FINITE HALF SPACE MODEL: IMPLICATIONS FOR OCEANIC HEAT FLOW AND GLOBAL HEAT LOSS.

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Abstract

It is well known that Half-space cooling (HSC) model fails to provide a satisfactory account of the regional variations in heat flow and bathymetry of the oceanic lithosphere. In this work, we point out that this problem arise from inherent drawbacks in the physical basis of HSC model formulation and present a new model that can overcome such limitations. According to the new model the formation of lithosphere is assumed to take place as a boundary layer, arising from cooling and solidification of the magma rich mantle (MRM) layer on top of the asthenosphere. As a result, the development of the boundary layer take place in finite half-space, and not in semi-infinite half-space as stipulated in the HSC model.

The new model (designated FHS) formulation is compatible with the current knowledge of the spatial domains of the lithosphere and the magma rich mantle layer (MRM) at the top of the asthenosphere. It also takes into account the effects of latent heat of the MRM layer in the solution of the heat conduction equation used in the determination of cooling history of the lithosphere. Results of numerical simulations reveal that theoretical values derived from FHS model provide vastly improved fits to observational data for heat flow and bathymetry for the entire age range of the oceanic lithosphere, when compared with those that can be achieved using HSC and Plate models. More importantly, the improvements in model fits have been achieved without the need to invoke the ad-hoc hypothesis of large-scale hydrothermal circulation in the stable ocean basins. Implications of the new model results for understanding regional scale variations in global heat flow are discussed and the need to downsize the current estimates of global heat loss emphasized.

Introduction

It is common practice to make use of half-space cooling (HSC) model (e.g.: Parsons and Sclater, 1977; Schubert et al, 2001) as a convenient means of investigating regional variations of heat flow and bathymetry in oceanic regions. Nevertheless this model fails to provide a satisfactory account of some of the important features of large-scale variations in oceanic heat flow. One of the major problems is that the theoretical values of heat flow derived from the HSC model are much higher than the observed ones in regions close to the ridge axis. The problem is further complicated by the fact that HSC model predicts values systematically lower than the observed ones in stable ocean basins. In the present work, we point out that such problems arise from inherent drawbacks in the HSC model formulation. In the present work we present a new model (designated the finite half-space cooling model - abbreviated FHS) that can overcome the above mentioned problems and at the same time provide a satisfactory account of large scale variations in heat flow and bathymetry of the ocean floor.

As prelude to presentation of FHS model we discuss the basics of boundary layer formation in magma chambers, which seems to have been the backbone of all HSC models, including the historical attempt by Kelvin to determine the age of the Earth (Burchfield, 1975). Following this the details of the FHS model fits to observational data on heat flow and bathymetry are presented, along with results of numerical simulations exploring the influence of model parameters. Implications of the new model results for understanding regional scale variations in heat flow and bathymetry of ocean floor are discussed and the need to downsize the current estimates of global heat loss emphasized.

Boundary Layer Formation in Magma Chambers

The wide acceptance of the HSC model stems in part from its ability to account for the growth of solidified layers over molten magma bodies. Hence it is worthwhile examining the fundamentals of this problem of boundary layer formation before considering its application for oceanic lithosphere. Our starting point is the relation for energy balance of a volume element at the interface between the boundary layer and the magma rich layer. In the presence of latent heat effects, this balance may be written as (Carslaw and Jaeger, 1959; Ozisik, 1980):

$$\int_S q_i \, ds + \int_V \rho L \frac{dV}{dt} = \int_S q_s \, ds + \int_V \rho C_p \frac{dT}{dt} dV \quad (1)$$

where T is temperature, qi and q_s the influx and out flux of heat relative to the volume element, ρ the density, L the latent heat and t the time. A schematic illustration of this case is given in figure (1).

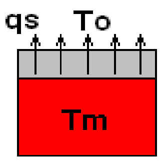

Figure (1) - A schematic illustration of solidification block.

