



Velocity analysis in homogeneous VTI media using SVD-semblance

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Abstract

Velocity analysis in prestack reflection data on CMP gathers is traditionally performed using semblance coherence measure with hyperbolic moveout curves. However, due to the assumptions made in its development, this measure is inaccurate in estimating parameters in sections with long-spread, anisotropy, and in the presence of amplitude variation with offset (AVO), especially in the presence of polarity reversals. Nonhyperbolic moveout approximations are commonly used to approximate moveout on surveys with long offsets in anisotropic media. Among these, the shifted hyperbola approximation is considered quite valid to model traveltimes reflections in CMP gather data with large offsets obtained in VTI media. Amplitude variations with offset, in the presence of reverse polarity may be introduced in velocity analysis, modifying the traditional semblance algorithm. Singular value decomposition (SVD) applied to the semblance reduces the influence of AVO even in the presence of noise. In this paper we perform velocity analysis based on semblance SVD with shifted hyperbola approximation in estimating parameters in anisotropic media VTI (transversely isotropic with a vertical symmetry axis) in data free and with noise. The results suggest the validity, accuracy and robustness, technique to estimate parameters in such media.

Introduction

Velocity analysis in seismic sections is performed using Coherence measures in general semblance. However, in seismic data with high amplitude variation along the seismic events and offset-depth ratio greater than one, the semblance loses precision in estimation of parameters. This has led to consider changes to the semblance in order to suit it to seismic sections with anisotropy, long offsets, variations in amplitude with the offset and reverse polarization.

Several nonhyperbolic moveout curves in CMP sections with long offsets have been proposed to approximate traveltimes in VTI media (Tsvankin and Thomsen (1994); Alkhalifah and Tsvankin (1995); Fomel (2004)). Methods for performing semblance-based velocity analysis using nonhyperbolic approximations, have been presented by Alkhalifah (1997) and Grechka and Tsvankin (1998). Using approximations by rational interpolation Douma and Calvert (2006) and Douma and Baan (2008) presented an accurate method for performing semblance-based velocity analysis in homogeneous VTI layered media.

Sarkar et al. (2001) and Sarkar et al. (2002) presented AB and AK-semblance algorithms in order to make the semblance, sensitive to variations in phase and amplitude with offset. Their experiments showed that the new measures were robust to deal with AVO anomalies and reverse polarity. Yan and Tsvankin (2008) extended the AK-semblance algorithm for VTI media and large offsets using nonhyperbolic moveout equation of Alkhalifah and Tsvankin (1995). Using singular value decomposition of the matrix formed by the windowed data in semblance, Ursin et al. (2014) showed an adaptation to semblance, precise and robust enough to handle data with large amplitude variations with the offset.

In this paper, we present the velocity analysis based on SVD-semblance algorithm with nonhyperbolic moveout approximation shifted hyperbola in order to measure the accuracy and robustness in the parameter estimation in homogeneous VTI media.

Wave propagation in VTI media

TI media are characterized by five elastic parameters, density-normalized: $a_{11}, a_{13}, a_{33}, a_{44}$ and a_{66} . The P-wave propagation in VTI media has phase velocity equation, as a function of the phase angle θ given by Gassmann (1964); Thomsen (1986):

$$\frac{v^2(\theta)}{v_{Pz}^2} = 1 + \varepsilon \sin^2 \theta + \sqrt{1 + \frac{4 \sin^2 \theta}{f} (2\delta \cos^2 \theta - \varepsilon \cos 2\theta) + \frac{4\varepsilon^2 \sin^4 \theta}{f^2}}, \quad (1)$$

where ε and δ are dimensionless parameters Thomsen (1986) and $f = 1 - \frac{v_{S_z}^2}{v_{Pz}^2}$, with $v_{Pz} = \sqrt{a_{33}}$ and $v_{S_z} = \sqrt{a_{44}}$ the P- and S-wave vertical velocity, respectively.

