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Structure enhancing filtering with the structure tensor

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ABSTRACT

The structure tensor is a very versatile tool. It can be used to detect edges, estimate coherency and local slopes. In this work we use the structure tensor to estimate local slopes for data filtering purposes. We investigate the use these slopes to apply an structure-preserving filter to an synthetic seismic dataset.

INTRODUCTION

The structure tensor was applied to seismic data analysis and filtering many times before. Bakker (2002) gives a very comprehensive description of the applications of structure tensors to seismic data filtering. They can also be used to identify and create clusters of areas of interest in seismic data (Faraklioti and Petrou, 2005) and to edge-preserving smoothing by diffusion filtering of seismic data (Hale, 2009; Lavialle et al., 2007).

We use the structure tensor to estimate local slopes of the data. With this slopes we can predict each trace from its nearest neighbors by using plane-wave structure prediction (Fomel, 2010). Then, we stack the predicted traces with the original ones, accomplishing structure filtering (Liu et al., 2010). We also test the possibility of using stacking weights to improve the structure preservation.

THE STRUCTURE TENSOR

The structure tensor is obtained by simple windowed smoothing operations and simple differentiation of the image. It's commonly used to detect lines and regions of interest in images. The structure tensor is known by different names depending on the application field: *gradient structure tensor*, *second-moment matrix*, *scatter matrix*, *interest operator* and *windowed covariance matrix* (Faraklioti and Petrou, 2005).

The first order structure tensor is obtained by a first order Taylor expansion from the squared difference function. For the sake of simplicity we omit the deduction of the tensor (see e.g. (Faraklioti and Petrou, 2005) for detailed derivation).

Let $P(t,x)$ be the recorded wave field at a given time t and position x. The structure tensor is given by:

$$D(t_0,x_0) = \begin{pmatrix} \left\langle \frac{\partial P}{\partial x}^2 \right\rangle & \left\langle \frac{\partial P}{\partial x}\frac{\partial P}{\partial t} \right\rangle \\ \left\langle \frac{\partial P}{\partial x}\frac{\partial P}{\partial t} \right\rangle & \left\langle \frac{\partial P}{\partial t}^2 \right\rangle \end{pmatrix}, \qquad (1)$$

where, the symbol $\langle\cdot\rangle$ represents the average value produced by the smoothing procedure under a given window, around (t_0, x_0). The window size, in this case, is called *integration scale*. The local smoothing window used when the derivatives are calculated is called *local scale*. This is done because derivatives estimated from raw data are very noisy (Faraklioti and Petrou, 2005).

We can observe that the structure tensor is symmetric and semi-positive definite. This means that all the eigenvalues are real and positive. The structure tensor eigenvalues and eigenvectors can be used to detect lines, borders and regions with constant image intensity. The expected behavior for each one of those scenarios is summarized in table 1.

Local Structure	Eigenvalues
constant intensity	$\lambda_1 \approx \lambda_2 \approx 0$
line	$\lambda_1 \gg 0\ \lambda_2 \approx 0$
corner	$\lambda_1 \gg 0\ \lambda_2 \gg 0$

Table 1: Local structure conditions and expected relationships between eigenvalues of the structure tensor matrix(Faraklioti and Petrou, 2005).

The eigenvalues of the matrix 1 are the roots of the characteristic equation:

$$\begin{aligned} &\lambda^2 - \left(\left\langle \frac{\partial P}{\partial x}^2 \right\rangle + \left\langle \frac{\partial P}{\partial t}^2 \right\rangle \right) \lambda \\ &\quad + \left\langle \frac{\partial P}{\partial t}^2 \right\rangle \left\langle \frac{\partial P}{\partial x}^2 \right\rangle - \left\langle \frac{\partial P}{\partial x}\frac{\partial P}{\partial t} \right\rangle = 0. \end{aligned} \qquad (2)$$

