Global optimization for AVO inversion: a genetic algorithm using a table-based ray-theory algorithm

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Abstract

Amplitude Variation with Offset (AVO) inversion provides estimates of the P-wave velocity, S-wave velocity and density of a stratified medium. Global optimization is desirable for the inversion to account for the multi-parametric behavior of the AVO inversion which is strongly affected by the initial estimates of the model rock properties. We carried out an analysis to verify the dependency between P-wave, S-wave velocity and density in the recovered parameters using empirical relations as constraints. In inversion schemes, the forward modeling is often the most time-consuming process. To reduce computation time, we have implemented a genetic algorithm using a *table-based* ray-theory algorithm to allow for a large amount of vertically inhomogeneous models in the global search. Our results show that the genetic algorithm was capable of recovering the physical parameters with good agreement for examples using the empirical constraints. However, it sometimes converged to solutions which were far from the correct answer, but were good models to explain the observed dataset. The forward modeling algorithm has shown excellent performance to be used in global optimization schemes, because it allows the use of a large number of members in the population of the genetic algorithm.

Introduction

The methodology of AVO has been widely used in the industry as a direct hydrocarbon indicator. A more quantitative interpretation can be achieved by means of AVO inversion for the rock properties. For the forward modeling part, one classically calculates the reflection coefficients for plane waves as a function of incident angle (offset) using the formulas of Knott (1899) and Zoeppritz (1919). For the reverse model, Rosa (1976) derived and verified the ill-posed nature of the inversion of Zoeppritz equations for rock properties, meaning that we have multiple combinations of input to produce the same output. Besides, due to its multi-parametric formulation, the solution space is very complex with many local minima, which makes it difficult to find correct solutions.

For those reasons, Stoffa and Sen (1991), Mallick (1995) and others suggest global optimization schemes to treat the AVO inversion. However, the process of global optimizations requires many forward models which considerably increase the computation time. Many of the works published in the literature use forward modeling algorithms based on the wave equation, which demand great computational power.

This study is based on the premise that unconsolidated sediments with small rock-property changes between layers can be modeled with ray theory without a loss of resolution. For computational ease, we have developed a forward modeling algorithm called *table-based ray theory* in order to speed up the computation of synthetic seismograms for global optimization. The genetic algorithm is based on findings of Stoffa and Sen (1991) and Sen and Stoffa (1992) and uses the Gardner et al. (1974) and Castagna et al. (1985) empirical relationships as constraints to verify the dependency among the parameters.

Synthetic common-midpoint seismogram: forward modeling

The forward modeling algorithm implemented is a so-called *table-based ray tracing*. It assumes an Earth model composed of n horizontal isotropic layers. The parametric equations to compute the two-way traveltimes and offset as a function of a constant ray parameter are given by (Slotnick, 1959)

$$x = 2\sum_{i=1}^{n} \frac{h_i p v_i}{(1 - p^2 v_i^2)^{1/2}} , \qquad (1)$$

$$t = 2\sum_{i=1}^{n} \frac{h_i}{v_i(1 - p^2 v_i^2)^{1/2}} , \qquad (2)$$

where h_i, v_i and p are, respectively, the layer thickness and velocity of the ith layer, and the ray parameter defined by

$$p = \frac{\sin(\theta_i)}{v_i} . \qquad (3)$$

Here θ_i is the angle between the seismic ray and the vertical in the ith layer. Using a sonic log or other vertical velocity information, we build three different two-dimensional tables (*offset, traveltime* and *reflection coefficient*) starting with equations 1 and 2.

To start the process, the P-wave velocity as a function of depth is transformed to time. The same applies to the S-wave velocity and density. For the 2D traveltime table, there are 90 columns associated with the ray parameter when the incident angle in the first medium varies from $0°$ to $89°$. The rows are associated with the t_o times in increments of the time sample interval. Now, the 2D tables for offset and

traveltime can easily be determined using equations 1 and 2 respectively. Since the velocity function is sampled in equal increments of the sample rate the same way the 2D tables are, the computation of the incident angle for each cell in the 2D table is possible with equation 3.

