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ANALYSIS OF SEISMIC DATA IN THE τ-p DOMAIN

PAUL L. STOFFA[1]

Universidade Federal da Bahia — Instituto de Geociências e Instituto de Física, Programa de Pesquisa e Pós-graduação em Geofísica Rua Caetano Moura, 123 — Federação, Salvador, BA, Brazil

Densely sampled surface seismic data can be transformed to the τ-p domain to derive information about subsurface propagation velocities and to develop images of the subsurface that depend upon the angle of incidence. For one dimensional earth structures this plane wave decomposition of the observational data makes it possible to derive interval velocities estimates using pre- and postcritical reflections and refractions with little or no limiting assumptions. Once the velocity structure is obtained, it can be used to conrrect the seismic arrivals to their two-way normal incidence travel time in a manner analogous to that employed in traditional seismic processing, but without limiting assumptions. Amplitude dependence as a function of incidence angle can also be preserved. 1D acoustic and elastic synthetic seismic data can also be computed in this domain and require a similar transformation that superimposes the individual plane wave response functions. Numerical difficulties encountered in computing both transforms illustrate the need for fine two dimensional sampling. These aliasing artifacts occur because of finite spatial and temporal sampling and limited receiving aperture. They can be minimized by employing modern data acquisition systems but can often be reduced by simple windowing procedures during the transformation. But, in many applications such as velocity analysis and imaging, these artifacts will not be coherent along the imaging trajectories and their impact is minimal.

INTRODUCTION

Seismic data are acquired and analyzed to derive information about the subsurface. Two primary objectives are the construction of a structural image which delineates changes in material properties and the identification of these properties, i.e., structural and stratigraphic inversion. Changes in material properties are defined by reflection coefficients, and the subsurface property often the most useful to identify is the intrinsic propagation velocity. Velocity is a key to seismic interpretation because it is often indicative of the type of subsurface material and because it is required to properly form the subsurface structural image.

The complete seismic interpretation problem is complex. The subsurface media vary in three-dimensions, and two wave types propagate, compressional and shear. Energy becomes trapped within geologic units and these reverberations often obscure weak primary reflection events of interest. Energy not only reflects from the discontinuities in subsurface properties, it also scatters and refracts.

Our ability to derive information about the subsurface is limited by the acquisition methodology available or economically feasible. For example, three-dimensional areal surveys have only recently been employed, and these are usually approximations to true three-dimensional data acquisition. In most acquisition programs, the cost does not justify areal acquisition and relatively sparse linear profiles are acquired.

During routine seismic data acquisiton, a source of energy and a linear receiving array are deployed at predetermeded positions along the profile line (Fig. 1). The linear array of detectors receive the reflected and refracted energy from each source activation. The receiving array typically is 3.6 km in length and the distance between source locations is often 25 m. This common source data can be focused during processing to minimize the distorting effects of subsurface dip and roughness. The original received data from multiple source locations which have Common Mid Points, CMP's, are gathered into an ensemble for further processing. This simple procedure makes it possible to significantly simplify the analysis problem which would be encountered in analyzing the original data (Fig. 1).

Originally, the receiving apertures employed were usually less than or equal to the depth of subsurface targets of interest. This restricted the observed events to relatively small subsurface angles of incidence. As digital recording technology improved and it became possible to faithfully record hundreds of channels of data at sample rates greater than 1 msec, high temporal and spatial resolution became possible. This also made it possible for larger offsets to be acquired, e.g., from 2.4 to 4.8 km, without significantly degrading the spatial resolution. This trend

1 Now at The University of Texas at Austin, Institute for Geophysics and Department of Geological Sciences, 8701 Mopac Boulevard, Austin, Texas 78759-8345.