In anisotropic media the group velocity equation and the group angle equation, are difficult to explain. However, its square magnitude, V , and the group angle Θ , can be obtained from the phase velocity, v , and phase angle, θ , respectively, as Berryman (1979):

$$V^2(\Theta) = v^2(\theta) + \left[\frac{dv(\theta)}{d\theta} \right]^2 \quad (2)$$

and

$$\tan(\Theta) = \frac{\tan(\theta) + \frac{1}{v(\theta)} \frac{dv(\theta)}{d\theta}}{1 - \frac{\tan(\theta)}{v(\theta)} \frac{dv(\theta)}{d\theta}}, \quad (3)$$

Considering a flat reflective interface to a depth z , the wave propagation traveltimes in CMP gather as a function of the offset x in homogeneous VTI media, is nonhyperbolic and it is given by:

$$t^2(x) = \frac{4z^2 + x^2}{V^2(\Theta)}, \quad (4)$$

However, due to their representation depends explicitly of the group velocity, moveout approximations are often

used on the steps of seismic processing. Moveout approximations based on Taylor series expansion are widely used in seismic processing. The *shifted hyperbola* approximation Malovichko (1978) is an accurate moveout curve of the fourth order Castle (1994), ie it is nonhyperbolic. Fomel (2004) explained, analytically, the shifted hyperbola approximation for homogeneous VTI media as:

$$t_p^2(x) \approx \frac{3+4\eta}{4(1+\eta)} t_{ph}^2(x) + \frac{1}{4(1+\eta)} \sqrt{t_{ph}^4(x) + \frac{16\eta(1+\eta)}{(1+2\eta)} \frac{t_{pz}^2 x^2}{v_{pn}^2}}. \quad (5)$$

where $t_{ph}^2(x) = t_{pz}^2 + \frac{x^2}{(1+2\eta)v_{pn}^2}$, with: t_{pz} the P-wave two-way vertical traveltime, $v_{pn} = v_{pn}(1+2\delta)$ is the normal moveout velocity, and $\eta = \frac{\epsilon-\delta}{1+2\delta}$ the anelliptic parameter Alkhalifah and Tsvankin (1995).

Velocity analysis in VTI media

Seismic data in a CMP gather with N traces, based-semblance velocity analysis is performed to estimate t_z and v_n , maximizing the functional Taner and Koehler (1969):

$$S(t_z, v) = \frac{\sum_{\tau=t_z-T/2}^{t_z+T/2} [\sum_x \mathbf{D}(\tau, x)]^2}{N \sum_{\tau=t_z-T/2}^{t_z+T/2} \sum_x \mathbf{D}^2(\tau, x)}, \quad (6)$$

where τ is the two-way vertical traveltime in a time window, T , centered at time t_z and $\mathbf{D}(t, x)$ is a data window sampled by a moveout curve $t(\tau, v)$ with v being the stacking velocity. Generally semblance uses hyperbolic moveout curve and is accurate in estimating parameters for data without AVO and offset-depth ratio x/z values at most one.

However, current seismic processing techniques have used as a model of subsurface anisotropic media, especially anisotropic VTI media. This has required adjustments to processing steps in such a way that they can incorporate the wave propagation characteristics in such lithologic models. VTI media are widely used as a model of the subsurface seismic exploration because they describe more accurately the wave propagation in hydrocarbon reserves, for example, in shales and periodic thin layering (thin isotropic layers).

Semblance SVD homogeneous VTI media

Consider now the $N_t \times N_x$ data matrix $\mathbf{D}(\tau, x)$:

$$\mathbf{D} = \mathbf{W} + \mathbf{N}, \quad (7)$$

where \mathbf{W} is the $N_t \times N_x$ signal matrix and \mathbf{N} is the $N_t \times N_x$ noise matrix. In this way, the semblance 6 is rewrite as:

$$S = \frac{\|\mathbf{W}\|_F^2}{\|\mathbf{D}\|_F^2}, \quad (8)$$

where $\|\mathbf{D}\|_F$ is the Frobenius norm, defined as:

$$\|\mathbf{D}\|_F = \sum_{i=1}^N \sum_{j=1}^N \|\mathbf{D}\|^2, \quad (9)$$

and \mathbf{W} is the signal matrix:

$$\mathbf{W} = \mathbf{s}\mathbf{e}^T, \quad (10)$$

which is the product between the signal vector \mathbf{s} and the transposed of $N_x \times 1$ scattered vector: $\mathbf{e}^T = [1, 1, \dots, 1]$.

