Correlation properties studies in complex networks of worldwide earthquakes and simulated data

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Abstract

We studied the correlation properties of worldwide and synthetic earthquake networks by analyzing their assortativity. The actual seismological data were obtained from the World Earthquake Catalog of Advanced National Seismic System for 2002 to 2016, and divided in shallow earthquakes (depth up to 70 km) and deep earthquakes (depth greater than 70 km). Synthetic data were produced from simulations using a modified version of the Olami-Feder-Christensen model. To calculate the correlation measures for all networks, it was used two methodologies of connections between the network elements. The results for shallow earthquakes and synthetic data indicate assortative correlation, while the deep earthquake networks are neutral.

Introduction

The complex network theories started to be used in the seismological study in Abe & Suzuki (2004a, 2004b). The authors constructed earthquake networks for seismic data from California and Japan, taking into account spatial and temporal information of successive earthquakes. It is important to highlight that there are different network construction methods and analyses, and the results found in the various models show that the earthquake networks have similar behavior to other networks found in nature for many different phenomena.

A handy tool in studying earthquakes is the use of computer simulations. In this way, the use of complex networks for the statistical study of earthquakes can be done by using actual earthquake data or by using synthetic data generated through computer simulation models. One of the most used simplified models is the one created by Olami, Feder and Christensen (OFC model), which reproduces several characteristics of earthquakes (Christensen & Olami, 1992a, 1992b; Olami et al. 1992).

Previous works have analyzed seismological data (from actual and synthetic earthquake catalogs) performing some of the most common and fundamental features of complex networks, such as the degree distribution, the clustering coefficient, and the average shortest path. However, another interesting characteristic to be studied in earthquakes is the correlation property. Correlation properties can be analyzed in complex networks using a measure named *assortativity*, which indicates a type of connection preference that elements tend to have when connecting to each other.

In this work, we analyze the assortativity features for the networks created from worldwide seismic events using two different models of connections for two different datasets: one for shallow earthquakes and the other for deep earthquakes. We differentiate the seismic events concerning their depths since shallow and deep earthquakes are mechanically different from each other (Frohlich, 1989, 2006). Therefore, we make the data division by depth to observe whether it would exist differences in the properties of the networks created with these two datasets. We adopted the division used in Frohlich (1989) and Spence et al. (1989): shallow earthquakes are those with depth up to 70 km, and deep earthquakes are the ones located deeper than this value. Furthermore, a network of successive connections was also created using earthquake data simulated with a modified version of the Olami-Feder-Christensen (OFC) model (Ferreira et al., 2015). The results were compared with those of the actual seismic events.

Correlation Properties

Assortativity (or assortative mixing) refers to the tendency of nodes in a network to connect to other nodes with similar properties. Here, we focus on assortativity in terms of a node's degree, k. The analysis of this property allows us to investigate the relation between the connectivity degrees of the nodes that link to each other. A statistical measure that is commonly used to analyze this preference is the nearest-neighbors average connectivity of nodes (Pastor-Satorras et al., 2001; Vázquez et al., 2002; Barabási and Pósfai, 2016), expressed as

$$k_{nn}(k) = \sum_j jP(j|k) \qquad (1)$$

where $P(j|k)$ is the conditional probability that an arbitrary selected edge links a j-degree node with a k-degree node. This function considers the average degree of the neighbors of a node as a function of its degree k.

When it is independent of k, the network has no obvious correlation of degree and is called neutral. If, however, $k_{nn}(k)$ increases with k, the network is assortative. This means that the hubs (nodes with high degrees) of the network tend to connect to other hubs and nodes with low degrees tend to be linked to other low degrees nodes. On the other hand, if $k_{nn}(k)$ decreases with k, the network is disassortative, i.e., the hubs prefer to link to nodes with low degrees (Barabási and Pósfai, 2016). Thus, the nearest-neighbors average connectivity of nodes can help to detect the presence or absence of correlations in real networks.

The degree correlation coefficient, which is the Pearson correlation coefficient between the degrees found at the two ends of the same link, is a complementation of the analysis of the nearest-neighbors average connectivity of nodes, and gives to us a quantitative characterization. We calculate this coefficient by

$$r = \sum_{jk} \frac{jk(e_{jk} - q_j q_k)}{\sigma^2}, \qquad (2)$$

where e_{jk} is the probability of finding a node with degrees j and k at the two ends of a randomly selected link, q_k is the probability of existing a node with degree k at the end of a randomly selected link, and σ^2 is the variance of q_k.

