflection data. They are used in, for example, migration (Alkhalifah and Larner, 1994; Vestrum et al., 1999; Mukherjee et al., 2001), moveout correction and velocity analysis (Tsvankin and Thomsen, 1994; Alkhalifah and Tsvankin, 1995; Fomel, 2003) and remigration (Fomel, 1994; Hubral et al., 1996; Schleicher and Aleixo, 2007).

Various authors proposed a shifted-hyperbola approximation (Malovichko, 1978; Claerbout, 1987; Castle, 1994). This equation describes a hyperbola that is symmetric about the t-axis and has asymptotes that intersect the time axis $x = 0$ at a time $t = \tau_s$ that is different from the zero-offset traveltime τ_0. The shifted hyperbola proposed by (Claerbout, 1987) contains a free parameter, called a, that can be used to find the best fitting traveltime approximation. The shifted hyperbola's parameter can be related to the anisotropy parameter η (Siliqi and Bousquié, 2000; Ursin and Stovas, 2006), generating a VTI approximation for the traveltime.

However, for a homogeneous transversely isotropic medium with a vertical symmetry axis (a VTI medium) the hyperbolic approximation is only valid for small offsets, and the velocity coefficient is an NMO velocity that differs from the vertical velocity (Thomsen, 1986). Tsvankin and Thomsen (1994) give a forth-order approximation, but this equation rapidly loses accuracy with increasing offset. Alternatively, they proposed a continued-fraction approximation, that is valid for long offsets (Tsvankin and Thomsen, 1994; Alkhalifah and Tsvankin, 1995),

$$t^2(x) = 1 + x^2 - \frac{2\eta x^4}{1 + (1 + 2\eta)x^2}. \quad (1)$$

Everywhere in this paper, we use the normalized half-offset, $x = 2h/\tau_0 v_{nmo}$, and the normalized traveltime $t(x) = \tau(x)/\tau_0$.

Using another methodology, Stovas and Ursin (2004) derive a different continued-fraction approximation for the traveltime function,

$$t^2(x) = 1 + x^2 - \frac{G x^4}{1 + (1 + 4G)x^2}, \quad (2)$$

where G is a parameter that depends of the anisotropic parameters ϵ and δ. It has the form

$$G = \frac{2(\epsilon - \delta)}{(1 + 2\delta)^2}\left[1 + \frac{2\gamma_0^2\delta}{\gamma_0^2 - 1}\right],$$

where γ_0 is vertical P-wave velocity over vertical S-wave velocity. Note, however, that this approximation is only valid for short and intermediate offsets.

Zhang and Uren (2001) observed that the ray velocity in general transversely isotropic (TI) media can be approximated by a simple equation. Based on this equation, they provide a traveltime approximation for P-waves in homogeneous TI media as

$$t^2(x) = \frac{1}{2}\left[1 + x^2/Q + \sqrt{(1 + x^2/Q)^2 + 4A\,x^2/Q}\right], \quad (3)$$

where $Q = 1 + 2\eta$. They give no rule for how the anisotropy parameter A depends on the actual medium parameters.

Fomel (2004) provides a anelliptic approximations for qP velocities in VTI media. This is a long offset approximation for traveltime function in VTI media,

$$t^2(x) = \frac{3 + 4\eta}{4(1 + \eta)}\, t_h^2(x) + \frac{1}{4(1 + \eta)}\sqrt{t_h^4(x) + Rx^2}, \quad (4)$$

where $t_h^2(x) = 1 + x^2/Q$ and $R = 16\eta(1 + \eta)/Q$. Note that $t_h^2(x)$ is the hyperbolic part of equation (4), however using the horizontal velocity $v_h = v_{nmo}\sqrt{1 + 2\eta}$ rather than the NMO velocity. An approximation of similar quality was obtained by Fomel and Stovas (2007).