Both eigenvalues can be easily found by solving the previous equation. Its solution is given by:

$$\lambda = \frac{1}{2}\left(\left\langle \frac{\partial P}{\partial x}^2 \right\rangle + \left\langle \frac{\partial P}{\partial t}^2 \right\rangle\right) \pm \frac{1}{2}\sqrt{\left(\left\langle \frac{\partial P}{\partial x}^2 \right\rangle + \left\langle \frac{\partial P}{\partial t}^2 \right\rangle\right)^2 - \left\langle \frac{\partial P}{\partial x}\frac{\partial P}{\partial t} \right\rangle} \qquad (3)$$

By looking at equation 3 we can notice that both eigenvalues are positive, and satisfy the relation $\lambda_1 \geq \lambda_2 \geq 0$. In order to avoid loss of significant digits, it is wise to compute the eigenvalues as follows:

$$\lambda_1 = \frac{1}{2}\left(\left\langle \frac{\partial P}{\partial x}^2 \right\rangle + \left\langle \frac{\partial P}{\partial t}^2 \right\rangle\right) + \frac{1}{2}\sqrt{\left(\left\langle \frac{\partial P}{\partial x}^2 \right\rangle + \left\langle \frac{\partial P}{\partial t}^2 \right\rangle\right)^2 - \left\langle \frac{\partial P}{\partial x}\frac{\partial P}{\partial t} \right\rangle}, \qquad (4)$$

and

$$\lambda_2 = \frac{\left\langle \frac{\partial P}{\partial x}^2 \right\rangle \left\langle \frac{\partial P}{\partial t}^2 \right\rangle - \left\langle \frac{\partial P}{\partial x}\frac{\partial P}{\partial t} \right\rangle^2}{\lambda_1}. \qquad (5)$$

EIGENVALUES AND LOCAL SLOPES

Let e_1 and e_2 be eigenvectors corresponding to eigenvalues λ_1 and λ_2, respectively. The eigenvector e_2 is perpendicular to data gradient, thus being parallel to the structures on the seismic image (Hale, 2009).

We can estimate the data local slope by using the inclination of e_2. The inclination of e_2 is given by:

$$\tan\sigma = \frac{\lambda_2 - \left\langle \frac{\partial P}{\partial x}^2 \right\rangle}{\left\langle \frac{\partial P}{\partial x}\frac{\partial P}{\partial t} \right\rangle}. \qquad (6)$$

Since e_1 is orthogonal to the data structures, we can also use it to estimate the local slope, by employing the following equation:

$$\tan\sigma = -\frac{\left\langle \frac{\partial P}{\partial x}\frac{\partial P}{\partial t} \right\rangle}{\lambda_1 - \left\langle \frac{\partial P}{\partial x}^2 \right\rangle}. \qquad (7)$$

STRUCTURE PREDICTION FILTERING

There are many ways to accomplish structure-enhancing filtering of a seismic image, like diffusion filtering of seismic data (Lavialle et al., 2007) or steering Gaussian elongated windows along local slopes patterns (Haglund, 1992). For simplicity, we choose to filter along the structures using plane-wave prediction (Liu et al., 2010).

A trace can be predicted by shifting it according to the local seismic event slopes. Consider the prediction operator $\mathbf{P}_{i,j}(\sigma_i)$ as an operator for prediction of trace j from trace i, according to the local slope pattern σ_i (see e.g. Fomel (2002) and Fomel (2010) for further details). It's possible to predict a trace from a distant neighbor by simple recursion. So, predicting trace k from trace 1 is simply:

$$\mathbf{P}_{1,k} = \mathbf{P}_{k-1,k} \cdots \mathbf{P}_{2,3}\,\mathbf{P}_{1,2}. \qquad (8)$$

In this work we propose the use of the structure prediction with the dips estimated by the structure tensor, instead of using the ones estimated with plane-wave destructors. After estimating the slopes, we predict a trace from its neighbors and stack the predicted traces with the original trace. In that way we accomplish the structure filtering (Liu et al., 2010).

TESTS WITH SYNTHETIC SEDIMENTARY DATA

We tested the ability of the structure tensor to estimate local slopes for filtering purposes with a synthetic data for a sedimentary model, showed in figure 1. This dataset, proposed by Claerbout (1992), is composed by 200x200 pixels, spaced by 8 m in the x axis and 4 ms in the t axis.