The value of r varies from -1 (perfect disassortativity) to 1 (perfect assortativity). If r = 0, then the network has no assortative (or disassortative) mixing and, therefore, is neutral.

Data

The worldwide earthquakes dataset was separated in shallow earthquakes and deep earthquakes, with 80520 and 21226 seismic events, respectively. Only earthquakes with magnitude m ≥ 4.5 in the Richter scale were considered.

To generate the synthetic seismic catalog, we used a modified version of the original OFC model (Olami et al.,1992). The original model can be represented by a bi-dimensional square $\ell \times \ell$ lattice of $N = \ell^2$ blocks (sites) interconnected by springs, where each block is also connected through a spring to a single rigid driven plate and by friction to other rigid fixed plate on which they stay. This is the regular topology of the lattice. Due to the relative motion between the plates (imposed by the model), all the blocks will be subjected to an elastic force which tends to put them in motion and other frictional force opposite to the first. When the resulting force in one of the blocks is greater than the maximum static friction force, the block slides and relaxes to a position of zero force, so that there is a rearrangement of forces in its first neighbors, which can cause other slippages and the emergence of a chain reaction. The first block to move is the epicenter of the earthquake and we measure the magnitude s of this earthquake by the number of blocks that skidded.

In this work, we used a lattice with small-world topology instead of the regular one. As done by Ferreira et al. (2015), this new small-world topology is built from the regular topology, where each edge of the network is reconnected randomly with probability p, keeping fixed the original degree of each site. Our analysis was conducted using p = 0.001 for a lattice of size ℓ = 400, a dissipation coefficient α = 0.20, and the number of events generated was 2×10^7, after the transient regime.

Methods

We followed the definition used in Ferreira et al. (2014) to construct the network of global epicenters: the surface of the planet is divided into equal square cells of size $L \times L$, with L= 20 km, and a cell becomes a node of the network every time the epicenter of an earthquake is located therein. To create the links between the nodes, we used two methodologies, which are described below.

Successive model

This method is the same created by Abe and Suzuki (2004a, 2004b) and employed in Ferreira et al. (2014). It consists of connecting a node to its subsequent one, in the temporal order, by a directed edge. Thus, in this model, the construction of the network considers that each earthquake is related to the one that happens right after it in the temporal series, regardless of the time difference between them.

Time window model

This refined model was proposed in Ferreira et al. (2018) and showed evidences of being a better approach to construct networks of earthquakes from all over the world. It consists of defining a *time window, T*, which is placed on the chronologically ordered data, to create the links between the nodes. The first node inside the window is connected to all other nodes within that window by directed edges. Thereafter, the window is moved forward to the next event and the connections procedure is repeated. Figure 1 illustrates an example of this process.

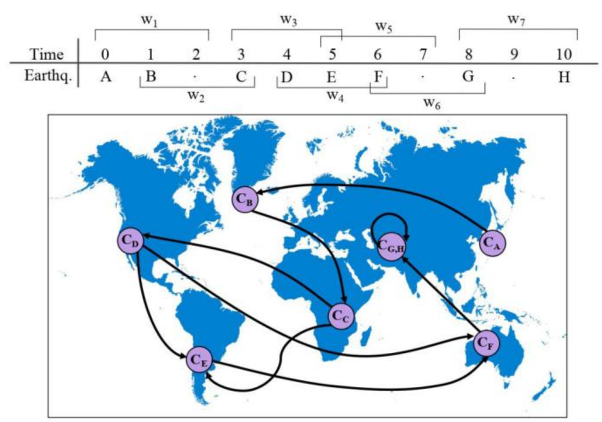


Figure 1 - Network's construction for the time window model. The time windows are represented by w_i, where i is the window number, and all the time windows must have the same value (in this example, T = 2, in arbitrary units). Events in the same window are connected as explained in the text. We can see that there are 8 earthquakes (A, B, C, D, E, F, G, H), but the epicenters network has only 7 nodes (C_A, C_B, C_C, C_D, C_E, C_F, C_G), because $C_G = C_H$. It can also be observed that the link between C_G and C_H is a self-link.

We built networks using both the successive and the time window model for the shallow and deep earthquakes collected. The time window, *T*, values were *T* = 3800s, for shallow seismic events, and *T* = 16500s, for deep earthquakes. These values were calculated in Ferreira et al. (2018, 2020), respectively.