The shifted hyperbola has the general form

$$t(x) = 1 + \frac{1}{S}\left[\sqrt{1 + S\,x^2} - 1\right]. \qquad (5)$$

Several different values for parameter S have been proposed in the literature. Malovichko (1978) considered a layered medium and expressed the parameter S as that $S = \mu_4/\mu_2^2$, where μ_j is the jth velocity momentum. Claerbout (1987) suggests to use a free parameter $a = 1/(1-S)$ in the shifted hyperbola to fit the approximation to the observed traveltime. He gives no interpretation of a in terms of medium parameters. In the shifted hyperbola of Castle (1994), S is no longer a constant but is allowed to vary with offset, i.e.,

$$S(x) = \frac{x^2 - 2(t-1)}{(t-1)^2}.$$

For VTI media, Siliqi and Bousquié (2000) and Ursin and Stovas (2006) expressed the parameter S as $S = 8\eta + 1$.

The nonhyperbolic moveout equations for VTI media expressed in terms of V_{nmo} and η can be also applied to a horizontal orthorhombic layer by making both parameters functions of azimuth (Xu et al., 2003). These parameters are given by (Xu et al., 2003)

$$V_{nmo}^{-2}(\alpha) = \frac{\sin^2(\alpha-\varphi)}{[V_{nmo}^{(1)}]^2} + \frac{\cos^2(\alpha-\varphi)}{[V_{nmo}^{(2)}]^2} \qquad (6)$$

$$\begin{aligned}\eta(\alpha) &= \eta^{(1)}\sin^2(\alpha-\varphi) + \eta^{(2)}\cos^2(\alpha-\varphi) \\ &\quad -\eta^{(3)}\cos^2(\alpha-\varphi)\sin^2(\alpha-\varphi),\end{aligned} \qquad (7)$$

where α is the source-to-receiver azimuth with respect to the aquisition frame, and φ is the azimuth of the $[x_1, x_3]$ symmetry plane of the orthorhombic medium (assuming that one of the symmetry planes is horizontal) and parameters $\eta^{(1)}$, $\eta^{(2)}$, $\eta^{(3)}$, $V_{nmo}^{(1)}$ and $V_{nmo}^{(2)}$ are given in Grechka and Tsvankin (1999).

$$\eta^{(i)} = \frac{\epsilon^{(i)} - \delta^{(i)}}{1 + 2\delta^{(i)}},\ \text{for } i = 1, 2, \qquad (8)$$

$$\eta^{(3)} = \frac{\epsilon^{(1)} - \epsilon^{(2)} - \delta^{(3)}(1 + 2\epsilon^{(2)})}{(1 + 2\epsilon^{(2)})(1 + 2\delta^{(3)})}, \qquad (9)$$

$$V_{nmo}^{(i)} = v_{p0}\sqrt{1 + 2\delta^{(i)}},\ \text{for } i = 1, 2. \qquad (10)$$

Using $\eta(\alpha)$ from equation (7) in the traveltime approximation (1), it becomes (Vasconcelos and Tsvankin, 2004)

$$t^2(x) = 1 + x^2 - \frac{2\eta(\alpha)x^4}{1 + [1 + 2\eta(\alpha)]x^2}. \qquad (11)$$

While equation (6) for the NMO ellipse is exact even for arbitrary anisotropy and heterogeneity, equation (7) is based on the weak-anistropy approximation for a single orthorhombic layer. Vasconcelos and Tsvankin (2004) found that equation (7) remains suffciently accurate for a stack of horizontal orthorhombic layers with a uniform orientation of the vertical symmetry planes.

Correspondingly, Elapavuluri and Bancroft (2006) extended the shifted hyperbola traveltime approximation to orthorhombic media. They propose to use for parameter S the value $S = 1 + 4\eta$, where η is given in equation (7).

In this paper, we give an overview over a collection of traveltime approximations found in the literature and compare their quality. Moreover, we propose some new traveltime approximations based on the approximations found in the literature. The main advantage of our approximations is that some of them are have rather simple analytic expressions that makes them easy to use, while achieving the same quality as the better of the established formulas.

NEW TRAVELTIME APPROXIMATIONS

In this section, we study a few additional traveltime approximations. Most of them are obtained by further approximation of one or several of the above formulas, mainly the one of Fomel (2004). Others are the result of adaptations that are based on the numerical experiments.