In this figure the rectangle in red color represents the part of the magma chamber in the liquid state. The upper ash coloured rectangle represents the boundary layer of thickness (L), Tm the temperature of the liquid magma chamber, T0 the surface temperature of the confining boundary layer and qs the heat flux through the boundary layer at its surface. The thickness (L) is a function of the time elapsed (t). The conditions imposed in obtaining the solution of equation (1) are:

1- The base of the magma chamber remains at depth values much larger when compared with the thickness of the boundary layer. This condition constitutes essentially the semi-infinite half-space approximation;

2- All of the thermal energy released during solidification is lost by conduction, and consequently there are no variations in the internal energy of the volume element;

3- The out flow of heat is one-dimensional, vertically upwards. The implication is that lateral temperature variations may be neglected.

Under these conditions equation (1) reduces to:

$$\int_V \rho \, L \, \frac{dV}{dt} = \int_S q_S \, ds \qquad (2)$$

For one dimensional heat transfer equation (2) may be rearranged as:

$$\rho \; L \, \frac{dz}{dt} = \lambda \, \frac{dT}{dz} \qquad (3)$$

The relation for the thickness of the boundary layer is:

$$dz = \sqrt{(T_m - T_o) dt / \rho \; L} \qquad (4)$$

According to equation (4) the boundary layer growth is directly proportional to the square root of time elapsed, provided the physical properties are constants and the medium is homogeneous. The solution (4) has been used in modelling the growth of boundary layers in magma chambers. An example is illustrated in Figure (2) for the case of lava lakes of Hawaii (Turcotte and Schubert, 1967).

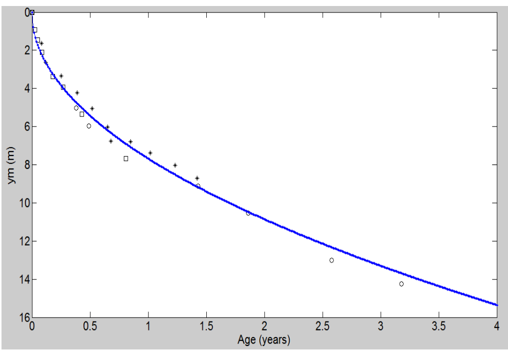

Figure (2) – Model fit to Boundary Layer Formation over Magma Chambers in lakes of Hawaii.

In this figure the curve in blue color refers to the results of model simulation assuming a solidification temperature (Tm) of 1300K and latent heat of solidification of 700 kJ / kg. As mentioned earlier, the solution of equation (4) is based on the condition that the thickness of the magma chamber is much larger than that of the overlying boundary layer. This is a reasonable assumption in dealing with the problem of solidification of lava lakes of Hawaii, but as discussed below, leads to physically meaningless results when applied to formation of oceanic lithosphere.

HSC Model of the Oceanic Lithosphere.

In the HSC model the lithosphere is considered as the boundary layer of the mantle convection system. It grows in thickness continuously as it moves away from the up-welling limb of the mantle convection system. The geometry assumed in the HSC model is illustrated in Figure (3).

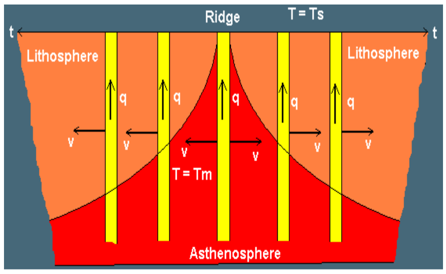

Figure (3) – Geometry in HSC model.

The basic assumption in HSC model is that the underlying magma rich asthenospheric mantle (designated hereafter as MRM) may be considered as a semi-infinite half space, for modeling effects of heat transfer. Nevertheless, it is important to point out that this approximation holds good only in cases where the thickness of the MRM layer are large relative to those of the overlying lithosphere. The conductive cooling is assumed to be one-dimensional, implying that the material beneath the boundary layer is homogeneous and have no lateral variations in its temperatures.