For the data generated with the OFC model, each epicenter was defined as a node and we constructed a network using the successive model of connections, as it was done in Ferreira et al. (2015).

Results

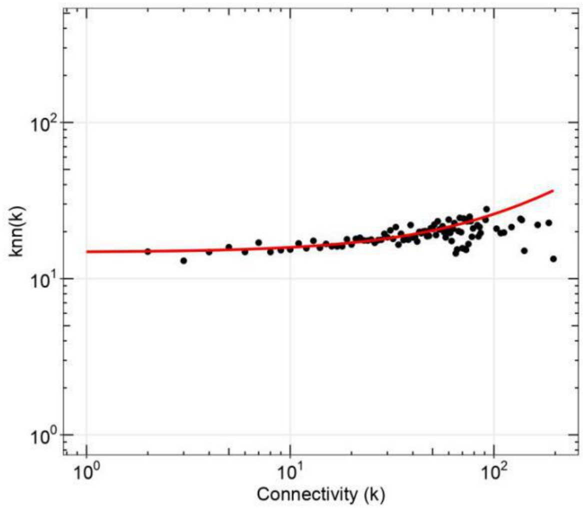
To analyze the assortativity of our earthquake networks, we calculated the nearest-neighbors average connectivity of nodes, $k_{nn}(k)$ (Eq. 1), using the degree k of the nodes.

Figure 2 shows a comparison between the network of shallow earthquakes built using the successive model and the time window model. It is observed that, in both distributions, the nearest-neighbors average connectivity of nodes, $k_{nn}(k)$, increases linearly with k, which means these networks are assortative. Therefore, the nodes with a high degree connect, on average, to nodes with a high degree. This result was the same found in networks of earthquakes from California and Japan (Abe & Suzuki, 2006), which makes sense since most earthquakes that occur in these areas have depths up to 70 km (shallow earthquakes).

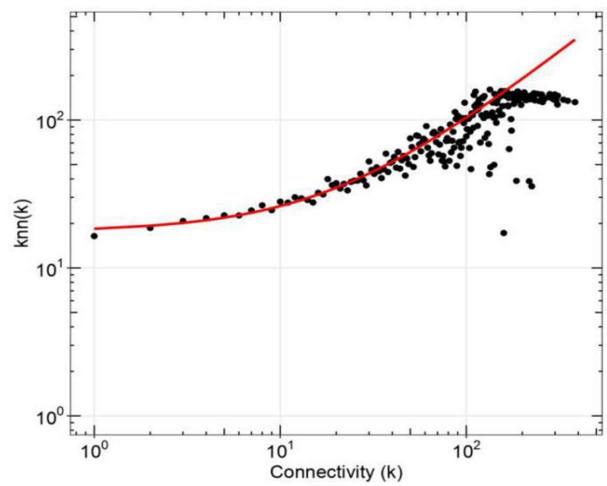
However, the network constructed with the time window model is more assortative than the one built with the successive model. It is interesting because Ferreira et al. (2018, 2020) showed that the time window model gives results that make more sense than the successive model (e. g., it naturally identifies the world's places with more occurrence of seismic events). The high assortative value found implies that areas of the world with intense shallow seismic activity are not only correlated but strongly correlated.

The results for the networks of deep earthquakes are shown in Figure 3. In both cases, successive and time window models, the networks are neutral, i.e., $k_{nn}(k)$ is independent of k. It means the world's geographical regions with greater deep seismic activity are correlated both with each other and with areas of less occurrence of deep earthquakes, without preference.

Finally, as shown in Figure 4, we found that the nearest-neighbors average connectivity of nodes, $k_{nn}(k)$, for the network of earthquakes simulated with the modified OFC model has an increasing behavior with k, in agreement with the results found for shallow earthquakes. Similar results were also found in a previous work conducted for networks built using the standard OFC model (Peixoto & Prado, 2006).