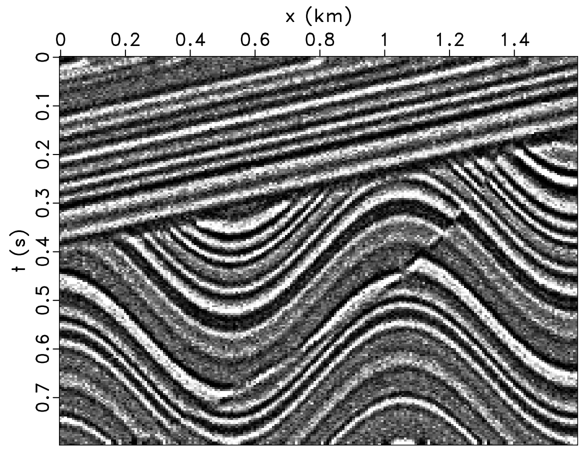

Figure 1: Synthetic data for a sedimentary model with noise.

The first step to obtain the slopes is the eigenvalues calculation. For this procedure the local and integration scales were windows with 5x5 samples each. The window samples had Gaussian like weights, following the expression $e^{-(\alpha t^2+\beta x^2)/16}$, with $\alpha = 1$ (ms)$^{-2}$ and

$\beta = 1$ (m^{-2}). For simplicity, t and x were considered integers with unitary spacing inside the window and origin at the window center.

The slopes were estimated using the first eigenvalue and the equation 7. We choose to use λ_1 to obtain the slopes, by using equation 4. If calculated without proper care, λ_2 may suffer from loss of significance. By calculating it using equation 5 the dips obtained with equation 6 are equivalent to the ones obtained with λ_1 and equation 7.

The slopes obtained with the structure tensor are based on sums over data derivatives. This derivatives can be a little noisy, even after applying smoothing procedures. A possible workaround is to further smooth the data before differentiation, taking care to not blur the reflector's edges too much.

That can be done by changing the local and integration scales. A bigger local scale make the structure tensor ignore smaller details. The integration scale should reflect the characteristic size of the texture of interest (Weickert, 1999), in this case it should reflect the reflectors size. Instead of increasing the scale's size, we choose to smooth the slopes obtained three times with a triangular smoothing window of 7x7 samples, obtaining the slopes showed in figure 2.

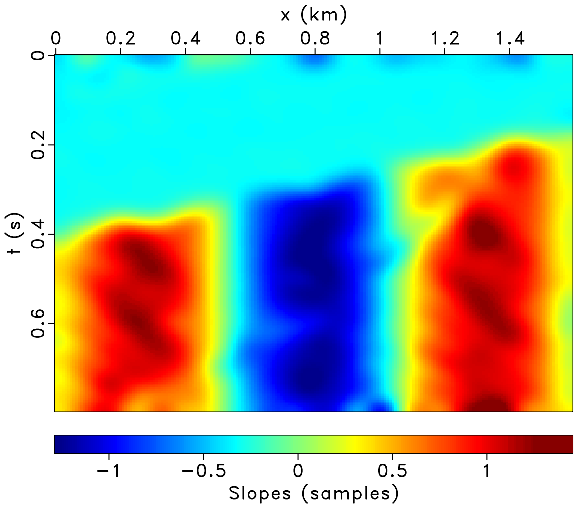

Figure 2: Smoothed slopes estimated using the structure tensor.

We used the estimated slopes to predict each trace from its three nearest neighbors, generating the data cube depicted in figure 3. At this point it is possible accomplish structure filtering by simply stacking the traces of the data cube. By doing so, we have the filtered data showed in figure 4. The noise was clearly attenuated, but the fault and the interface between the folded layers and the plane layers was smeared. This effect is very clear when we calculate the difference between the original and filtered data (figure 5).

