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APPLICATION OF A STRIP SOURCE OF HEAT FOR MEASURING THERMAL CONDUCTIVITY OF SEMI-INFINITE SOLIDS

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During the last few decades the transient line source method for measuring thermal conductivity of soft materials has been used for measuring conductivity of solid rocks. More recently, the methods has been adapted for rocks with semi-infinite geometry but the mathematical formalism of infinite medium with cylindrical symmetry, has still been used. In order to overcome this difficulty we propose a simple theoretical model for heat generation over the surface of a semi-infinite medium. The model consist in substituting the infinite line source by a strip source of limited width. The solution found was applied to measure thermal conductivity of solid rocks with semi-infinite geometry. The measurements were carried out by using a line source apparatus constructed in our laboratory. The construction of this apparatus is described in detail and was made with inexpensive and commercially available materials.
Tests with a standard specimen of fused silica showed for thermal conductivity a discrepancy of about 6% relative to the values expected by mathematical formulae. This can be ascribed to heat losses into the material of the sensor block.

Durante as últimas décadas o método transiente da fonte linear de medida de condutividade térmica de materiais pastosos tem sido usado para medir condutividade de rochas sólidas. Mais recentemente, o método tem sido adaptado para rochas com geometria semi-infinita mas usando ainda o formalismo matemático de meio infinito com simetria cilíndrica.
Para superar esta dificuldade propomos um modelo teórico simples para geração de calor na superfície de um meio semi-infinito. O modelo consiste em substituir a fonte linear infinita por uma fonte planar de largura limitada. A solução encontrada foi aplicada para medir condutividade térmica de rochas sólidas com geometria semi-infinita. As medidas foram executadas com um aparelho de fonte linear construído em nosso laboratório. A construção deste aparelho é descrita em detalhe e foi feita com materiais de baixo custo disponíveis no comércio.
Testes com uma amostra padrão de sílica fundida mostraram uma discrepância para a condutividade térmica em torno de 6% relativa aos valores esperados pela fórmula matemática. Atribui-se esta discrepância às perdas de calor para o material do bloco sensor.

INTRODUCTION

The line source method has increasingly been used for measuring thermal conductivity of rocks during the last few decades (Hamza et al., 1980; Carvalho et al., 1980; Carvalho, 1981; Marangoni & Hamza, 1983; Vacquier, 1985). This technique has largely been preferred over the steady-state divided-bar method in cases where shorter times to perform measurements are required. However, the line source method presents some undesirable aspects related to the preparation of the samples such as the difficult work required to drill holes of small diameter in consolidated rocks and the existence of high contact resistance between the sample and the needle probe.

In a recent work, Carvalho (1981) proposed a new method for measuring the thermal conductivity of solid rocks. The method is a variation of that described by Von Herzen & Maxwell (1959) and consists of inserting a needle-probe into a sandwich consisting of an insulating plate and a polished face of a rock sample whose thermal conductivity is to be measured. This proposal is very promising since it appreciably reduces the thermal contact resistance problem as well as the need for elaborate work involved in the preparation of samples. The theoretical basis of the methos used by Carvalho (1981) is that developed by Blackwell (1954) in which the needle-proble is modeled as an infinitely long continuous cylindrical source of heat in an homogeneous and infinite medium. According to this

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theoretical description the probe is supposed to generate heat at a constant rate Q per unit length and unit time and the temperature T near the cylindrical source is given by:

$$T = \frac{Q}{4\pi K} \ln (4\alpha t/Ab^2) \quad (1)$$

Where

t is time;
K, α are the thermal conductivity and thermal diffusivity of the infinite medium;
b is probe radius, and
A = 1.7811, a constant.

This relationship is valid for large values of t and gives, for a plot of T versus $\ln t$, a straight line whose slope is $Q/4\pi K$. From this slope the value of K can easily be found provided Q is known. Values obtained by Carvalho (1981) for thermal conductivity of several samples showed a divergence from values measured by other methods by a factor of about 2. As noted by Carvalho (1981) the theoretical approximation used is that of a continuous cylindrical source in an infinite medium, while the experimental method corresponds to that of a continuous cylindrical source in a semi-infinite medium. Consequently an "ad hoc" factor of 2 was used to adjust the experimental data to values found by steady-state standard methods.

In this paper the solutions obtained for the problem of a continuous strip source of heat in a semi-infinite medium is considered as a more appropriate approximation to the experimental set up used by Carvalho (1981). A similar apparatus was constructed and tested with a standard specimen of fused silica. Measurements of thermal conductivities for different rocks were also carried out. It was verified that the theoretical approximation mentioned above gives, for the rock samples that were tested, values of thermal conductivity in agreement with typical values found in the literature.