Analytical expressions for temperature variation in such a boundary layer may be obtained as solution to the relevant one-dimensional heat conduction equation:

$$\frac{\partial T}{\partial t} = \kappa \, \frac{\partial^2 T}{\partial z^2} \qquad (5)$$

where κ is the thermal diffusivity of the lithospheric material. If the characteristic wavelength of the problem addressed in equation (5) is assumed to be large ($z \to \infty$) its solution may be expressed as (Carslaw and Jaeger, 1959):

$$\frac{T(z,t) - T_0}{(T_m - T_O)} = erf \left(\frac{z}{2\sqrt{\kappa \; t}} \right) \qquad (6)$$

where erf is the error function. Equation (6) represents the fundamental solution of the HSC model. In obtaining the solution of equation (6) the effects of latent heat of the magmatic material below the boundary layer have been ignored. The relation for surface heat flux derived from equation (6) is:

$$q(t)_{z=0} = \frac{\lambda \, (T_m - T_0)}{\sqrt{(\pi \; k \; t)}} \qquad (7)$$

where λ is the thermal conductivity of the lithosphere. Figure (4) illustrates the fit of HSC model fit to oceanic heat flow data. As can be seen from this figure the HSC model is incapable of providing an adequate fit to experimental data. It predicts heat flow much higher than the observed values in areas of young ocean crust (with ages less than 60 My). Also, the HSC model predicts heat flow values much lower than the observed values for old ocean crust (with ages greater than 80 My).

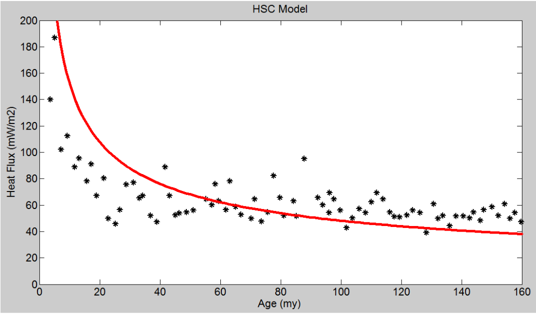

Figure (4) – Fit of HSC model fit to oceanic heat flow data.

The reason for the poor fit of HSC model to observational heat flow data set can be understood by considering the nature of boundary conditions imposed in obtaining the solution (equations 6 and 7). For example, the condition that the MRM layer beneath the lithosphere extends downwards to infinity in the z- direction is incompatible with thicknesses of the top part of the asthenosphere estimated from seismic data. In other words, the upper limit of integration used in the solution of differential equation of HSC model is incompatible with the vertical dimensions of the problem domain. The infinite half-space approximation also implies that the energy reservoir for heat supply at the bottom of lithosphere is large. An immediate consequence is that it leads to overestimates of heat flow for times short compared to the time constant of the overlying boundary layer. Another problem with the solution of equation (7) is that cooling proceeds indefinitely for all times. As a result the solidification isotherm migrates continuously to deeper depth levels, which leads to model heat flow values lower than the observed ones, for times large compared to the time constant of the lithosphere. In addition, the physical basis of the HSC model is deficient in the sense that it does not take into account the effects of latent heat in the solidification processes at the lower boundary of the lithosphere.

Finite Half Space Model of the Oceanic Lithosphere

It is clear that we need to look for a new model which can account for variations in MRM layer thicknesses compatible with results of deep seismic investigations of the lithosphere. In the formulation of such a model we note that the zone of upwelling material beneath the ridge axis has finite width. According to mass balance considerations the material in the upwelling zone is the same that moves laterally away from the ridge zone. Hence the thickness of the MRM layer, where lateral flow takes place, cannot be greater than the width of the zone of upwelling. In the case of laterally symmetric flows the thickness of the MRM layer beneath the boundary layer can only be half that of the upwelling zone. However, the boundary layer growth by solidification takes place at the expense of material in the MRM layer, which is confined between the lithosphere at the top and the relatively stagnant asthenosphere at the bottom. As a result there is a continuous decrease, with distance from the ridge axis, in the thickness of the MRM layer. At large distances from the ridge zone the thickness of the lithosphere approaches an asymptotic limit determined by the initial thickness of the MRM layer. In other words, the boundary layer development may be considered as taking place in finite half-space, and not in semi-infinite half-space as stipulated in the HSC model. A schematic illustration of the geometry of this model, designated as the finite half-space model (abbreviated FHS) is provided in Figure (5).