Different expressions for the traveltimes can be obtained from differently grouping terms before approximating the square roots in the above formulas. For small values of ϵ, we have up to the first order $\sqrt{1+\epsilon} \approx 1 + \epsilon/2$.

After algebraic calculations we can identify a class of good approximations. All approximations in this class have the form

$$t^2(x) \approx t_h^2(x) + B_i(\eta)\,x^2/t_h^2(x), \qquad (12)$$

where the factor $B_i(\eta)$ can be represented as

$$\begin{aligned} B_1(\eta) &= 2\eta/Q, \\ B_2(\eta) &= 2\eta/(1+\eta)Q, \\ B_3(\eta) &= 2\eta/(1+\eta)^2, \\ B_4(\eta) &= 2\eta/Q^2, \\ B_5(\eta) &= 8\eta(1+\eta)/5Q. \end{aligned} \qquad (13)$$

Another class of approximations has the form

$$t(x) \approx t_h(x) + A_i(\eta)\,x^2/t_h^3(x). \qquad (14)$$

Depending on the approximation involved, the factor $A_i(\eta)$ relates to η through one of the following expressions:

$$\begin{aligned} A_1(\eta) &= \eta/Q, \\ A_2(\eta) &= \eta/(1+\eta)Q, \\ A_3(\eta) &= \eta/Q^2, \\ A_4(\eta) &= \eta/(1+\eta)^2, \\ A_5(\eta) &= (2\eta/Q\sqrt{1+\eta/3+4\eta}), \\ A_6(\eta) &= 2\eta/Q\sqrt{(1+\eta)(3+4\eta)}, \\ A_7(\eta) &= 2\eta/Q\sqrt{3+4\eta}, \\ A_8(\eta) &= 4\eta(1+\eta/5Q). \end{aligned} \qquad (15)$$

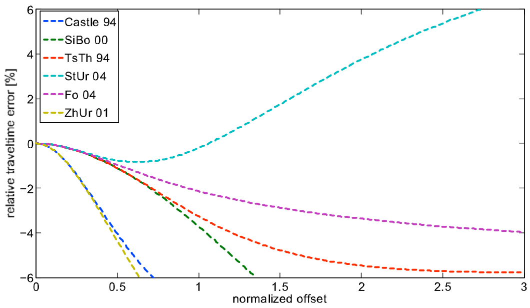

Figure 1: Relative error of VTI traveltime approximations (1) [TsTh 94], (2) [StUr 04], (3) [ZhUr01], and (4) [Fo 04], and shifted hyperbola (5) with $S = S(x)$ [Castle 94] and $S = 1 + 8\eta$ [SiBo 00].

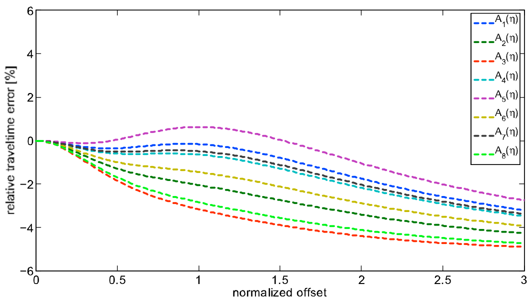

Figure 2: Relative error of VTI traveltime approximations of the type $t(x) \approx t_h(x) + A_i(\eta)\,\dfrac{x^2}{t_h^3(x)}$.

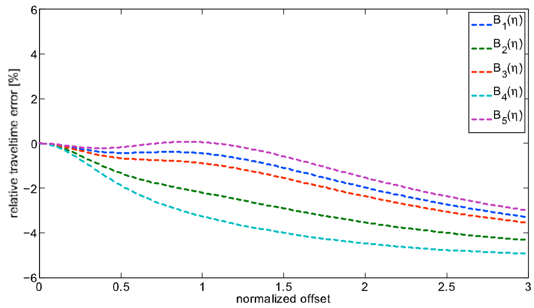

Figure 3: Relative error of VTI traveltime approximations of the type $t^2(x) = t_h^2(x) + B_i(\eta)\,\dfrac{x^2}{t_h^2(x)}$.