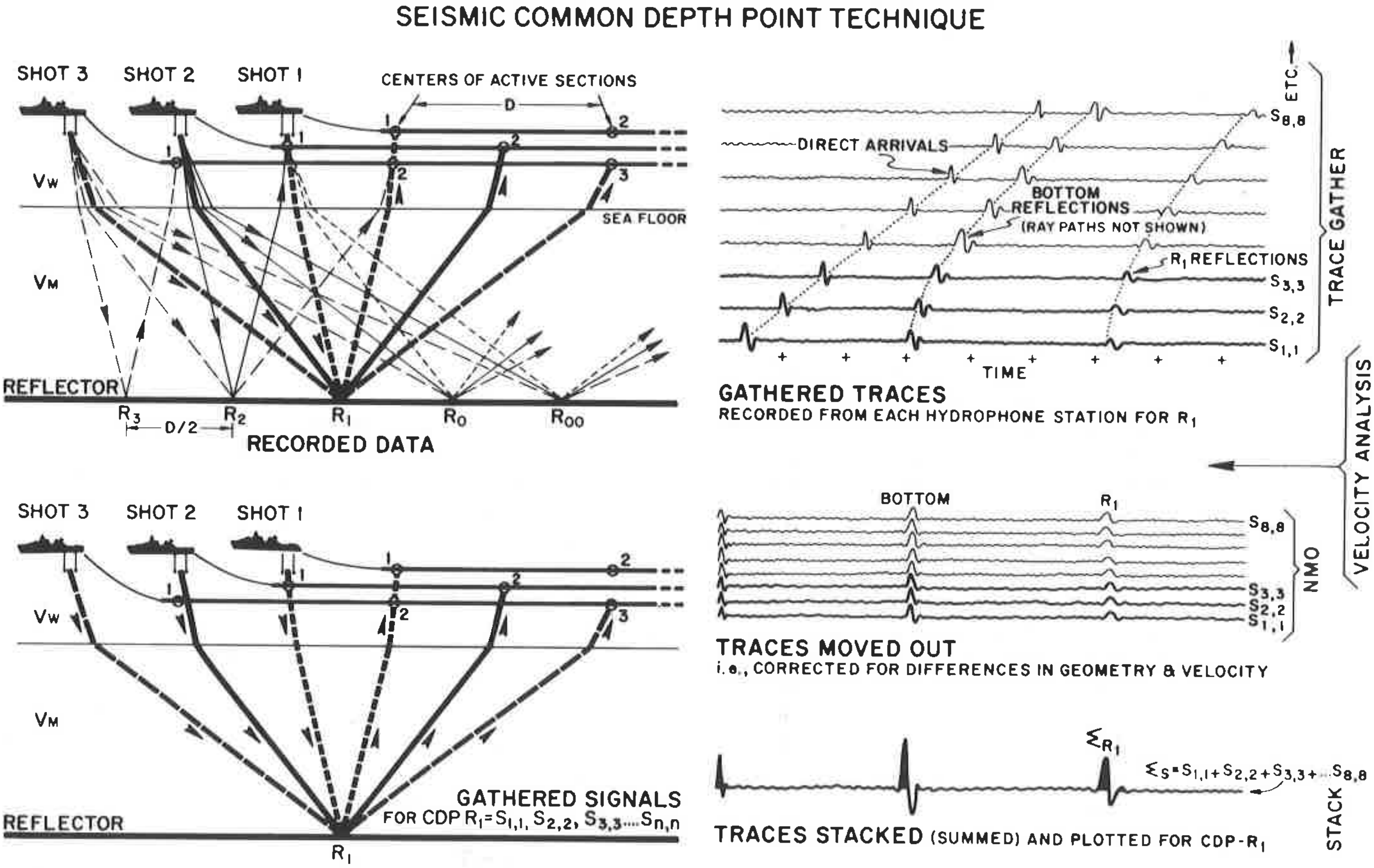


Figure 1 — A schematic diagram of the basic steps employed in marine common depth point (or more accurately, common midpoint) data acquisition and processing. During acquisition, common shot point data are recorded (upper left) as the ship advances a distance of D/2 along the profile line between each shot. During processing traces which have a common surface midpoint from multiple shots are combined into a CDP gather (lower left). This subsurface focusing minimizes the effect of dipping horizons and topography during the later processing stages. The increase in traveltime with offset or normal moveout (upper right) can be approximated by the hyperbolic traveltime equation for small offsets. The stacking velocities which correct for normal moveout are used to align the seismic events for all offsets (center right) before combining (stacking) the events into a single trace with increased signal to noise ratio (lower right).

toward larger offsets makes it possible to record seismic events other than near-normal-incidence reflections.

