Fourier finite difference method for reverse time migration

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ABSTRACT

In this paper we propose two novel solutions for the two way wave equation for reverse time migration. The first one is an interpolation procedure which is based on the exact solutions of the wave equation for the constant velocity case. The time advanced wavefields for several reference velocities are computed in the Fourier domain and then interpolated in the space domain. Next, we propose a Fourier finite difference method (FFD) which combines the spectral method and a finite difference solution which is similar to the FFD method used for the one way wave equation. In the numerical examples we show their applicability and robustness and conclude that the FFD method is equivalent to the multiply velocity method but is significantly less expensive.

INTRODUCTION

Reverse time migration (RTM), using the two-way acoustic wave equation is not a new concept. It was introduced in the late 1970's ((Hemon, 1978)). But despite its advantages in depth imaging ((Baysal et al., 1983), (Whitmore, 1983), (McMechan, 1983), (Loewenthal and Mufti, 1983)) it was not used in practice due its high computational requirements. Now, computer technology has improved and 3D prestack RTM is being used to address the imaging challenges posed by sub salt and other complex subsurface targets ((Dussaud et al., 2008)).

By using the full wave equation, RTM implicitly includes multiple arrival paths and has no dip limitation, enabling the imaging of complex reflectors. RTM images are formed by crosscorrelating the source wavefield and the receiver wavefields after forward and reverse time propagation, respectively at each time step. While RTM is now computationally feasible it is still expensive. One reason is that the most common implementations use small time steps to avoid numerical instability and reduce dispersion. However, many new algorithms are being developed to overcome this problem. A two-step marching method was introduced by (Soubaras and Zhang, 2008) which allows a large extrapolation time. (Zhang and Zhang, 2009) proposed a one-step extrapolation method which is implemented based on the optimized separable approximation (OSA) ((Song, 2001)). (Pestana and Stoffa, 2009) introduced RTM using recursive time stepping based on the Rapid expansion method (REM) ((Tal-Ezer et al., 1987)).

In this paper we propose two novel solutions for the time stepping problem. The first is an interpolation procedure which uses different velocities and is based on the exact solution of the wave equation for the constant velocity case. These time advanced wave fields are computed in the Fourier domain and then interpolated in the space domain. Next, we propose a Fourier finite difference method (FFD) for the two way wave equation. The idea is almost the same one proposed by (Song and Fomel, 2010), but the derivation of the method and its implementation are completely different. This method combines the spectral method and finite differences together and is similar to the method of (Ristow and Ruhl, 1994) for one way wave equations.

We present the first prestack RTM results obtained by the proposed methods using the Marmousi synthetic data as an example. The numerical results show that the proposed methods have the ability to image steeply dipping reflectors and complex structures using a larger time step than is commonly used in finite-difference schemes. Further, we show that the FFD method is equivalent to the multiple velocity method but is significantly less expensive.

THEORY AND METHOD

We consider the following acoustic wave equation:

$$\frac{\partial^2 P(\mathbf{x},t)}{\partial t^2} = -L^2 P(\mathbf{x},t) \tag{1}$$

where $-L^2 = v^2(\mathbf{x})\,\nabla^2$, $v(\mathbf{x})$ is the velocity of propagation, vector position defined by $(\mathbf{x}) = (x,y,z)$, and $\nabla^2 = \left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}\right)$ is the Laplacian operator in Cartesian coordinates.

With the initial conditions $P(\mathbf{x}, t=0) = P_0$ and $\frac{\partial P(\mathbf{x},t)}{\partial t}\Big|_{t=0} = \dot{P}_0$, the formal solution of the wave equation ((Pestana and Stoffa, 2009)) is

$$P(\mathbf{x}, t+\Delta t) + P(\mathbf{x}, t-\Delta t) = 2\,\cos(L\Delta t)\,P(\mathbf{x},t) \tag{2}$$

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For variable media, we propose to approximate the cosine term in equation (2) as:

$$\cos(L\Delta t) \approx \sum_{n=1}^{N} a_n(v)\, b_n(k) \tag{3}$$

where $a_n(v)$ and $b_n(k)$ are real functions of velocity and wavenumber, respectively. The function $a_n(v)$ is a spatial function and it is computed using *optimal* reference velocities: here we used the same procedure proposed by (Bagaini et al., 1995) to estimate in the interval $[v_m, v_M]$ (where v_m and v_M are the minimum and the maximum velocity values of the entire field). The optimal reference velocities $(v_1, \ldots, v_N)$ are computed using the statistical entropy of the velocity distribution. Each weighting function for each reference velocity $(a_n(v(\mathbf{x}))$ is computed during the migration procedure.