THEORY

The experimental set-up using a needle-probe sandwiched between an insulating plate and a rock sample whose thermal conductivity one intends to measure (see Fig. 1) is assumed, in our approximation, as a continuous strip source which supplies heat to a semi-infinite homogeneous medium. This may be justified if we imagine that the heat generated in a needle-probe of radius a can only flow into a half-space, it thus touch this half-space along a strip of width 2a. The insulating plate is assumed to be a medium of thermal conductivity $K_i = 0$. For practical purpose this means that the rock sample has thermal conductivity $K >> K_i$. As a theoretical approximation consider the semi-infinite medium $z > 0$, on which heat is being supplied at the rate q per unit time per unit area over the infinite strip $-a < x < a$, $-\infty < y < \infty$, $z = 0$. If the region $z > 0$ is initially at temperature $T = 0$, the temperature T at the point x of the surface $z = 0$ at a time $t > 0$ is given by (Carslaw & Jaeger, 1959):

$$T = \frac{qa}{K} \sqrt{(\tau/\pi)} \left\{ \text{erf}\left(\frac{a+x}{2a\sqrt{\tau}}\right) + \text{erf}\left(\frac{a-x}{2a\sqrt{\tau}}\right) - \frac{a+x}{2a\sqrt{(\pi\tau)}} E_i\left[-\left(\frac{a+x}{2a\sqrt{\tau}}\right)^2\right] - \frac{a-x}{2a\sqrt{(\pi\tau)}} E_i\left[-\left(\frac{a-x}{2a\sqrt{\tau}}\right)^2\right]\right\} \quad (2)$$

Where

$\tau = \alpha t/a^2$;
K, α are the thermal conductivity and thermal diffusivity of the semi-infinite medium, and
a is the semi-width of the strip.

For small values of ξ the error function $\text{erf}\xi$ and the exponential integral function $E_i(-\xi)$ may be approximated by (Abramowitz & Stegun, 1970):

$$\text{erf}\xi \cong \frac{2}{\sqrt{\pi}} \xi \quad (3)$$

$$E_i(-\xi) \cong \gamma + \ln \xi \quad (4)$$

Where

$\gamma = 0.5772$, is the Euler constant.
If we take these approximations and the measurements are carried out at x=0, equation (2) can be written as

$$T = \frac{Q}{2\pi K} \ln (B\alpha t/a^2) \quad (5)$$

Where

$Q = 2aq$, is the power input per unit length per unit time, and
B = 16.595, is a constant.

This relationship is valid for t large compared with a^2/α. It was experimentally observed that a plot of T versus $\ln t$ has a linear asymptote for times greater than about 40 seconds. The inverse of the slope of the straight line, when multiplied by $Q/2\pi$ gives directly the thermal conductivity of the sample.

Observing equations (1) and (5) it is easily seen that the slope of the curve T versus $\ln t$ for the latter is twice of that for the former. The physical reason is that in the mathematical approximation leading to equation (1) heat diffusion occurs through an infinite medium with cylindrical symmetry, while in the approximation leading to (5), heat diffusion takes place only through the half-space occupied by the sample. As a result the thermal conductivity measurement performed with the semi-infinite medium and analysed with the infinite medium theoretical approximation will give a half of the correct value.

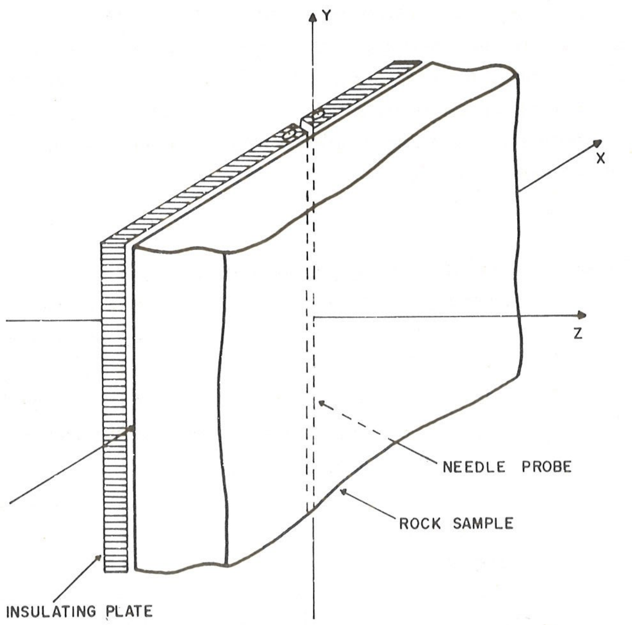

Figure 1 — Sketch showing the geometry of the half-space strip source device.