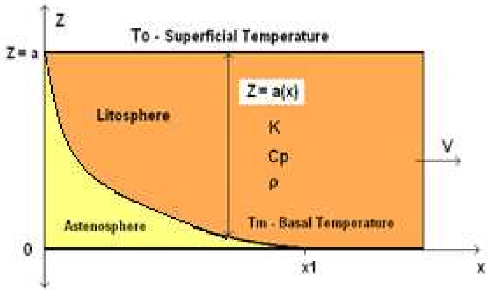

Figure (5) – Geometry in FHS model.

The physical basis of such a problem may be described in terms of magma accretion at the base of the lithosphere. Recently Hamza et al (2010) presented a model that takes into consideration the thermal consequences of variable rates of magma accretion at the lower boundary of the lithosphere. According to this model (designated VBA) variability in the rate of magma accretion at the base of the lithosphere has direct influence on surface heat flux and bathymetry and has been found to be capable of accounting for the main features in the respective observational data sets. Nevertheless, VBA model does not address explicitly the thermal effects of latent heat at the lithosphere – asthenosphere boundary. In the present work, we propose a modified form of the VBA model that incorporate the effects of latent heat and is compatible with results of deep seismic investigations of the oceanic lithosphere.

The general relation for energy balance of the volume element in the FHS model, including the effects of latent heat is:

$$\int_S q_E \, ds + \int_V \rho_{MRM} \, L \frac{dV}{dt} = \int_V \rho_{Lit} \, C_p \frac{dT}{dt} dV \qquad (7)$$

In equation (7) ρ_{MRM} and ρ_{Lit} are densities of the MRM and lithosphere layers and q_E is heat flux at the top of the volume element. It may be rearranged as:

$$\int_S q_E \, ds = \int_V \left(\rho_{Lit} \, C_p - \rho_{MRM} \, L \frac{dV}{dT} \right) \frac{dT}{dt} dV \quad (8)$$

The expression dV/dT is the volumetric proportion of the reaction that occurs per unit change in temperature. Note that V represents percent volume of the reaction and hence does not have the usual unit of cubic meters.

For the case of one dimensional heat flux in the z direction equation (8) may be rewritten as:

$$\frac{\partial T}{\partial t} = \kappa_{mod} \frac{\partial^2 T}{\partial z^2} \quad (9)$$

where k_{mod} is the modified thermal diffusivity given by:

$$\kappa_{mod} = \frac{\lambda}{\rho_{Lit}\, c_p - \rho_{MRM}\, L\, dV/dT} \quad (10)$$

According to equation (10) inclusion of the effects of latent heat leads to larger values of effective thermal diffusivity.

We now introduce the assumption that the characteristic wavelength of the solution is related to the thickness of the stable lithosphere at large distances from the ridge axis. The solution of equation (9) in this case is:

$$\frac{T(z,t)-T_O}{(T_m - T_O)} = \frac{erf\left(z/2\sqrt{\kappa_{mod}\, t}\right)}{erf\left(a/2\sqrt{\kappa_{mod}\, t}\right)} \quad (11)$$

Equation (11) satisfies the initial and boundary conditions of the problem under consideration. Thus, at the ridge axis (ie: as t → 0) the temperature T → Tm for all depth levels, while at large distances the temperature variation with depth tends towards an asymptotic limit depending on the relative magnitude of z with respect to the initial thickness of the MRM layer. However, at the base of the MRM layer (ie: at z = a) the temperature is Tm for all times. A number of simulations were carried out in investigating the FHS model response to changes in parameter values. The values of the parameters employed in simulation are provided in Table (1).

The temperature field described by equation (11) is shown in figure (6).