Considering the representation of the cosine function given by (3), the time wave propagation can be performed in the following way:

$$P(\mathbf{x}, t{+}\Delta t){+}P(\mathbf{x}, t{-}\Delta t) = 2\left[\sum_{n=1}^{N} a_n(v) F^{-1} b_n(k)\right] F P(\mathbf{x}, t) \tag{4}$$

Considering the 2D case, each $b_n(k)$ is given by $b_n(k) = \cos(v_n\sqrt{k_x^2 + k_z^2}\,\Delta t)$ ((Pestana and Stoffa, 2009)) and for each marching time step, this method requires one fast Fourier transform (FFT) and N inverse fast Fourier transforms (IFFT).

Fourier Finite Difference Method

Now we rewrite the equation (1) in the following form:

$$P(\mathbf{x}, t + \Delta t) + P(\mathbf{x}, t - \Delta t) = \cos(L\Delta t)\sec(L_0\Delta t) T(\mathbf{x}, t) \tag{5}$$

where we have introduced the temporary wavefield $T(\mathbf{x}, t)$, that is given by:

$$T(\mathbf{x}, t) = 2\, \cos(L_0 \Delta t)\, P(\mathbf{x}, t) \tag{6}$$

with $L_0^2 = -\, v_0^2\, \nabla^2$ and v_0 is the minimum velocity of the media.

Using Taylor series, we can approximate both the $\cos(L\Delta t)$ and $\sec(L_0\Delta t)$ functions and substituting these approximations into equation (5) results in:

$$\begin{aligned} P(\mathbf{x}, t + \Delta t) + P(\mathbf{x}, t - \Delta t) &= \Big[1 + c_2(\mathbf{x}) K^2 \Delta t^2 \\ + \quad c_4(\mathbf{x}) K^4 \Delta t^4 &+ c_6(\mathbf{x}) K^6 \Delta t^6 + ...\Big] T(\mathbf{x}, t) \end{aligned} \tag{7}$$

where $K = \sqrt{-\nabla^2}$ or in the Fourier domain we have $K^2 = k_x^2 + k_z^2$

Thus the equation (7) is rewritten as

$$\begin{aligned} P(\mathbf{x}, t + \Delta t) &= T(\mathbf{x}, t) - P(\mathbf{x}, t - \Delta t) + \Big[c_2(\mathbf{x})\, K^2 \Delta t^2 \\ &+ c_4(\mathbf{x})\, K^4\, \Delta t^4 + ...\Big]\, T(\mathbf{x}, t) \end{aligned} \tag{8}$$

with the c coefficients given by: $c_2(\mathbf{x}) = \frac{v_0^2}{2}\big\{1 - \alpha^2(\mathbf{x})\big\}$, $c_4(\mathbf{x}) = \frac{v_0^4}{24}\big\{5 - 6\,\alpha^2(\mathbf{x}) + \alpha^4(\mathbf{x})\big\}$, $c_6(\mathbf{x}) = \frac{v_0^6}{720}\big\{61 - 75\,\alpha^2(\mathbf{x}) + 15\alpha^4(\mathbf{x}) - \alpha^6(\mathbf{x})\big\}$, and $\alpha(\mathbf{x}) = \frac{v(\mathbf{x})}{v_0}$.

We notice that for $\alpha(\mathbf{x}) = 1$ all coefficients are zero and we recover the exact solution of the wave wave for the constant velocity case.

We'd like also to mention that a similar approach was proposed by (Song and Fomel, 2010) and differs mainly in the weighting coefficients derived for the finite difference operators.