In routine applications it is common to exploit the limited aperture CMP seismic data typically acquired by using simplifying assumptions about the near normal incidence reflection field. For example, it is common to assume that the reflection events follow hyperbolic travel time, T(X), trajectories:

$$T^2 = T_0^2 + \frac{X^2}{V_s^2}$$

Knowledge of the two-way normal time, T_0, and hyperbolic stacking velocity, V_s, makes it possible to correct the arrival time of the reflections at each source-receiver offset to the normal ray path time, To. After this *"normal moveout"* correction, the data are summed (stacked) to improve both the signal-to-random and coherent noise ratio (Fig. 1). At each surface midpoint location the procedure is repeated, eventually producing an image which is related to the subsurface reflectivity along the profile line (Fig. 2). Also, the stacking velocities derived by analysis of the original CMP gathers are now known and, after some additional assumptions, can be related to the sobsurface propagation velocities.

For limited aperture CMP data this analysis method works reasonably well, since the assumptions employed are consistent with the data acquired. That is, since the aperture is limited, Dix's (1955) $T^2 - X^2$ approximate travel time equation is appropriate and hyperbolic stacking velocities derived from the data can be assumed equivalent to the root mean square, RMS, velocity. From these velocities and the two-way normal reflection times, the subsurface interval velocities can be estimated. But, for such limited aperture data, the potential for further subsurface resolution, particularly velocity resolution, is limited.

It is now widely recognized that large offset data can help in refining our estimates of material velocities and that changes in reflectivity with offset (or angle of incidence) can be useful in gaining knowledge about changes in material properties. To take advantage of both possibilities, a