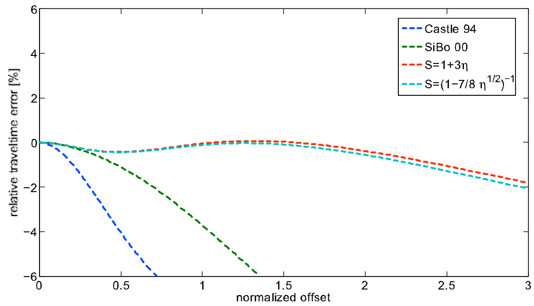

Figure 4: Relative error of shifted hyperbola approximations in VTI media, with $S = S(x)$ [Castle 94], $S = 1+8\eta$ [SiBo 00], and new shifted hyperbola approximations with $S = 1 + 3\eta$ and $S = (1 - 7/8\sqrt{\eta})^{-1}$.

NUMERICAL COMPARISONS

In this section, we compare the above VTI traveltime approximations for a homogeneous VTI layer above a horizontal reflector with the exact traveltime. Moreover, we compare the corresponding orthorhombic approximations for a homogeneous orthorhombic layer to that of Vasconcelos and Tsvankin (2004) (equation 11). Since we are comparing normalized traveltimes, the comparison is valid for arbitrary reflector depth and NMO velocity.

VTI media

The VTI medium used for the tests is the Greenhorn shale (Jones and Wang, 1981), the elastic parameters of which are i.e., $c_{11} = 14.47$ km^2/s^2, $c_{33} = 9.57$ km^2/s^2, $c_{13} = 4.51$ km^2/s^2, and $c_{55} = 2.28$ km^2/s^2, which were also used by Fomel (2004). In this medium, we have $\epsilon = 0.2560$, $\delta = -0.0505$ and $\eta = 0.3409$.

Figure 1 shows the relative error between the traveltime approximations (1)–(5) and the exact traveltime. For approximation (3), we used $A = 2\eta$, which turns expressions (3) and (4) to be identical after approximation of the square roots. We see that approximation (4) is the best of these, with its error never exceeding 4% in the depicted offset range between 0 and 3. In our experiments, the approximation of Fomel and Stovas (2007) produced indistinguishable results from that of Fomel (2004). The second-best is approximation (1) with a relative error below 6%. The errors of the other approximation exceeds 6% for rather small offsets.

In Figure 2 we present the approximations of the type $t(x) \approx t_h(x) + A_i(\eta)\,x^2/t_h^3(x)$. Again, all of these approximations are rather accurate. None of them exceeds a relative error of 5% in the chosen range of offsets. Moreover, these approximations possess quite simple expressions that may be advantageous for theoretical considerations. The best of these approximations with a maximum error of about 2.5% is the one given in $A_5(\eta)$.

Figure 3 depicts the approximations of the type $t^2(x) \approx t_h^2(x) + B_i(\eta)\,x^2/t_h^2(x)$. Note that the axes are the same as in Figure 1. As we can see, these are rather accurate approximations. None of these approximations exceeds a relative error of 5%, the best one being equation for $B_5(\eta)$, the error of which remains below 3%.

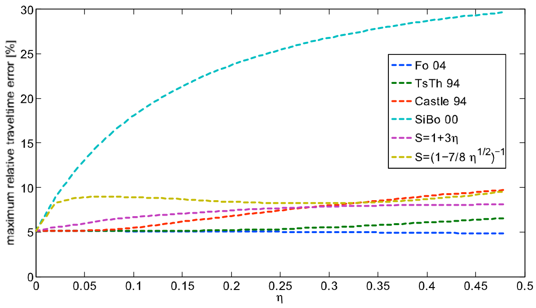

Figure 5: Maximum relative error of (4) [Fo 04], (1) [TsTh 94], shifted hyperbola approximations with $S = S(x)$ [Castle 94], $S = 1 + 8\eta$ [SiBo 00], and new shifted hyperbola approximations with $S = 1 + 3\eta$ and $S = (1 - 7/8\sqrt{\eta})^{-1}$ in VTI media.

