Evaluation of Lichtenecker's Mixing Model for Effective Permittivity at 50MHz

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Abstract

Mixing models can be used to study the contribution of each phase to the bulk dielectric permittivity of porous materials. They are particularly useful when studying the electromagnetic properties of soils and vadose zone using TDR or GPR techniques. The objective of this research was to evaluate the Lichtenecker Mixing Model using data from a 50MHz impedance sensor. Laboratory data was used to estimate the solid phase permittivity and the α parameter. The estimated α and solid phase permittivity values were within the ranges reported in the literature. Results show that the use of mixing models and inverse modeling using nonlinear regression can be a useful approach for estimating soil electromagnetic properties.

Introduction

The effective or bulk dielectric permittivity (ε_b) of a porous material can be modeled using a class of models known as mixing models. A mixing model predicts ε_b based on the individual permittivity for each phase (ε_i). One of the simplest forms of mixing models is given by (Brovelli & Cassiani, 2008):

$$\varepsilon_b^{\alpha} = \sum_{i=1}^{n} \phi_i \varepsilon_i^{\alpha} \qquad (1)$$

where ϕ_i is the volume fraction of each phase and α is a dimensionless fitting parameter. In soils, rocks and other porous materials containing a solution at different degrees of saturation the model is composed of three phases, each with a distinct dielectric permittivity: the solution phase (ε_w) the matrix or solid phase (ε_s) and the gas phase (ε_a). Under such conditions, Equation 1 assumes the following form (Zakri et al., 1998):

$$\varepsilon_b^{\alpha} = \theta \varepsilon_w^{\alpha} + (1-\phi)\varepsilon_s^{\alpha} + (\phi-\theta)\varepsilon_a^{\alpha} \qquad (2)$$

where ϕ is the porosity of the material (cm^3 cm^{-3}), and θ is the volumetric solution content (cm^3 cm^{-3}). Equation 2 is also known as the Lichtenecker or Lichtenecker-Rother equation (Zakri et al., 1998; Brovelli & Cassiani, 2008). According to the literature, the α parameter varies between -1 and 1, with each end of the range being associated with the equivalent of an electrical circuit wired in parallel (α = 1) or in series (α = -1) (Figure 1). A special solution of Equation 2 can be found for isotropic media, when α is empirically set to 0.5 (Equation 3) (Brovelli & Cassiani, 2008).

$$\varepsilon_b^{0.5} = \theta \varepsilon_w^{0.5} + (1-\phi)\varepsilon_s^{0.5} + (\phi-\theta)\varepsilon_a^{0.5} \qquad (3)$$

Equation (3) is known as the Complex Refractive Index Model (CRIM) and has found extensive usage as a mixing model for estimating the effective permittivity and modeling the individual conductivity of specific phases using Time Domain Reflectometry (TDR) and Ground Penetrating Radar (GPR) techniques (Huisman et al., 2003).

The objective of this research was to investigate the ranges of solid phase permittivity (ε_s) and α exponent for three contrasting soils estimated by inverse modeling using Equations 2 and 3.

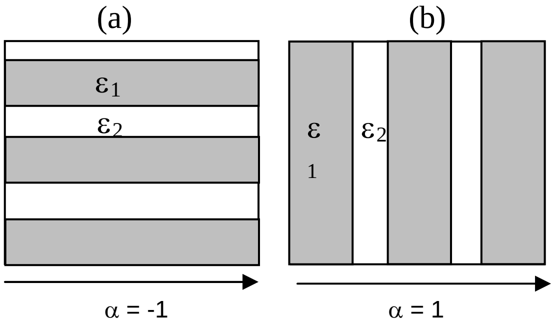

Figure 1. Parallel (a) and series (b) arrangements of an idealized 2-phase porous material with respect to the electrical current direction (indicated by the arrow).