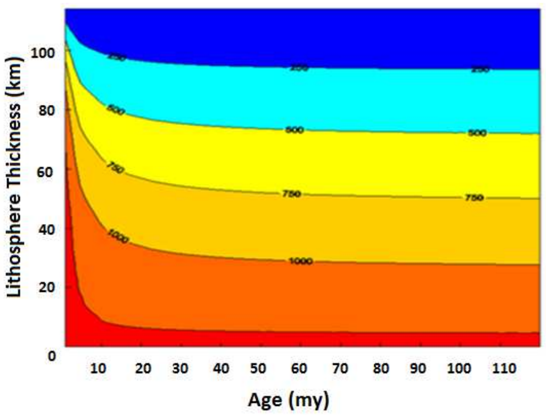

Figure (6) – Temperature field.

An important feature easily discernible in this figure is that the region where significant temperature variations occur is restricted to young ocean floor with age less than 30 My.This result is significantly different from that obtained in the HSC Model, where anomalous temperatures extend to ages in excess of 55 My (Cardoso and Hamza, 2006; Cardoso et al 2009). Another important point is that the FHS model (Equation 11) leads to a family of solutions depending on the value chosen for the initial thickness of the MRM layer below the lithosphere. The solution used in the HSC model (Equation 6) is a particular case of the more general solution obtained in the FHS model (Equation 11), in which the thickness of the underlying MRM layer is very large (ie: as a → ∞).

Ocean Floor Heat Flux

The equation for surface heat flux obtained from equation (11) is:

$$q(t)_{z=0} = \frac{\lambda(T_m - T_0)}{\sqrt{(\pi\, k_{mod}\, t)} \times erf\left(a/2\sqrt{k_{mod}\, t}\right)} \quad (12)$$

In equation (12) the thermal effects of latent heat are taken into account through the use of the term κ_{mod} for the effective thermal diffusivity. The solution for the case where latent heat effects can be ignored may be obtained by using thermal diffusivity κ, as defined in equation (5), in place of κ_{mod}.

The FHS model fits to observational heat flow data for oceanic regions are illustrated in Figure (7) for two cases: one in which effects of latent heat are included (the blue curve) and a second one in which latent heat effects are ignored (the red curve). Note that the heat flow values are systematically low in the first case when compared with the second, for the entire age range considered. The reduction in heat flow is a consequence of the enhanced thermal diffusivity which leads to relatively low temperature gradients and consequently low heat flow.

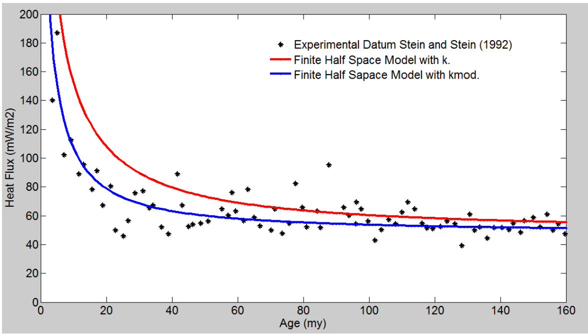

Figure (7) – FHS model fits to observational heat flow data for oceanic regions.

It is clear from Figure (7) that the fit to observational heat flow data is better when latent heat effects are included. The fit is valid for the entire age range of the oceanic lithosphere. More importantly, it has been achieved without the need to introduce artificial adjustments based on the ad-hoc hypothesis of regional hydrothermal circulation in stable ocean crust.

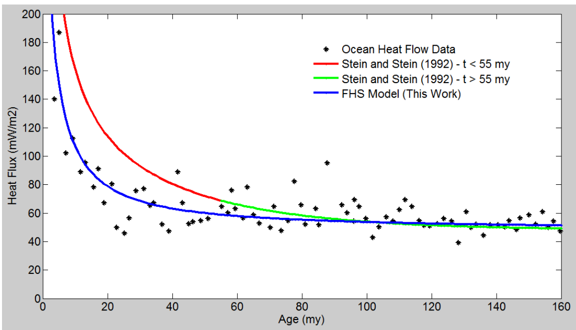

Figure (8) – Comparison between FHS model and GDH model for ocean floor heat flux.