Calculating the SVD (*singular value decomposition*) of the data matrix Golub and Van Loan (1996):

$$\mathbf{D} = \mathbf{U}\mathbf{\Sigma}\mathbf{V}^T = \sum_{i=1}^{N_t} \mathbf{u}_k \sigma_k \mathbf{v}_k^T, \quad (11)$$

where \mathbf{u}_k and \mathbf{v}_k are the $N_t \times 1$ and $N_x \times 1$ orthonormal vectors, respectively, and σ its singular values chosen such that: $\sigma_1 \geq \sigma_2 \geq \dots \geq \sigma_{N_t}$, to $N_t \leq N_x$. The semblance can then be rewritten as Ursin et al. (2014):

$$S = \frac{\sum_{k=1}^{N_t} \sigma_k^2 \|\mathbf{D}\mathbf{v}_k\|^2}{N_x \sum_{k=1}^{N_t} \sigma_k^2}. \quad (12)$$

However, assuming that the shape signal remains the same in all traces, but with the amplitude varying with the offset, the signal matrix can take the form:

$$\mathbf{W} = \mathbf{s}\mathbf{a}^T, \quad (13)$$

where $\mathbf{a}^T = [a_1, a_2, \dots, a_{N_x}]$, and \mathbf{a} a vector with amplitudes in each channel. Golub and Van Loan (1996) showed that a least squares estimation of an array, is a first self-image. Thus, the signal matrix can be decomposed as $\mathbf{W} = \mathbf{u}_1 \sigma_1 \mathbf{v}_1^T$, and semblance operator modified as Gersztenkorn and Marfurt (1999); Ursin et al. (2014):

$$S_{SVD} = \frac{\sigma_1^2}{\sum_{k=1}^{N_t} \sigma_k^2}, \quad (14)$$

hereinafter called SVD semblance.