Methods

Thirty "undisturbed" soil cores (5.37 cm inner diameter × 6 cm tall) and three bulk soil samples were collected on June 10, 2005 at the Plant Sciences experimental farm at the University of Tennessee, Knoxville. The sampling was performed in areas with different soil series, covering three contrasting soil textural classes:

Clay (Typic Paleudult), Sandy Loam (Humic Hapludult), and Silty Clay Loam (Fluvaquentic Eutrudept), according to the USDA system. All samples were collected at a depth of 20 to 25 cm. The bulk soil samples were air dried, broken apart by hand, sieved with a 2 mm mesh sieve, and packed to give ten "disturbed" cores for each soil texture. The disturbed cores were the same size as the undisturbed cores. Duplicate samples of the disturbed and undisturbed cores were saturated from the bottom up, with saline solutions at five concentrations, namely distilled-deionized water (~0 or control), KCl at 0.01 and 0.02 Mol L^{-1}, and $CaCl_2$ at 0.01 and 0.02 Mol L^{-1} for three days. A Hydra Probe sensor (Stevens Water Monitoring System Inc., 2007) was inserted into one end of each sample. The samples were then placed horizontally on load cells (Transducer Techniques, model LSP-1), and the Hydra Probes and load cells were connected to dataloggers (VITEL VX1100 and Campbell 21X micrologger, respectively). The real component of the soil bulk permittivity (ε_b) was measured by the Hydra Probe, while the change in weight of the samples over time, due to air drying, was measured by the load cells and used to calculate the volumetric water content. The permittivity of the air (ε_a) was assumed to be unity (Kraus, 1992), while that of the solution (ε_w) was measured by immersing the Hydra Probes in each solution, resulting in values 84 for distilled-deionized water, 77.9 and 82.4 for KCl at 0.01 and 0.02 Mol L^{-1}, and 77.9 and 82.2 for $CaCl_2$ at 0.01 and 0.02 Mol L^{-1}, respectively. All statistical analyses were performed using the SAS® Statistical Analysis System software package.

Results

Data from the drying experiments for each soil core were fitted to Equation 3 using nonlinear regression. The analyses of variance (ANOVA) from the nonlinear regressions show that all of the fits were significant (*Pr > F < 0.0001*). The default nonlinear regression convergence criterion was also met in all cases. The average approximate coefficient of determination (R^2) was 0.98 with a minimum of 0.96 and a maximum of 0.99. The average (and standard deviation in parentheses) predicted solid phase permittivity (ε_s) was 7.56(±2.24) dS m^{-1} for the Clay soil, 6.33(±0.87) dS m^{-1} for the Sandy Loam soil and 9.38(±1.94) dS m^{-1} for the Silty Clay Loam soil (Figure 2).

When both coefficients from Equation 2 (ε_s and α) were estimated by nonlinear regression, convergence was not achieved for all samples. The nonlinear estimation procedure failed to converge in 20% of Clay soil samples and in 40% of Silty Clay Loam soil samples even though a robust fitting technique and a larger number of iterations were used (Leao et al., 2005). Convergence problems are usually caused by adding more parameters to a nonlinear model, resulting in the model having more parameters than are necessary to fit the empirical data, or when the model does not conform to the shape of the dataset. For the remaining samples all of the fits were significant (*Pr > F < 0.0001*) with average, minimum and maximum R^2 values of 0.99, 0.96 and 0.99, respectively. The average (and standard deviation) values for the estimated ε_s were 5.08(±2.69) dS m^{-1} for the Clay soil, 5.01(±0.71) dS m^{-1} for the Sandy Loam soil, and 4.59(±3.09) dS m^{-1} for the Silty Clay Loam soil (Figure 2). The average (and standard deviation) values for the α parameter were 0.57(±0.15) for the Clay soil, 0.59(±0.07) for the Sandy Loam soil, and 0.66(±0.15) for the Silty Clay Loam soil. The ε_s values estimated using Equation 3 were significantly different than those estimated using Equation 2 (*Pr* > |*t*| < 0.0001).

The average values discussed above were presented by soil type because it was expected that both ε_s and α would primarily be a function of soil type, being directly influenced by clay content and mineralogy. To assess if the estimated coefficients were influenced by other variables, an ANOVA was performed with the coefficients estimated from Equations 2 and 3 as independent variables and soil, disturbance and salt treatment and their interactions as class variables.

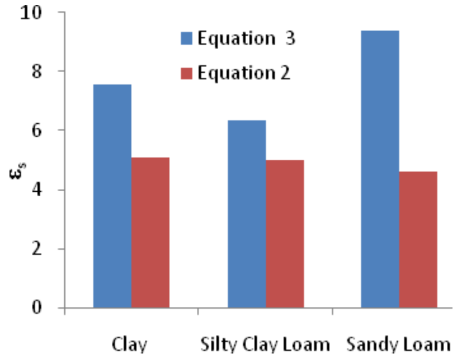

Figure 2. Average solid phase permittivity for each soil fitted using Equations 2 and 3.