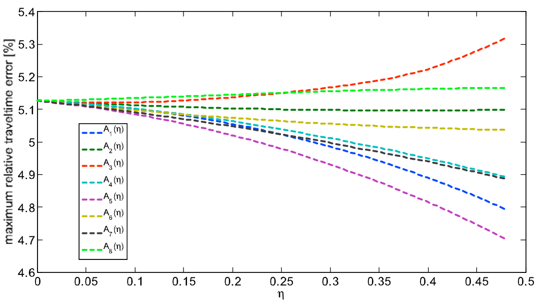

Figure 6: Maximum relative error of VTI traveltime approximations of the type $t(x) \approx t_h(x) + A_i(\eta)\,\dfrac{x^2}{t_h^3(x)}$.

In Figure 4 we compare the shifted hyperbola approximations from the literature with the ones obtained with modified expressions for S. Our tests indicated that the choices $S = 1 + 3\eta$ and $S = (1 - \frac{7}{8}\sqrt{\eta})^{-1}$ yield highly accurate traveltime approximations, in this example with a maximum error below 2%.

In Figures 5-7 we compare the maximum relative error of these traveltime approximations in the offset range $[0, 3]$ under variation of the parameter η. Figure5 shows the comparison of the standard traveltime approximations (1)–(5), as well as the shifted hyperbola approximation using $S = 1 + 3\eta$ and $S = (1 - 7/8\sqrt{\eta})$. We observe that the standard shifted hyperbola approximations are not very accurate, but the formulas of Tsvankin and Thomsen (1994) and Fomel (2004), as well as the shifted hyperbolas with the new values for S produce relatively small maximum errors.

Figure 6 depicts the corresponding errors for the traveltime approximations of the type (14). In this figure, we observe that all these approximations have a maximum error of about 5% for all values of η. The maximum errors of approximations $A_2(\eta)$, $A_6(\eta)$ and $A_8(\eta)$ are almost completely constant. Approximations $A_1(\eta)$, $A_4(\eta)$, $A_5(\eta)$ and $A_7(\eta)$ have slightly decreasing maximum errors for increasing η. The best approximation for larger η is approximation $A_5(\eta)$.

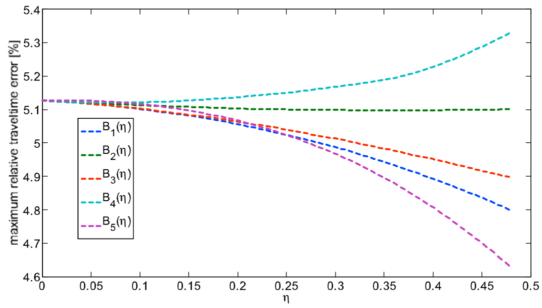

Figure 7: Maximum relative error of VTI traveltime approximations of the type $t^2(x) = t_h^2(x) + B_i(\eta)\dfrac{x^2}{t_h^2(x)}$.

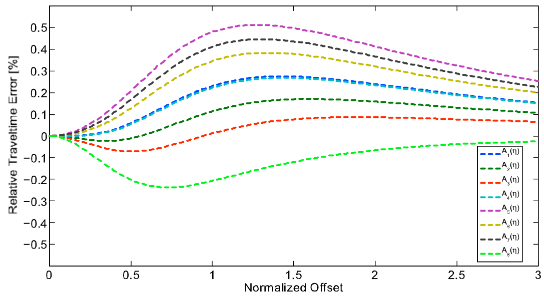

Figure 8: Relative error of orthorhombic traveltime approximations of the type $t(x) \approx t_h(x) + A_i(\eta)\,\dfrac{x^2}{t_h^3(x)}$.

Figure 7 allows to draw almost the same conclusions about the approximations of type (12). We observe that all maximum errors are close to 5%. Approximation $B_2(\eta)$ has practically constant error. Approximations $B_1(\eta)$, $B_3(\eta)$ and $B_5(\eta)$ have slightly decreasing errors. The best approximation for larger η is approximation $B_5(\eta)$.