(a)

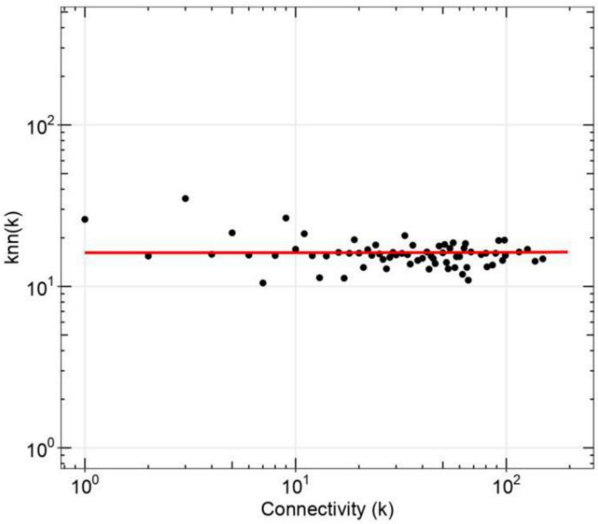


(b)

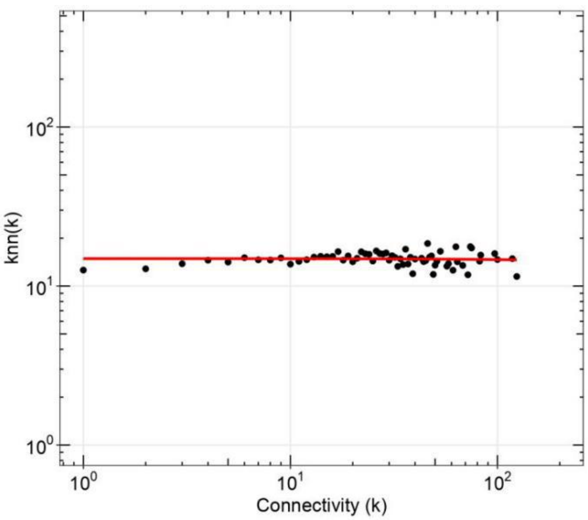
Figure 2 - Nearest-neighbors average connectivity of nodes, $k_{nn}(k)$, for the network of shallow earthquakes using (a) the successive model and (b) the time window model. It can be observed that both distributions follow a crescent linear fit (red line). These plots show that both networks have assortative mixing, being the network constructed with the time window model much more assortative.

We have also calculated the degree correlation coefficient, r, (Eq. 2) for each of our networks, and the results are shown in Table 1.

The networks of shallow events and the one constructed with synthetic data are assortative, since the values of r found are positive. Furthemore, the networks of deep earthquakes present $r \approx 0$, indicating that they are neutral. Therefore, all these results agrees with our findings for the nearest-neighbors average connectivity of nodes, $k_{nn}(k)$.



(a)



(b)

Figure 3 - Nearest-neighbors average connectivity of nodes, $k_{nn}(k)$, for the network of deep earthquakes constructed with (a) the successive model and (b) the tome window model. No correlation between $k_{nn}(k)$ and k is presented in the distributions, which means that both networks are neutral.

Futhermore, in Figures 5(a) and 5(b) are presented the geospatial image of the network of shallow and deep earthquakes, respectively, created with the time window model, where 2% of nodes with the highest degree (hubs) in the networks are displayed. For shallow events, these hubs hold 16% of all links of the network. We observe that hubs are not connected only to other close hubs (as in the case of Japan, which holds more than one hub), but they are also linked across the planet.

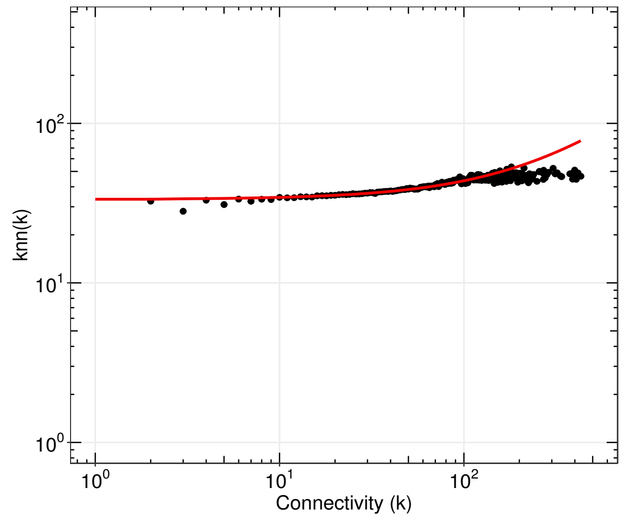


Figure 4 - Nearest-neighbors average connectivity of nodes, $k_{nn}(k)$, for the network of earthquakes generated with the modified OFC model using the successive model. This network has $k_{nn}(k)$ increasing linearly with k (red line); therefore, it is assortative.