FOURIER FINITE-DIFFERENCE IMPLEMENTATION

The equation (8) can be implemented in the Fourier domain and space domain in two steps as in the Fourier finite difference method introduced by ((Ristow and Ruhl, 1994)) for one-way depth extrapolation. First, the temporary field $T(\mathbf{x}, t)$ is computed in the Fourier domain. We need to transform $P(\mathbf{x}, t)$ to $\hat{P}(\mathbf{k}, t)$ using a fast Fourier algorithm (FFT). Then we multiply $\hat{P}(\mathbf{k}, t)$ by $\cos(L_0\Delta t)$ to get $\hat{T}(\mathbf{k}, t)$ and transform to $T(\mathbf{x}, t)$ by inverse FFT. We can apply (in the space domain) any conventional finite-difference scheme to compute $P(\mathbf{x}, t + \Delta t)$

Considering only the first order velocity correction term on the RHS of equation (8) we have:

$$P(\mathbf{x}, t{+}\Delta t) = T(\mathbf{x}, t) {-} P(\mathbf{x}, t{-}\Delta t) {-}\, c_2(\mathbf{x}) \Delta t^2 \nabla^2 T(\mathbf{x}, t) \tag{9}$$

where ∇^2 is the Laplacian operator and it can be computed using 4th or higher order finite-difference schemes and $c_2(\mathbf{x})$ is the perturbation velocity computed for each spatial position as given before.

RESULTS

We implemented the reverse time migration method using the interpolation and the FFD methods proposed here to migrate the Marmousi dataset. In the velocity model the velocities vary from 1500 m/s to 5500 m/s and the grid size for the migration result is $\Delta x = 25m$ and $\Delta z = 8m$. The methods are implemented in the Fourier domain and to avoid instability problems, we consider $r = \frac{v\,\Delta t}{\Delta x} < \frac{1}{\sqrt{1+\alpha^2}}$, where $\alpha = \Delta x / \Delta z$. In the interpolation results presented in Figure 1, we show that using 5 to 7 velocities as determined using (Bagaini et al., 1995)'s procedure produced a very good image of the Marmousi model. (For all results as part of the imaging condition we applied a Laplacian filter do remove the low wavenumber artifacts.) This method proved quite expensive for this example, because we needed 5 to 7 Fourier transforms for each time step, to obtain a reasonable result. However, using just one forward and back Fourier transform for each time step and with the additional velocity correction proposed, and implemented via finite differences, we were able to obtain a very good results for the Marmousi model (Figure 2).

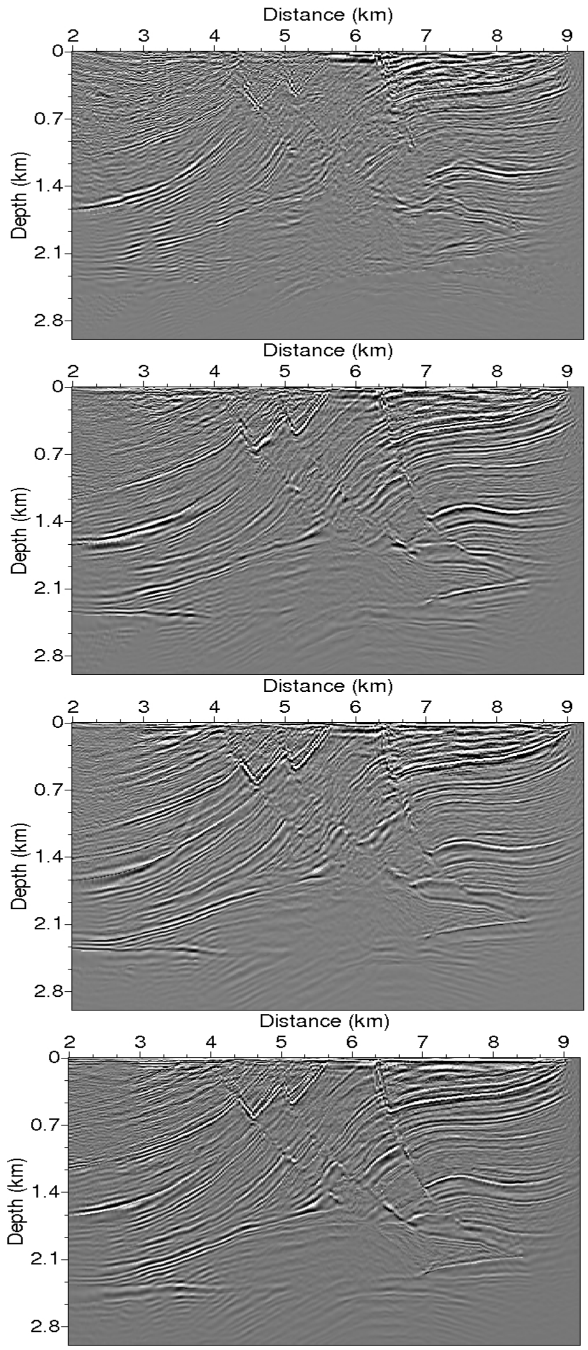

Figure 1: (a); Pre-stack RTM results using the interpolation method for Marmousi dataset. Migration result using 3 velocities (upper left), 5 velocities (lower left), 7 velocities (upper right) and 10 velocities (lower right)