The fit obtained in Figure (7) may be compared with that presented by Stein and Stein (1992) on the basis of what has been termed as Global Depth Heat Flow (GDH) reference model. The GDH is however a hybrid model, based on the sequential use of two different relations: HSC model fit for ages <55 My and Plate model fit for ages >55 My. The choice of the age value of 55 My is based purely on statistical considerations. This major difference between the FHS and GDH model fits is illustrated in Figure (8).

Fit to Ocean Floor Bathymetry.

Fits to ocean floor bathymetry rather than surface heat flow is often considered a relatively more rigorous test of the thermal models of the lithosphere. The relation for bathymetry in FHS model has been developed following the isostatic compensation scheme discussed in earlier studies (e.g. McKenzie, 1967; Sclater and Francheteau, 1970; Parsons and Sclater, 1977). The equation for bathymetry obtained from equation (11) is:

$$e(t)=d_r+\frac{\alpha a T_m \rho_{MRM}}{(\rho_{MRM}-\rho_{Wat})}\left[\frac{2\sqrt{k_{\mathrm{mod}}\,t}\left(1-e^{-\left(a/2\sqrt{k_{\mathrm{mod}}\,t}\right)^2}\right)}{a\sqrt{\pi}\;erf\left(a/2\sqrt{k_{\mathrm{mod}}\,t}\right)}\right] \qquad (12)$$

In this equation d_r is the depth of ocean floor at the ridge axis, α is the coefficient of linear thermal expansion, ***a*** the thickness of the lithosphere in stable ocean basins, ρ_{MRM} is the density of the material in the MRM layer and ρ_{wat} is the density of ocean water.

The fit obtained using equation (12) for ocean floor bathymetry is illustrated in Figure (9). As in the previous case for heat flow (see figure 7) model fits for two conditions are illustrated: the blue curve represents the case where latent heat effects are included. The red colored curve refers to the case where latent heat effects are ignored.

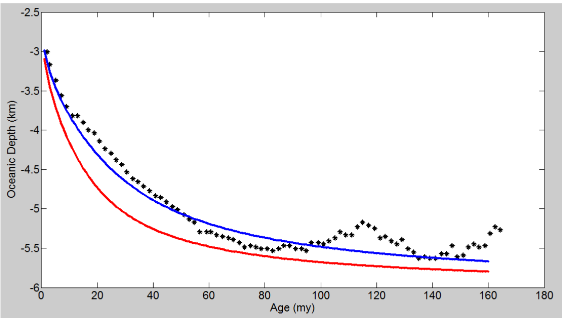

Figure (9) – FHS model fits to observational ocean floor bathymetry.

As in the case for heat flow discussed earlier (see Figure 7) the FHS model fit to bathymetry data is better when latent heat effects are included. The fit is valid for the entire age range of the oceanic lithosphere.

Hence the practice of using hybrid relations (HSC model fit for ages <20 My and Plate model fit for ages >20 My) proposed by Stein and Stein (1992) is unnecessary. This major difference between the FHS and GDH model fits is illustrated in Figure (10).

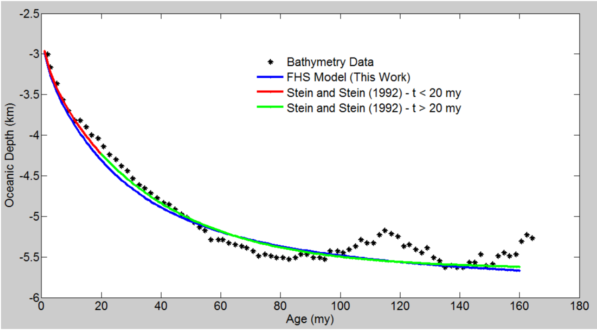

Figure (10) – Comparison between FHS model and GDH model for ocean bathymetry.