The ANOVA for the ε_s values estimated using Equation 3 was significant (*Pr > F* < 0.0001). However, of the 3 class factors included, only soil was significant. An evaluation of the ε_s values using the Duncan's Multiple Range test (Leao, 2009) for separation of means showed that all three soils were statistically different with regard to the ε_s values with averages decreasing from the Silty Clay Loam soil to the Sandy Loam soil. For parameters estimated using Equation 2 the ANOVA's were significant for ε_s (*Pr > F* = 0.0013), and α (*Pr > F* = 0.0224). As the probability values indicate, the degree of certainty was less than in the case of the data from Equation 3. This might be partly due to the missing coefficients resulting from failure to converge in all cases. For both coefficients from Equation 2 the only significant factor was disturbance. The average ε_s was higher for undisturbed conditions (6.04; ±2.14) and lower for disturbed conditions (4.07; ±1.84) (Figure 3).

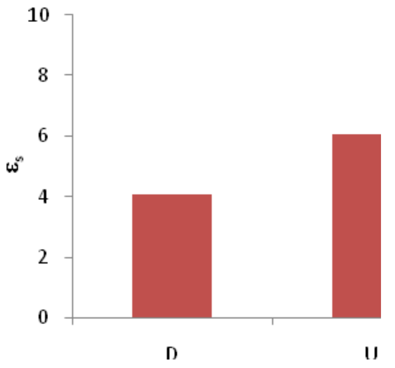

Figure 3. Average solid phase permittivity for the disturbed (D) and undisturbed (U) samples fitted using Equation 2.

The α coefficient was higher for the disturbed cores, with an average value of 0.64(±0.12), and lower for the undisturbed cores, with an average value of 0.55(±0.13) (Figure 4).

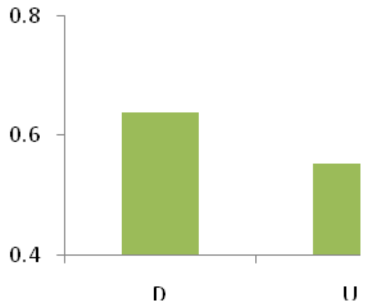

Figure 4. Average α for the disturbed (D) and undisturbed (U) samples fitted using Equation 2.

Discussion and Conclusions

The solid phase permittivity of common soil minerals was evaluated by Robinson (2004). The ε_S of quartz was 4.4(±0.3) while that of phyllosilicate minerals was slightly higher. Kaolininte had an average ε_S of 5.1(±0.7) and biotite mica had an average of 6.0(±0.5) while the 2:1 grade minerals illite and montmorillonite had ε_S values of 5.8(±0.2) and 5.5 (standard deviation not presented) respectively (Robinson, 2004). We found that independent of the method of estimation the Sandy Loam soil had a lower estimated ε_S value which was closer to the value measured for quartz by Robinson (2004). For the other two soils with a higher content of 2:1 grade minerals and kaolinite the values were slightly higher which was also in agreement with Robinson (2004). The mineralogy of the Clay soil is composed of 8% kaolinite and 9.2% of combined vermiculite, illite and mica; the Silty Clay Loam soil has 3.7% kaolinite and 9.7% of combined vermiculite, illite and mica, while the Sandy Loam soil has 1.2% kaolinite and 5.1% of combined vermiculite, illite and mica, all on a weight percent basis (Leao, 2009). The ε_S values are also close to the ranges reported for soils and minerals by Kraus (1992) and Cassidy (2009) indicating that inverse modeling of mixing models is an effective tool for estimating solid phase permittivity.

The α parameter estimated by the nonlinear regression procedure varied from 0.33 to 0.90 with average and standard deviation values of 0.60(±0.13). The values were within the ranges of +1 e -1 reported for the parameter (Brovelli & Cassiani, 2008) and the average was close to the value of 0.5 used in the CRIM model, indicating that the soils under evaluation were close to isotropic media. Somewhat surprisingly, the average α value for undisturbed conditions was closer to 0.5 than that for the disturbed samples, indicating that the undisturbed media had a higher degree of isotropy. Perhaps the packing process introduced a degree of anisotropy in the disturbed samples. In general, the results show that the use of mixing models and inverse modeling using nonlinear regression can be a useful approach for estimating soil electromagnetic properties.

Acknowledgements

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