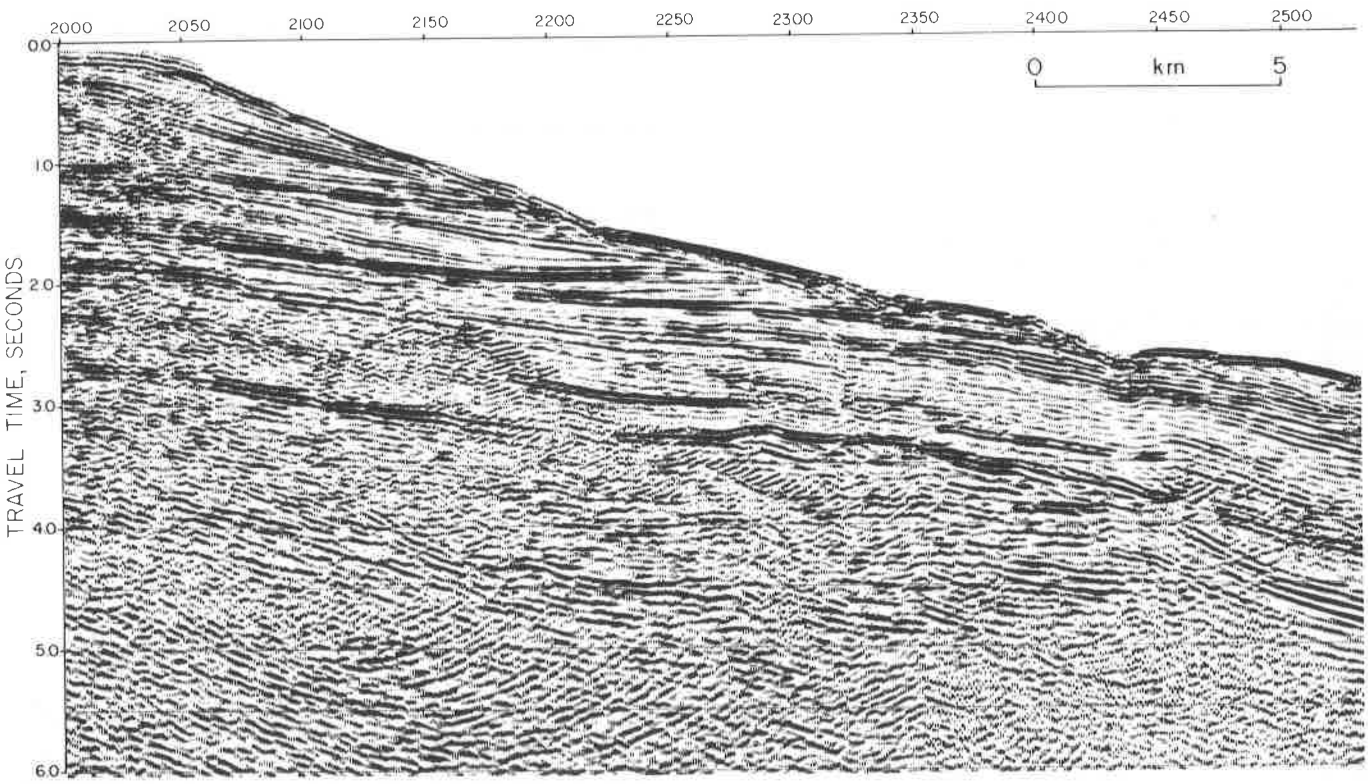


Figure 2 — Seismic record section formed using the procedures summarized in Fig. 1, 550 stacked seismograms are displayed. Additional processing procedures applied were: deconvolution to shape the source wavelet; band pass filtering; gain recovery; and migration.

new approach to the acquisiton, processing, and analysis of seismic data is required.

Assuming acquisiton methods in the future will routinely incorporate source-receiver offsets that are typically greater than the target of interest, existing processing methods will be found inadequate. An alternative and promising approach is to first perform a plane wave decomposition of the observed wave field. As we will see below, this will result in a data set that is composed of individual plane wave seismograms, each of which can be analyzed independently, or in combination with other plane wave seismograms. One way to accomplish this plane wave decomposition is to transform the observed data from the domain of source-receiver offset X and two-way travel time T to the domain of horizontal ray parameter p and vertical delay or intercept time τ. This transformation has already beem employed with success in several applications, particularly in the analysis of postcritical reflections and refractions (Diebold & Stoffa, 1981).

The τ-p parameterization occurs naturally for the case of a horizontally stratified earth model. It also readily explains why the CMP experimental geometry has proved successful in minimizing the effects of dip and subsurface structure. For one-dimensional structures, data transformed to this domain can be analyzed to derive velocity-depth data directly without the need of the intermediate stacking and RMS velocities. Also, all wave types, pre- and postcritical reflections and refractions can be employed in the velocity analysis. Finally, data can be combined directly in this domain to produce the subsurface image.

All currently employed X-T velocity analysis methods can be incorporated into equivalent methods or superior ones in the τ-p domain. This leads to the speculation that the τ-p domain may be where future "routine" processing is performed. We summarize below the relations between X-T and τ-p for a one-dimensional earth and consider some practical problems encountered in performing the transformation. We then review three methods of velocity analysis that can be performed in the τ-p domain.

THE RELATION OF τ-p To X-T SEISMOGRAMS

We assume a one-dimensional earth structure overlain by a half space. The potential of a unit amplitude point source excitation at the origin may be expressed as

$$\Phi_s(T,R) = \frac{1}{R}\,\delta\left(T - \frac{R}{\alpha_1}\right) \tag{1}$$

where $R = (X^2 + Y^2 + Z^2)^{1/2}$ and δ is Dirac's delta function. This spherical wave of velocity α_1, is a solution of the wave equation in a homogenous medium. Its Fourier transform (Ewing, et al. 1957) can be written as the Sommerfeld integral:

$$\frac{1}{R}\, e^{-j(\omega/\alpha_1)R} = \int_0^\infty J_0(kr)\, e^{-j\nu_1 |z|}\, \frac{k}{j\nu_1}\, dk \tag{2}$$

where

$$\nu_1 = \begin{cases} \left(\dfrac{\omega^2}{\alpha_1^2} - k^2\right)^{\frac{1}{2}} & \dfrac{\omega^2}{\alpha_1^2} \geqslant k^2 \\ -j\left(k^2 - \dfrac{\omega^2}{\alpha_1^2}\right)^{\frac{1}{2}} & \dfrac{\omega^2}{\alpha_1^2} \leqslant k^2 \end{cases}$$

and $r = (X^2 + Y^2)^{\frac{1}{2}}$. J_0 is the zeroth order Bessel function, ω is the angular frequency, and k is the horizontal wave number. Since

$$J_0(kr) = \frac{1}{2\pi} \int_0^{2\pi} e^{-jk(X\cos\rho + Y\sin\rho)} d\rho \tag{3}$$

it is clear that equation (2) is a decomposition of the spherical wave into plane wave components.