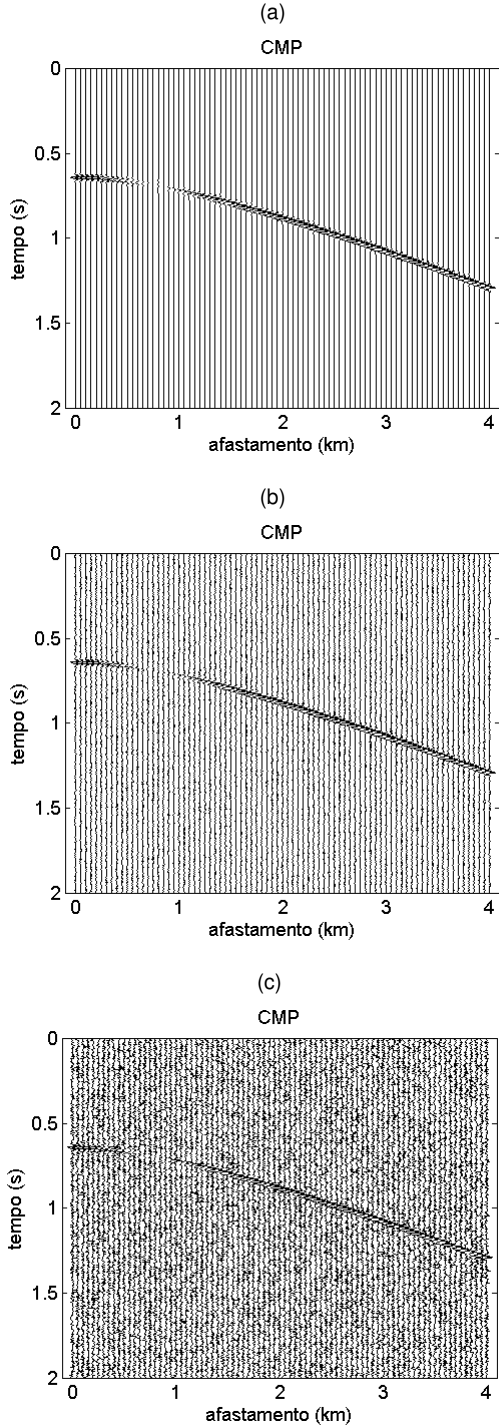
Velocity Analysis using SVD semblance

In this experiment are performed velocity analysis based on semblance SVD, with the *shifted hyperbola* nonhyperbolic approximation (5) considering as a model of subsurface the VTI shale Greenhorn Jones and Wang (1981), whose elastic parameters normalized by the density are: $a_{11} = 14.47 \text{ km}^2/\text{s}^2$, $a_{13} = 4.51 \text{ km}^2/\text{s}^2$, $a_{33} = 9.57 \text{ km}^2/\text{s}^2$ and $a_{55} = 2.28 \text{ km}^2/\text{s}^2$. The seismic sections was obtained considering a reflective interface located at a depth $z = 1,0 \text{ km}$, two way traveltime $t_z = 0,6464 \text{ s}$, velocities $v_{pn} = 2,9336 \text{ km/s}$, $v_{px} = 3,8045 \text{ km/s}$ and parameter $\eta = 0,3409$. The Figure 1 present the synthetic sismograms where the traveltime was the sampled at $\Delta t = 4 \text{ ms}$ and the signature source is the Gabor wavelet with frequency $f = 40 \text{ Hz}$. In Figure 1 (a), (b) and (c) there are variation in the noise level.

First, the velocity analysis is performed on data for AVO variation and free noise. The Figure 2 illustrate the v_{pn} versus v_{px} SVD semblance maps and the results are shown in Table 1. What can be seen is that the velocity analysis is quite accurate in parameter estimates, because the relative error in estimating the velocity v_{pn} does not exceed 0.4% and the relative error of the η parameter is less than 2%. It is noted also that the value of maximum SVD semblance is approximately 1.

Then, experiments are performed on data with AVO and variation in the values of s/n . The Figure ?? illustrate the v_{pn} versus v_{px} SVD semblance obtained using shifted hyperbola nonhyperbolic approximation to $s/n = 15$. The results are shown in Table ???. What can be observed, again is that the velocity analysis is quite accurate in parameter estimates, because the relative error in estimating the velocity v_{pn} does not exceed 0.4%, the

Figure 1: CMP sintético (a) sem ruído, (b) com razão sinal-ruído igual a 15 e (c) com razão sinal-ruído igual a 5, obtido no folhelho VTI Greenhorn para $x/z = 4, 0$.



relative error of the η parameter is less than 2% and the value of maximum SVD semblance is approximately 1.

Finally, the Figure ?? illustrate the v_{Pn} versus v_{Px} SVD semblance obtained using shifted hyperbola nonhyperbolic approximation to $s/n = 5$. The results are shown in Table ??. What can be observed, again is that the

Figure 2: v_{Pn} versus v_{Px} SVD semblance map, using *Shifted hyperbola* approximation (5) to estimating velocities and η parameter in datas with AVO and free noise, obtained in VTI Greenhorn shale, to $x/z = 4, 0$. Yellow circle - exact value and white losango - approximate value.