APPARATUS

A schematic diagram of the needle probe that was constructed in our laboratory is shown in Fig. 2a. A chromel-constantan thermocouple was used as the temperature sensor in place of a thermistor which is generally used. The chromel-constantan thermocouple shows a good linear behavior in the range 0-100°C, which is the range of temperature found in our experiments. The calibration of this sensor was performed with the reference junction at 0°C. In order to reduce the probe diameter, and thereby meet the line source approximation as described by Blackwell (1956), the chromel and constantan wire ends were soldered in opposition as shown in Fig. 2b. The thermocouple was used as a mechanical support for the coil of nichrome heater wire and it was varnish-coated to avoid any electrical contact with the heater wire. The electrical resistance of the nichrome coil was 74.1 ohm and the electrical insulation between this coil and the thermocouple wires were as high as 20 MΩ. The probe constructed in such a way had a total diameter of 1 mm, including electrical insulation, a length of 4.6 cm and was attached to a plate of amianthus. This plate acts both as a mechanical support and as an insulation medium which forces the flow of heat toward the rock sample. The samples were prepared with a polished plane surface in order to make good thermal contact with the plate of amianthus.

RESULTS AND COMPARISONS

Thermocouple emf were recorded in a continuous mode by a graphical potentiometer recorder which has 0.005 mV sensibility in the range 0-5 mV and a precision timer of 1/60 s. A typical record for measurements carried out with the equipment is shown in Fig. 3 where, with a 3W power input, the 3-minute time interval was found to be suitable for the determination of K. A plot of temperature versus log t for the same sample (fused silica) is shown in Fig. 4. As can be seen, a linear asymptote is closely approached after about 40 seconds. At smaller times the influence of thermal inertia of probe material, as well as the approximations (3) and (4) used in the mathematical theory of the method, do not permit a linear rise of temperature with $\ell n t$. It was observed during measurements that

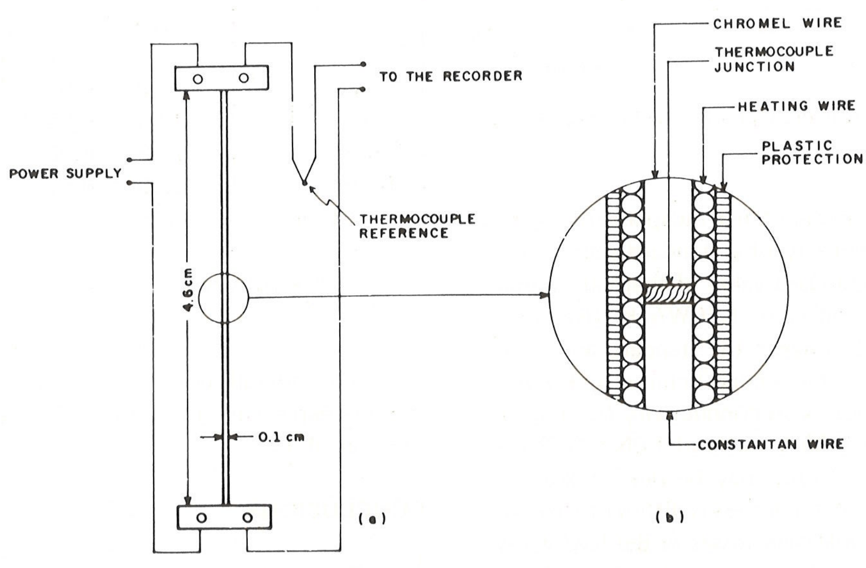

Figure 2 — a) Schematic diagram of the needle probe;
b) Expanded cross section showing heater wire and thermocouple configuration.

the linear behavior cited above, for any sample, always occurred after a time of about 30 to 40 seconds.