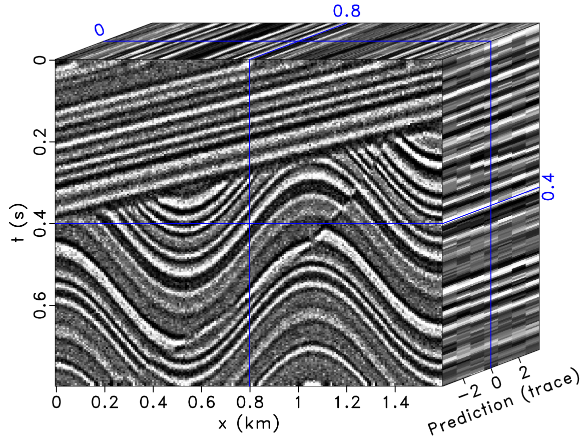

Figure 3: Predicted traces from input data (figure 1), using the slopes estimated in figure 2.

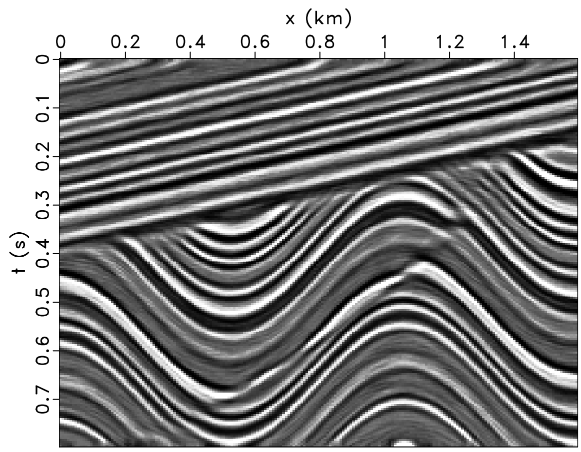

Figure 4: Filtered data using an simple mean filter.

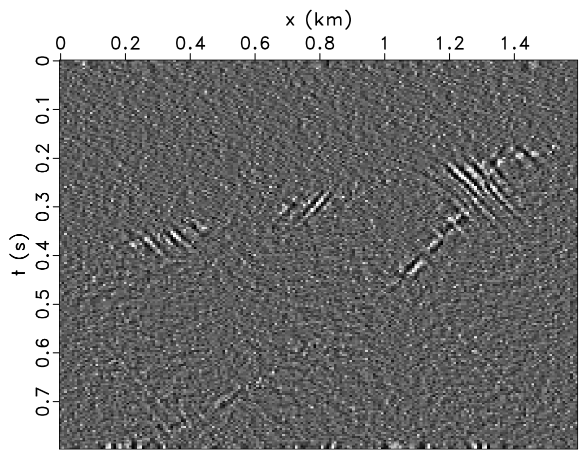

Figure 5: Difference between the original data (figure 1) and the data filtered with simple mean (figure 4).

SIMILARITY FILTERING WITH GAUSSIAN WEIGHTS

To prevent the blurring of data near faults and stratigraphic interfaces, we decided to improve the structure filtering by using similarity based filter weights for the stacking step (Liu et al., 2010). For the similarity weights, we use the definition of local similarity proposed by Fomel (2007). Those weights are showed in figure 6. To further improve the data staking we employed a Gaussian taper. This results lower weights in stacking for traces predicted from traces far from the original trace. The Gaussian weights are given by:

$$w_i = e^{-h_i^2/h_r^2} \,, \tag{9}$$

where h_i is the distance to trace i and h_r controls the shape of the Gaussian weight function (Liu et al., 2010). This approach is analogous to bilateral filtering (Tomasi and Manduchi, 1998), with the advantage of smooth variation of the similarity weights. The product of both weights is showed in figure 7.

Finally, the filtering using the stacking weights of figure 7 is showed in figure 8. We can see that the noise was attenuated, also there are very little smearing of the faults and other interfaces. This fact is further confirmed by the difference between the original data and the filtered data (figure 9).

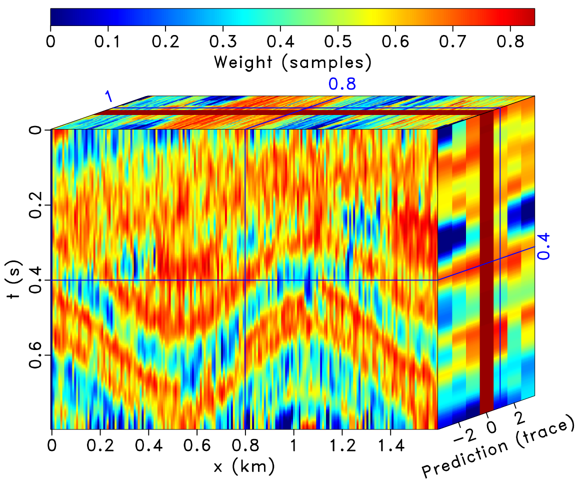

Figure 6: Similarity weights for the predicted traces (figure 1).