With the incidence angle known, the next step is to compute the 2D table of reflection coefficients R_P at each interface. This was accomplished with Shuey's approximation (Shuey, 1985) given by

$$R_P(\phi) = A + B\sin^2(\theta) + C\sin^2(\theta)\tan^2(\theta) , \qquad (4)$$

where ϕ is the incidence angle, θ is the average of the incidence and transmission angles, and the terms A, B and C are defined as

$$A = \frac{1}{2}\left(\frac{\triangle V_P}{V_P} + \frac{\triangle \rho}{\rho}\right) ,$$

$$B = \frac{1}{2}\frac{\triangle V_P}{V_P} - 2\left(\frac{V_S}{V_P}\right)^2\left(\frac{2\triangle V_S}{V_S} + \frac{\triangle \rho}{\rho}\right) , \qquad (5)$$

$$C = \frac{1}{2}\left(\frac{\triangle V_P}{V_P}\right) .$$

In these expressions, $\triangle V_P = V_{P2} - V_{P1}$, $\triangle V_S = V_{S2} - V_{S1}$, $\triangle \rho = \rho_2 - \rho_1$ are the differences between the P and S-wave velocity and density values, respectively, across an interface and V_P, V_S and ρ are their arithmetic averages.

In order to build a synthetic common-midpoint (CMP) section, we define which are the offsets to model and start a search row-by-row inside the *offset table* looking for the requested offsets. Once the cell is found for the specified offset, we go into the same cell position inside the *traveltime* and *reflectivity tables* to place the reflection coefficient from that position at the correct time arrival inside the trace.

Because the algorithm is very simple, it provides the potential of being adapted in the future to study more complex effects such as the stretch in normal moveout (NMO) correction, NMO without stretch (ray-trace NMO correction), array effects which affect the amplitude of the wave received by the streamer, extraction of AVO attributes, polar anisotropic media, etc.

Global optimization approach: Genetic Algorithm

The genetic algorithm (Figure 1) is a technique used to perform global optimizations based on the natural selection process (Holland, 1975). In order to simulate this process, the algorithm constantly modifies the initial pseudo-random population in order to reach local minima positions. Initially, each member of the pseudo-random population is a potential solution to the problem and after some iterations the genetic algorithm guides the population to the best fit positions.

The algorithm needs to *select* a percentage of the population to start the reproduction scheme where the *crossover* and the *mutation* are the two classic techniques to exchange genetic information between the members and also randomly change the genetic information to provide exploration of the solution space. In order to select the members of the population we need to measure the fitness between each potential solution and the data we want to optimize. For this purpose, we used the objective function
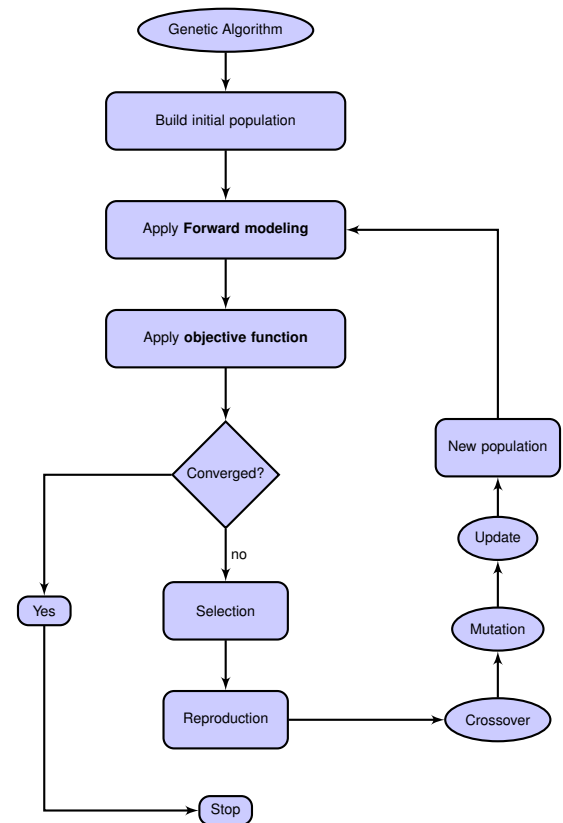

Figure 1: Flowchart of the genetic algorithm.

of Porsani et al. (2000) given by

$$h = \frac{2y^T x}{y^T y + x^T x} , \qquad (6)$$

where, x and y are the observed and modeled data in the time domain and x^T, y^T are their transposes. In order to constrain the inverse problem due to the dependency between S-wave velocity and density with P-wave velocity, we used the Gardner (Gardner et al., 1974) and Castagna (Castagna et al., 1985) relations to write S-wave velocity and density as a function of P-wave velocity.

Results

We used a synthetic model in order to verify the global optimization method for AVO inversion and its relationships with the prior information, i.e., the Gardner et al. (1974) and Castagna et al. (1985) constraints, and also to analyze the performance of the investigated forward modeling algorithm. The physical parameters for a simple synthetic model are described in the Table 1. The thickness between the two inner layers was chosen to be small enough to allow interaction between the wavelets.