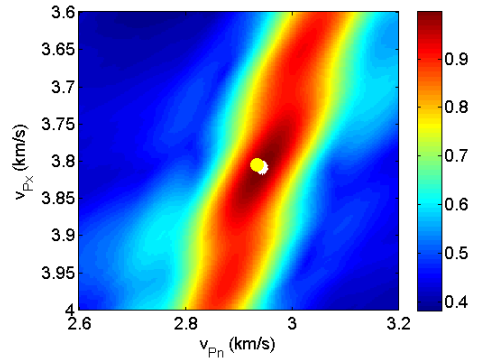


Table 1: relative error in estimated velocities v_{Pn} , v_{Px} and η parameter, and maximum SVD semblance; obtained from velocity analysis in data with AVO and free noise, in the VTI Greenhorn shale to $x/z = 4, 0$.

Error v_{Pn} (%)	Error v_{Px} (%)	Error η (%)	S_{max}
0.3449	0.1195	1.0964	0.9984

Figure 3: v_{Pn} versus v_{Px} SVD semblance map, using *Shifted hyperbola* approximation (5) to estimating velocities and η parameter in datas with AVO and $s/n = 15$, obtained in VTI Greenhorn shale, to $x/z = 4, 0$. Yellow circle - exact value and white losango - approximate value.

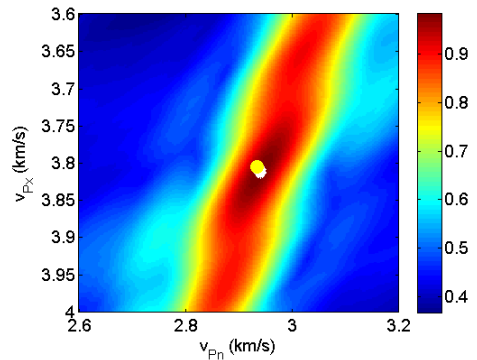


Table 2: relative error in estimated velocities v_{Pn} , v_{Px} and η parameter, and maximum SVD semblance; obtained from velocity analysis in data with AVO and $s/n = 15$, in the VTI Greenhorn shale to $x/z = 4, 0$.

Error v_{Pn} (%)	Error v_{Px} (%)	Error η (%)	S_{max}
0.2421	0.1723	0.3328	0.9855

velocity analysis is quite accurate in parameter estimates, because the relative error in estimating the velocity v_{Pn} does not exceed 0.7%, the relative error of the η parameter is approximately 4% and the value of maximum SVD semblance is approximately 1.

Figure 4: v_{Pn} versus v_{Px} SVD semblance map, using *Shifted hyperbola* approximation (5) to estimating velocities and η parameter in datas with AVO and $s/n = 5$, obtained in VTI Greenhorn shale, to $x/z = 4, 0$. Yellow circle - exact value and white losango - approximate value.

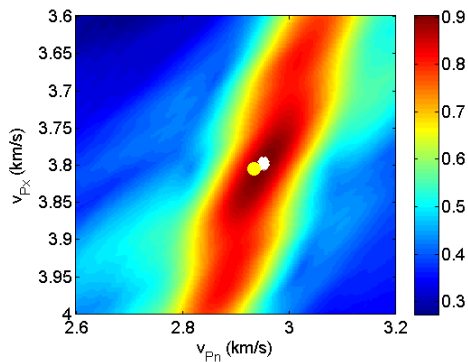


Table 3: relative error in estimated velocities v_{Pn}, v_{Px} and η parameter, and maximum SVD semblance; obtained from velocity analysis in data with AVO and $s/n = 5$, in the VTI Greenhorn shale to $x/z = 4, 0$.

Error v_{Pn} (%)	Error v_{Px} (%)	Error η (%)	S_{max}
0.2421	0.1723	0.3328	0.9855

Results and Conclusions

We have presented an adaptation of SVD semblance operator to estimate speed and parameter in VTI media. In general, the SVD semblance with *Shifted hyperbola* (5) nonhyperbolic approximation is a good option in velocity analysis to estimate velocities: v_{Pn} and v_{Px} , and η parameter in VTI data with AVO variation and polarity reversed. Experiments with and without noise and $x/z = 4, 0$, showed that the SVD semblance is quite accurate and robust in parameter estimates. Thus, we conclude that the SVD semblance addition to mitigating the effects of AVO variation also acts as a noise filter.

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