This plane wave description can also be used to describe the response of a layered medium observed at $z = 0$:

$$\Phi(\omega, X) = \int_0^{\infty} R(\omega,k)\, J_0(kX) \frac{k}{j\nu_1} dk, \tag{4}$$

(see Wenzel et al., 1982, Trietel et al., 1982 and Brysk & McCowan, 1986), where $R(\omega,k)$ is the plane wave impulse response for all layers beneath the upper half space and includes all internal multiples and compressional and shear wave conversions. (We have replaced r with X because we will only consider cylindrical symmetry). Also, $R(\omega,k)$ includes the time delay required for a plane wave to travel from the source to the first interface and back to the receiver at $z = 0$.

This description can be rewritten in terms of vertical delay time, $\tau = T - px$, and horizontal ray parameter, p. We change variavles from horizontal wavenumber k to horizontal ray parameter p, where $p = k/\omega$ and equation (4) becomes

$$\Phi(\omega, X) = \int_0^{\infty} (-j\omega)\widetilde{R}(\omega,p)\, J_0(\omega pX) \frac{p}{\eta_1} dp \tag{5}$$

where

$$\eta_1 = \begin{cases} \left(\dfrac{1}{\alpha_1^2} - p^2\right)^{\frac{1}{2}} & p \leqslant \dfrac{1}{\alpha_1} \\ -j\left(p^2 - \dfrac{1}{\alpha_1^2}\right)^{\frac{1}{2}} & p \geqslant \dfrac{1}{\alpha_1} \end{cases}$$

Although we could compute the Bessel function directly (Chave, 1983), we will use the standard high frequency approximation and consider only outgoing waves,

$$J_0(\omega pX) \cong \frac{1}{\sqrt{2\pi|\omega|\,pX}} e^{-j\omega pX} e^{j(\pi\omega/4|\omega|)} \tag{6}$$

We can rewrite (5) as

$$\Phi(\omega, X) = \frac{1}{\pi\sqrt{2X}} \int_0^{\infty} \frac{p^{\frac{1}{2}}}{\eta_1} \left[(-j\omega)\left(\frac{\pi}{|\omega|}\right)^{\frac{1}{2}} e^{+j(\pi\omega/4|\omega|)}\right] \widetilde{R}(\omega,p)\, e^{-j\omega pX} dp \tag{7}$$

and after an inverse Fourier transform we find,

$$\Phi(T, X) = \frac{1}{\pi\sqrt{2X}} \int_0^{\infty} \frac{p^{\frac{1}{2}}}{\eta_1} \left[-\frac{d}{dT} \frac{\theta(-T)}{(-T)^{\frac{1}{2}}}\right] * R(T - pX, p)\, dp \tag{8}$$

where $\theta(T)$ is the Heaviside or unit step function, R is the Fourier transform of $\widetilde{R}$ and * denotes convolution. The terms in brackets in equations (7) and (8) are Fourier transform pairs as shown by Chapman (1978), see also Wenzel et al. (1982) and Brysk & McCowan (1986). We recognize the vertical delay time, $\tau = T - px$, in equations (7) and (8). To derive the plane wave reflection response from seismograms, the above procedure is reversed:

$$\widetilde{R}(\omega, p) = \frac{\eta_1}{\pi\sqrt{2p}} \int_0^{\infty} X^{\frac{1}{2}} \left[j\omega\left(\frac{\pi}{|\omega|}\right)^{\frac{1}{2}} e^{-j(\pi\omega/4|\omega|)}\right] \Phi(\omega, X) e^{j\omega pX} dX \tag{9}$$

or

$$R(\tau, p) = \frac{\eta_1}{\pi\sqrt{2p}} \int_0^{\infty} X^{\frac{1}{2}} \left[\frac{d}{d\tau} \frac{\theta(\tau)}{\tau^{\frac{1}{2}}}\right] * \Phi(\tau + pX, X)\, dX \tag{10}$$

Equations (8) and (10) are transform pairs. Seismograms are derived from the plane wave reflection response, and the plane wave reflection response is derived from seismograms using these transforms. For detailed discussions, see Chapman (1978), Phinney et al. (1981), Wenzel et al. (1982), Treitel et al. (1982), Brysk & McCowan (1986).