Discussion and Conclusions

It is clear that the results obtained the FHS model of the present work provides vastly superior fits to oceanic heat flow and bathymetry data, relative to that can be achieved by HSC model. More importantly, such fits has been achieved without the need to invoke the hypothesis of regional scale hydrothermal circulation in crust of stable ocean basins.

According to Davies and Davies (2010) the HSC model fit has an upper limit of 66.5 My. On the other hand the FHS model provides vastly superior fits for ages up to 160 my. Thus it is unnecessary to make use of hybrid schemes (combinations of HSC and Plate models) with arbitrarily selected age values in obtaining fits for larger age ranges. In figure (11) we present a comparison between the FHS model of the present work and the different forms of the HSC models discussed in the literature (Parsons and Sclater, 1997; Stein and Stein, 1992; Jaupart et al, 2007).

Acknowledgments

The first author is recipient of a research scholarship granted under the program PCI – Programa de Capacitação Institucional. This work was carried out as part of research project with funding from Conselho Nacional de Desenvolvimento Científico – CNPq (Project No. 301865/2008-6; Produtividade de Pesquisa - PQ). We thank Dr. Iris Escobar (ON- Observatório Nacional) for institutional support.

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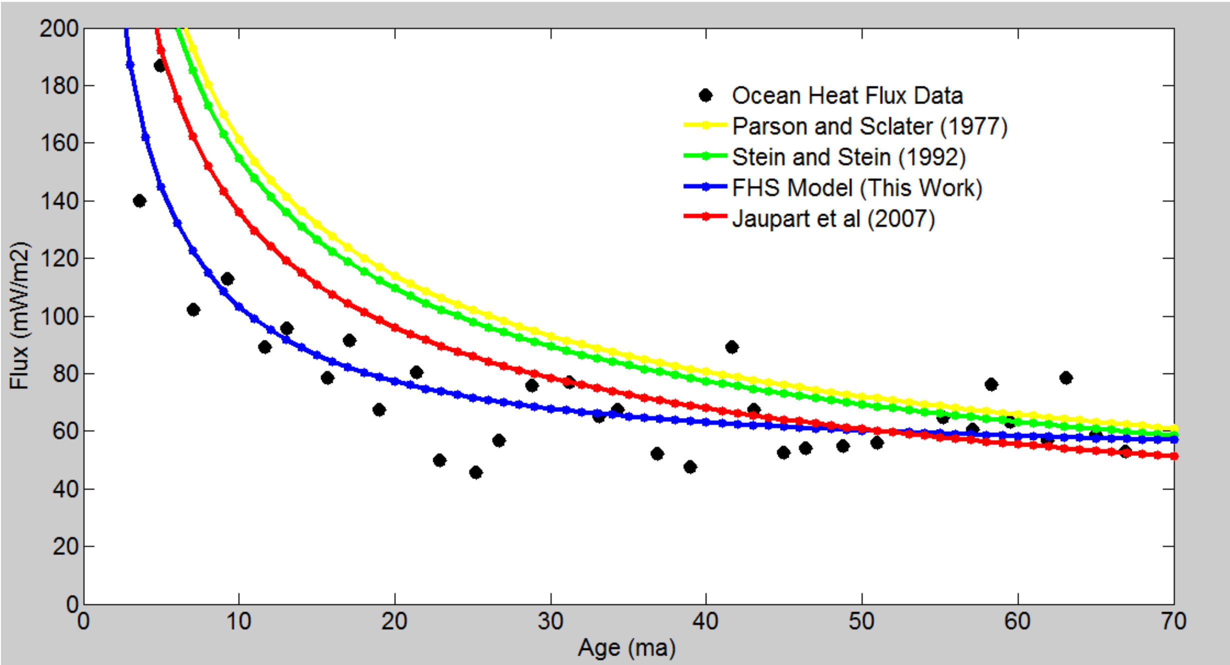

Figure (9) – Comparison between the FHS model of the present work and the different forms of the HSC models discussed in the literature (Parsons and Sclater, 1997; Stein and Stein, 1992; Jaupart et al, 2007).