CONCLUSIONS

The results of structure prediction filtering, with slopes from the structure tensor, were satisfactory. We can see on figures 4 and 8 that the noise was successfully attenuated. The smoothing took place along the layers, not across them, as expected. One of the problems encountered was the blurring of reflectors, as showed in figures

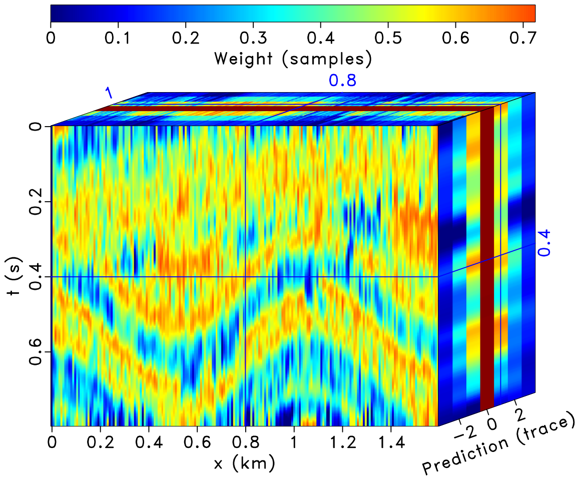

Figure 7: Similarity weights with Gaussian taper.

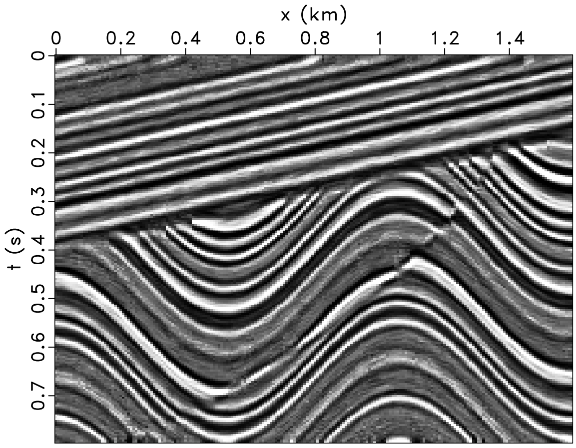

Figure 8: Filtered data using the improved similarity weights.

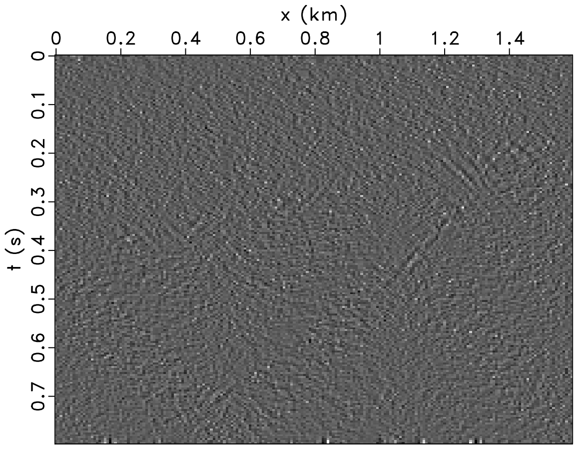

Figure 9: Difference between the original data (figure 1) and the data filtered with improved similarity weights (figure 8).

4 and 5. Blurring the reflectors implies in blurring the fault present in the image and the interface between the folded layers and the plane layers. To address this problem, we use the stacking weights proposed by (Liu et al., 2010). These weights help to preserve the sharpness of the reflectors (figures 8 and 9), acting almost like a bilateral filter. This filter is well known by its edge-preserving properties. In this case, the reflector's ends can be considered edges. The results could be further improved if the slopes estimation was enhanced at discontinuities, like faults or other types of seismic interfaces.

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