For the deep earthquakes case, the hubs do not concentrate a large number of connections between each other (only 3% of all links). This result implies that deep seismic events worldwide have no obvious correlation, making understanding their correlations more difficult than for the shallow ones.

Discussion and Conclusions

The assortativity of networks of worldwide and synthetic earthquakes was studied in order to characterize correlation properties better and understand the spread of information in the system of earthquakes.

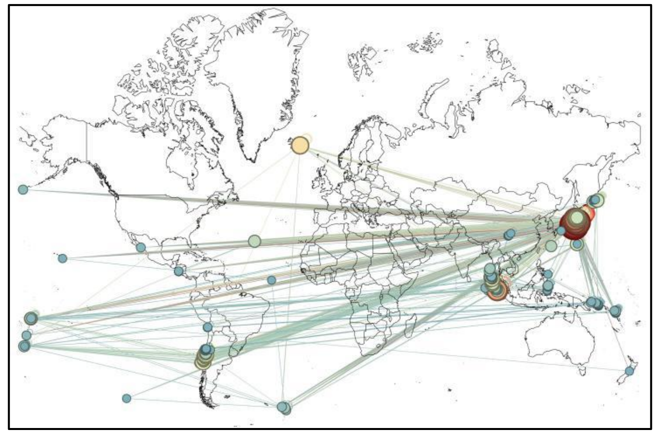
The assortative correlation exhibited in shallow and synthetic earthquakes networks is an exciting result. As the hubs are connected to other hubs in earthquake networks, the regions of the world where occurred large earthquakes tend to, on average, be linked to each other. Also, we observe agreement between real data for shallow events and synthetic data catalogs created with the improved version of the OFC model.

In the case of deep earthquakes, the neutral behavior found in the networks indicates the earthquakes correlate at random since, in neutral networks, the nodes are linked arbitrarily (Barabási & Pósfai, 2016).

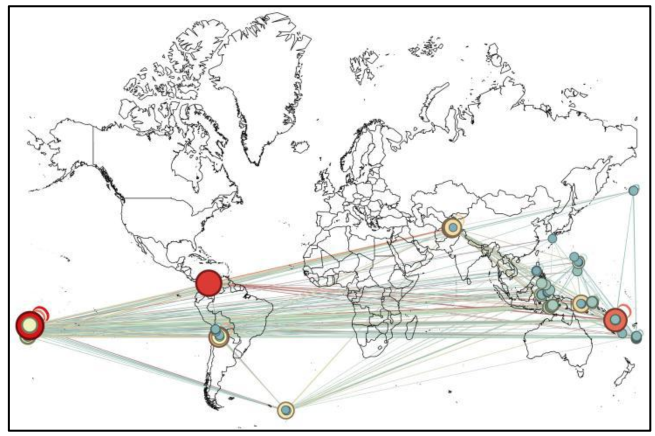
Our results suggest that shallow and deep earthquakes have different temporal and spatial correlation properties. While we have positive degree correlations for shallow earthquakes, these correlations seem not to exist for deep earthquakes. Because of that, the shallow earthquakes networks tend to link high-degree regions (regions with large earthquakes) with other high-degree areas, making it more difficult to change the seismological behavior of the earthquake networks even if a specific region stops having earthquakes for a period of time. Moreover, the results seem to indicate that, for shallow earthquakes, mainshocks may induce mainshocks in other areas, even if these areas are not close to each other. Another interesting feature revealed by our results from shallow earthquakes is a kind of "attracting dynamics". This feature causes the hubs to create several other hubs close to them; however, it does not prevent them from being connected to others further away. On the other hand, for deep earthquakes, our network analysis indicates that they connect randomly, i.e., with no specific preference.

Table 1 *- The number of nodes (N) and the values of the degree correlation coefficient (r) of the networks of worldwide and synthetic earthquakes used in this study.*

Network	*N*	*r*
Shallow earthquakes (successive)	28471	0.0711
Shallow earthquakes (time window)	23380	0.508
Deep earthquakes (successive)	8958	0.000763
Deep earthquakes (time window)	7675	0.0152
OFC model earthquakes	115510	0.0750



(a)



(b)

Figure 5 - Geospatial images of the networks constructed with the time window model between 2002 and 2016, for (a) shallow earthquakes and (b) deep earthquakes. The 2% of nodes with the highest degrees (hubs) in each network are shown, as well as the links between them. Larger and reddish cells have a higher number of connections.

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