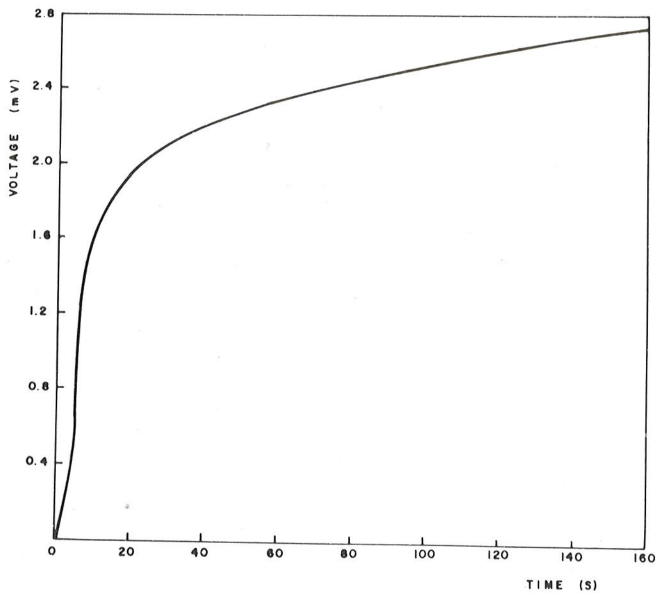

Figure 3 -- Thermocouple voltage versus time record for the fused silica (thermocouple voltage is proportional to the temperature at the middle of the strip source).

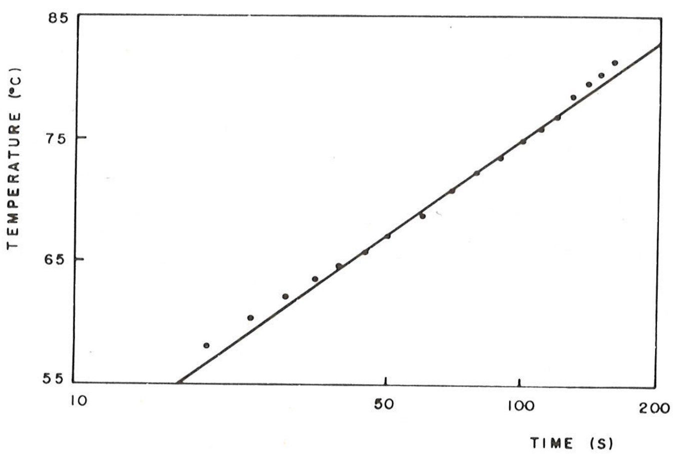

Figure 4 — Temperature versus log t plot for the fused silica.

Table 1 shows the results of five independent measurements of thermal conductivity of a standard specimen, a disk of fused silica. The standard value of thermal conductivity of fused silica at 50°C is 1.40 W/mK (Ratcliffe, 1959). The difference (f) between the standard and experimental values are given in the second column of Table 1. The values obtained for the mean conductivity ($\overline{K}$) and for difference ($\overline{f}$) are 1.32 ± 0.03 W/mK and 1.06 ± 0.03 respectively. This correction factor may be due to the fact that thermal conductivity of amianthus is different from zero, as well as due to possible heat losses in the lead wires.

In order to test the probe performance, thermal conductivities of different rock samples were measured and the results compared with values reported by Kappelmeyer & Haenel (1974). The results of the comparative study given in Table 2 show that measured values are within the range of compiled values for diorite, gabbro and granite. In the case of limestone the measured values of thermal conductivity is in good agreement with the average value of 3.20 W/mK obtained by T.B. Costa (personal communication) using steady-state divided bar method for limestone samples obtained from a well at Mossoró-RN, only a few kilometers away from where samples have been picked up for the present study.

Table 1 — Results of five independent measurements of thermal conductivity of a standard specimen of fused silica using the apparatus described in the present work. f is the factor by which K measured must be multiplied in order to obtain 1.40 W/mK, the standard value of thermal conductivity of fused silica at 50°C.

K measured (W/mK)	f
1.27	1.10
1.34	1.04
1.30	1.08
1.33	1.05
1.34	1.04
$\overline{K}$ = 1.32 ± 0.03	$\overline{f}$ = 1.06 ± 0.03

Table 2 — Comparison of thermal conductivities measured by the apparatus with those compiled from published values. σ is the standard deviation.

Sample	K measured (W/mK) ± σ	K compiled[1] (W/mK)
Basalt	2.42 ± 0.08	1.33 – 2.29
Diorite	1.84 ± 0.08	1.91 – 2.08
Gabbro	2.18 ± 0.08	1.93 – 2.51
Granite	2.56 ± 0.08	2.16 – 3.25
Limestone	3.15 ± 0.10	1.70 – 2.68

1 From Kappelmeyer & Haenel (1974).

The overall error, including errors in power input thermocouple emf and time records is estimated as being of order of 3%.

CONCLUSIONS

The mathematical description for the temperatura variation with time due to a strip source of heat on the surface of a semi-infinite solid has been used for measuring the thermal conductivities of consolidated rocks. The values found for a standard specimen of fused silica shows a difference of about 6% relative to the expected value at 50°C of 1.40W/mK given by Ratcliffe (1959).

The results obtained for different rock samples, as measured with our apparatus, show agreement with typical values for thermal conductivity found in the literature.

This equipment can easily be constructed with inexpensive commercially available materials.

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