CONCLUSIONS

Reverse time prestack depth migration is an important solution for the most complex seismic imaging challenges, particularly sub salt areas where other imaging techniques may fail to provide acceptable seismic images. In this paper, two novel methods are proposed for prestack RTM.

For variable velocity media we proposed the spatial interpolation of exact wave equation solutions for constant velocity time extrapolations computed in the Fourier domain. However, for complex structures we need a high number of reference velocities to obtain reasonable seismic images. To overcome this increase in computation burden, we also proposed a second method which is a FFD solution for the two wave equation. The results obtained with the Marmousi dataset demonstrated its applicability and robustness.

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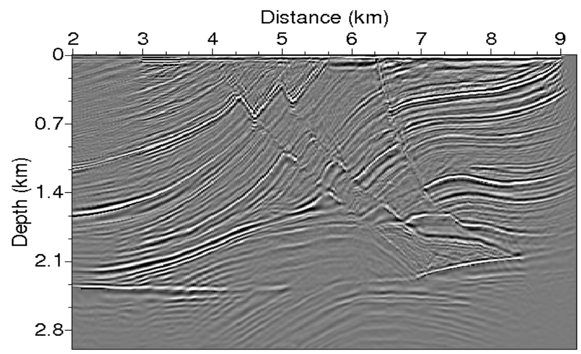

Figure 2: Marmousi pre-stack result using the FFD reverse time migration with the first order velocity correction term.

REFERENCES

Bagaini, C., Bonomi, E., and Pieroni, E., 1995, Data parallel implementation of 3D PSPI: 65th Annual Inter. Meeting, SEG, Expanded Abstracts, pages 188–191.

Baysal, E., Kosloff, D. D., and Sherwood, J. W. C., 1983, Reverse time migration: Geophysics, **48**, 1514–1524.

Dussaud, E., Symes, W., Williamson, P., Singer, P., Denel, B., and Cherret, A., 2008, Computational strategies for reverse-time migration: 78th Annual Inter. Meeting, SEG, Expanded Abstracts, pages 2267–2271.

Hemon, C., 1978, Equations d'onde et modeles: Geophysics Prospecting, **26**, 790–821.

Loewenthal, D., and Mufti, R. I., 1983, Reversed time migration in spatial frequency domain: Geophysics, **48**, 627–635.

McMechan, G. A., 1983, Migration by extrapolation of time-dependent boundary values: Geophysics Prospecting, **31**, 413–420.

Pestana, R., and Stoffa, P., 2009, Rapid expansion method (REM) for time stepping reverse time migration (RTM): 79th Annual Inter. Meeting, SEG, Expanded Abstracts, pages 2819–2823.

Ristow, D., and Ruhl, T., 1994, Fourier finite-difference migration: Geophysics, **59**, 1882–1893.

Song, X., and Fomel, S., 2010, Fourier finite-difference wave propagation: 80th Annual Inter. Meeting, SEG, Expanded Abstracts.

Song, J., 2001, The optimized expression of a high dimensional function/mainfold in a lower dimensional space: Chinese Bulletin, **46**, 977–984.

Soubaras, R., and Zhang, Y., 2008, Two-step explicit marching method for reverse time migration: 78th Annual Inter. Meeting, SEG, Expanded Abstracts, pages 2272–2276.

Tal-Ezer, H., Kosloff, D., and Koren, Z., 1987, An accurate scheme for forward seismic modeling: Geophys. Prosp., **35**, 479–490.

Whitmore, D. W., 1983, Iterative depth migration by backward time propagation: 53rd Annual Inter. Meeting, SEG, Expanded Abstracts, pages 382–385.

Zhang, Y., and Zhang, G., 2009, One-step extrapolation method for reverse time migration: Geophysics, **74**, A–29–A33.