We modeled a synthetic data set with the table-based ray-theory algorithm using the parameters from Table 1, a zero-phase Ricker wavelet with central frequency of 40 Hz, and a time sample rate of 2 ms. These parameters were also used in the modeling for the inversion. In order to perform the global optimization, each CMP gather modeled during each genetic algorithm run is NMO corrected. The parameters used in the *genetic algorithm* are given by
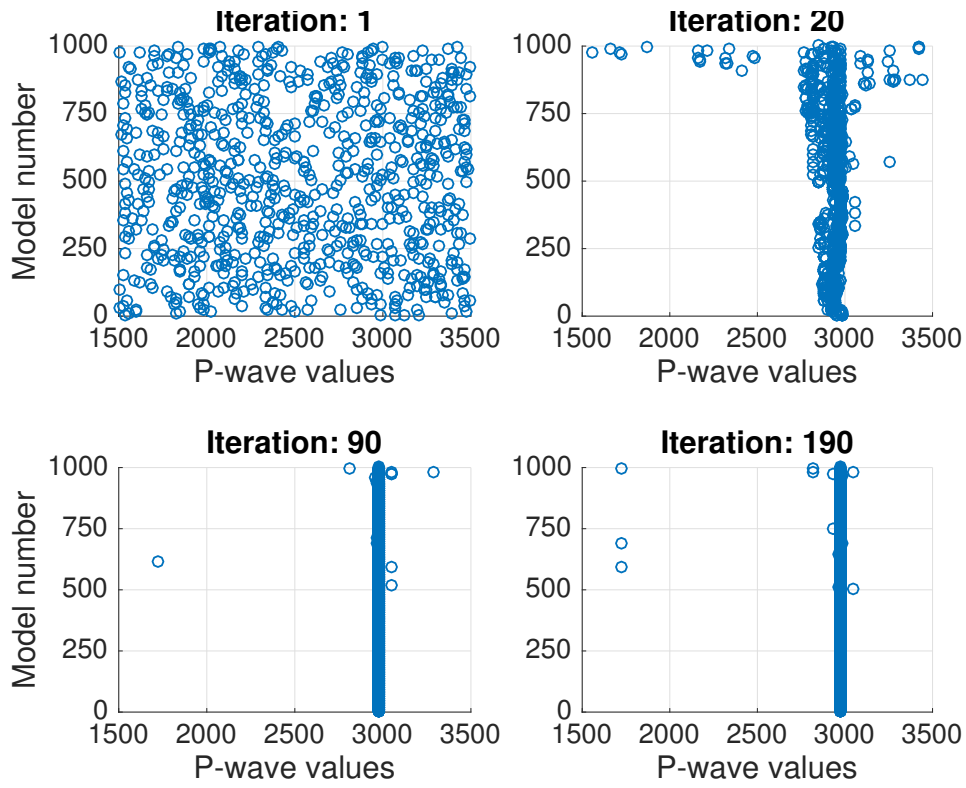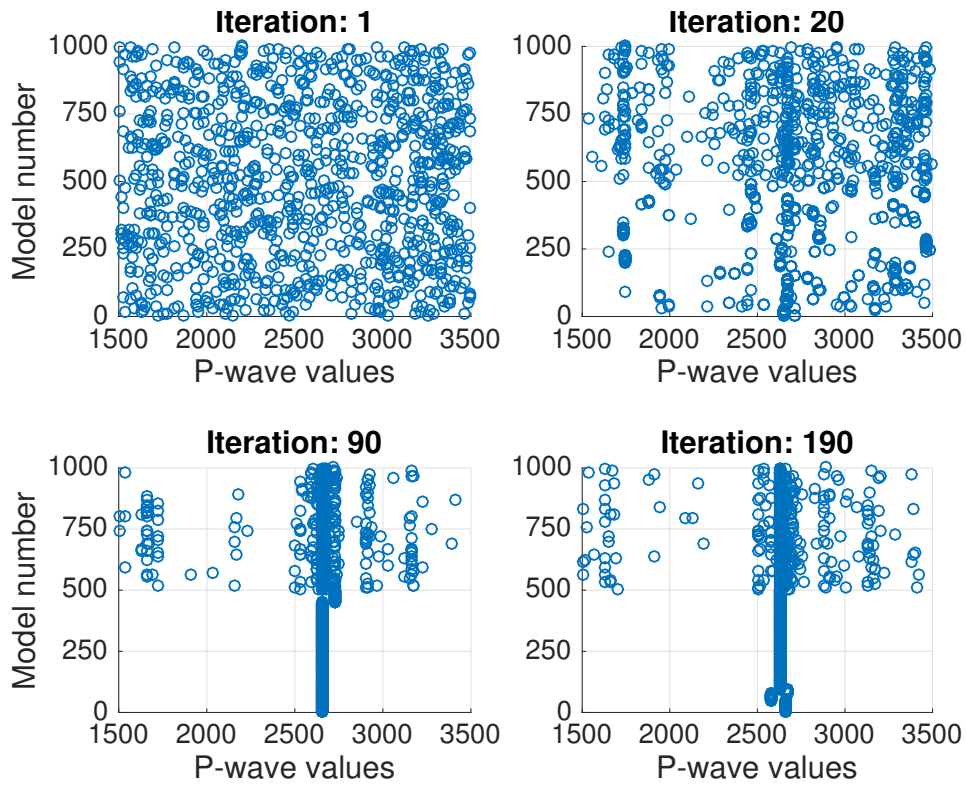
Figure 2: Evolution of the genetic algorithm solutions for the P-wave velocity parameter in the fourth layer when running with (a) and without (b) the Gardner et al. (1974) and Castagna et al. (1985) constraints.