Orthorhombic media

In Figure 8 we present the approximations of the type $t(x) \approx t_h(x) + A_i(\eta)\,x^2/t_h^3(x)$ for orthorhombic media. Again, all of these approximations are rather accurate. None of them exceeds a relative error of 0.5% compared with approximation (11) in the chosen range of offsets. Moreover, these approximations possess quite simple expressions that may be advantageous for theoretical considerations. The best of these approximations with a maximum error of about 0.1% compared with approximation (11) is the one given in $A_3(\eta)$.

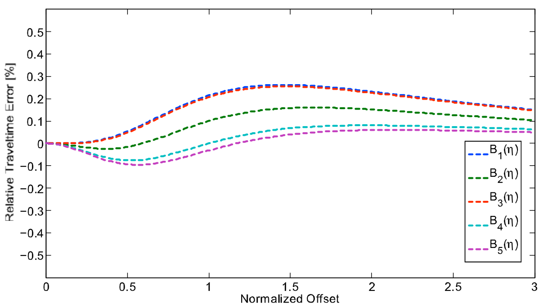

Figure 9: Relative error of orthorhombic traveltime approximations of the type $t^2(x) = t_h^2(x) + B_i(\eta)\dfrac{x^2}{t_h^2(x)}$.

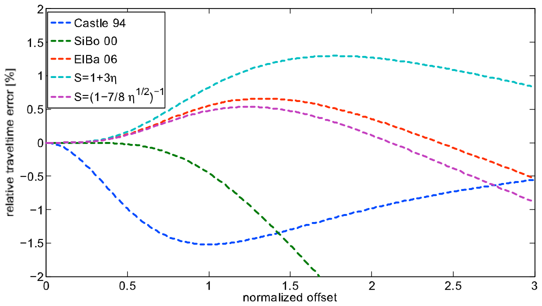

Figure 10: Relative error of shifted hyperbola approximations with $S = S(x)$ [Castle 94], $S = 1 + 8\eta$ [SiBo 00], $S = 1 + 4\eta$ [ElBa 06], and new shifted hyperbola approximations with $S = 1 + 3\eta$ and $S = (1 - 7/8\sqrt{\eta})^{-1}$, in orthorhombic media.

Figure 9 depicts the approximations of the type $t^2(x) \approx t_h^2(x) + B_i(\eta)\, x^2/t_h^2(x)$. None of these approximations exceeds a relative error of 0.4% compared with approximation (11), the best one being equation for $B_2(\eta)$, the error of which remains below 0.1% compared with approximation (11).

In Figure 10 we compare the shifted hyperbola approximations from the literature with the ones obtained with modified expressions for S. Our tests indicated that the choices $S = 1 + 4\eta$ and $S = (1 - \frac{7}{8}\sqrt{\eta})^{-1}$ yield highly accurate traveltime approximations, in this example with a maximum error below 0.5% compared with approximation (11).

CONCLUSION

Accurate traveltime approximations for large offsets are very important for many tasks of seismic processing. The conventional hyperbolic approximation, which is still used by many processing algorithms for moveout correction, time migration, multiple attenuation and velocity analysis, is inaccurate as soon as anisotropy, wave-mode conversions or significant medium heterogeneity are involved.

Many different formulas to approximate far-offset traveltimes have been proposed in the literature Tsvankin and Thomsen (1994) and Fomel (2004). Most of these are rather complicated algebraic expressions that are hard to use.

In this paper, we have studied the quality of several of these approximations for a homogeneous VTI or orthorhombic medium above a horizontal reflector. Moreover, by further approximation of the formulas from the literature, as well as by combining some of their properties, we have presented a number of new traveltime approximations.

Our numerical comparisons show that it is possible to find traveltime formulas of a much simpler type that provide equal or even better approximations to the true traveltime than those proposed in the literature. The formulas that provided the best approximations to the true traveltime in a homogeneous VTI medium are the shifted hyperbola with a different choice for the free parameter and the hyperbolic traveltime with an additional, rather simple correction term.

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