Table 1: Model parameters to test the inversion scheme.

Layers	Thickness (m)	Vp (m/s)	Vs (m/s)	Density (g/c^3)
1	1000	2000	551.72	2.07
2	50	2800	1241.38	2.25
3	50	2300	810.34	2.14
4	500	3000	1413.80	2.29

Table 2: Recovered parameters of the model with and without constraints.

Parameters	Synthetic Model	Recovered Model (with constraints)		Recovered Model (without constraints)	
		Parameters	Relative Error (%)	Parameters	Relative Error (%)
P-wave (m/s)	2000	2000	0.00	2002	-0.10
	2800	2788	0.43	2699	3.61
	2300	2280	0.87	2394	-4.09
	3000	2971	0.97	2658	11.40
S-wave (m/s)	551.72	552.71	-0.07	778	-41.01
	1241.38	1231	0.84	1327	-6.90
	810.34	793	2.14	1106	-36.49
	1413	1389	1.75	1358	3.95
Density (g/c^3)	2.07	2.069	0.01	1.535	25.83
	2.25	2.247	0.13	1.739	22.71
	2.14	2.134	0.24	1.541	28.01
	2.29	2.284	0.25	1.934	15.31
Correlation Coef.	1	0.999887645		0.999847412	

a *selection rate* of 50%, $P_{mutation}$ = 0.1, P_{update} = 0.47, $P_{crossover}$ = 0.90, 1000 members in the population and a stopping criterion of 200 iterations.

To study the quality of the inversion, we concentrate on the parameters of the fourth layer of the model in Table 1. Figure 2 shows the evolution of the genetic algorithm solutions for the inverted P-wave velocity parameter in the fourth layer when running with and without the Gardner et al. (1974) and Castagna et al. (1985) constraints.

As we can see in Figure 2(a), when using the constraints, the algorithm is guiding, already after a few iterations, all the members of the population close to the correct position in the solution space. However, Figure 2(b) shows that without the additional constraints to restrict the solutions, the algorithm is leading a significant part of the population to particular solutions that are not very close to the correct one. Nonetheless, these solutions are good answers to the objective function of the inverse problem as we can see in Table 2.

In Table 2 we have compiled the recovered parameters for the models with best value of the objective function that where recovered with and without the constraints on the parameters. In the process, the algorithm generated 301.000 forward synthetics in 25 minutes. Even with more than 20% of relative error in the average, the best model recovered without the constraints still produces a very similar CMP gather as we can verify by the correlation coefficient value. This is a consequence of the ill-posed nature of the underlying inverse problem as verified by Rosa (1976).

Conclusions

We have presented a table-based ray-tracing forward modeling algorithm that is very fast in calculating the traveltime and reflection coefficient vs. offset information for many vertically inhomogeneous models. By providing good performance with restricted computer power, this algorithm has shown excellent potential to be used in a genetic global optimization scheme for AVO inversion. In the inversion procedure, parameter constraints were helpful to reach the correct global minimum. However, the result without constraints indicate that the procedure may still converge to incorrect results that satisfy the objective function with quite a low residual. In future research, the genetic algorithm will be formulated inside the Bayesian framework to provide solid statistical information about the distribution of the solutions during the iterations. It is expected to be able to visualize distinguishable concentrations of particular solutions which could be useful to determine possible scenarios for the recovered parameters.

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