Several methods can be used to calculate the plane wave elastic reflection response $R(\tau,p)$ for a layered half-space. For example, we can compute its Fourier transform $R(\omega,p)$, directly using the method of Thomson (1950) and Haskell (1953) as modified by Dunkin (1965) or the approach of Kind (1976). Alternatively, the time domain formulation of Frasier (1970) might be used. If we are interested in only acoustic propagation, the recursive formulation of Tolstoy & Clay (1966) can be implemented. We describe briefly this formulation since it quickly illustrates the relationship between the vertical wave number and τ-p response.

In Tolstoy & Clay's (1966) formulation, the response observed above an interface due to an underlying one-dimensional structure is

$$R_i = \frac{c_i + R_{i+1}\, e^{2ikz_{i+1}\Delta z_{i+1}}}{1 + c_i R_{i+1}\, e^{2ikz_{i+1}\Delta z_{i+1}}}$$

where kz_{i+1} is the vertical wave number, Δz_{i+1} is the thickness in the underlying layer and c_i is the reflection coefficient at the i^{th} interface. Since

$$k_{z_{i+1}} = k_{i+1} \cos\theta_{i+1} \,,$$

$$k_{i+1} = \omega / v_{i+1} \,,$$

$$\cos\theta_{i+1} = (1 - p^2 v_{i+1}^2)^{\frac{1}{2}}, \text{ and}$$

$$2\Delta z_{i+1} / v_{i+1} = \Delta\tau_{i+1}(0)$$

where $\Delta\tau_{i+1}(0)$ is the two-way time at normal incidence, $p = 0$, we have:

$$R_i(p) = \frac{c_i(p) + R_{i+1}(p) e^{i\omega\Delta\tau_{i+1}(p)}}{1 + c_i(p) R_{i+1}(p) e^{i\omega\Delta\tau_{i+1}(p)}} \qquad (11)$$

where $\Delta\tau_{i+1}(p) = \Delta\tau_{i+1}(0)\ (1 - p^2 v_{i+1}^2)^{\frac{1}{2}}$. This recursive formulation can be used to quickly generate the acoustic τ-p response for a 1-D acoustic earth structure (Fig. 3). In Fig. 3, the refractions from the gradient zone and the postcritical reflections in this acoustic model exhibit the proper phase and the internal multiples from the reflections and the gradient zone are present. (Note that gradient zones were approximated by layers of one unit sample in two-way normal time).

Equation (11) indicates that all multiple arrivals will be periodic in the τ-p domain. These arrivals are generated as feedback controlled by the denominator term. Alam & Austin (1981) exploited this periodicity to deconvolve unwanted multiple energy from seismic data. In the X-T domain, multiple arrivals are only periodic at zero offset (actually $p = 0$) and are difficult to remove by single trace filtering methods.

After the plane wave response is computed for a particular structure using any one of the above formulations we must then integrate over all ray parameters. To calculate seismograms from a discrete τ-p response $R(i, p)$, where i is the time sample index, requires a discrete form of equation (8). Before the integration we apply the operator $(d/dT)(\theta(-T)/(-T))^{\frac{1}{2}}$ by convolution in the time domain or its transform by multiplication in the frequency domain. The seismogram $\Phi(T,X)$ is

$$\Phi(T,X) = \frac{1}{\sqrt{2\pi X}} \int_0^\infty R'(T - pX, p) \frac{p^{\frac{1}{2}}}{\eta_1} dp \qquad (12)$$

where

$$R'(\tau,p) = \frac{1}{2\pi} \int_0^\infty |\omega|^{\frac{1}{2}} e^{+j(\pi\omega/4|\omega|)} \tilde{R}(\omega,p) e^{j\omega\tau} d\omega$$

In discrete form, the seismogram $\Phi(i, X)$ is found by approximating the integral of (12) by the sum

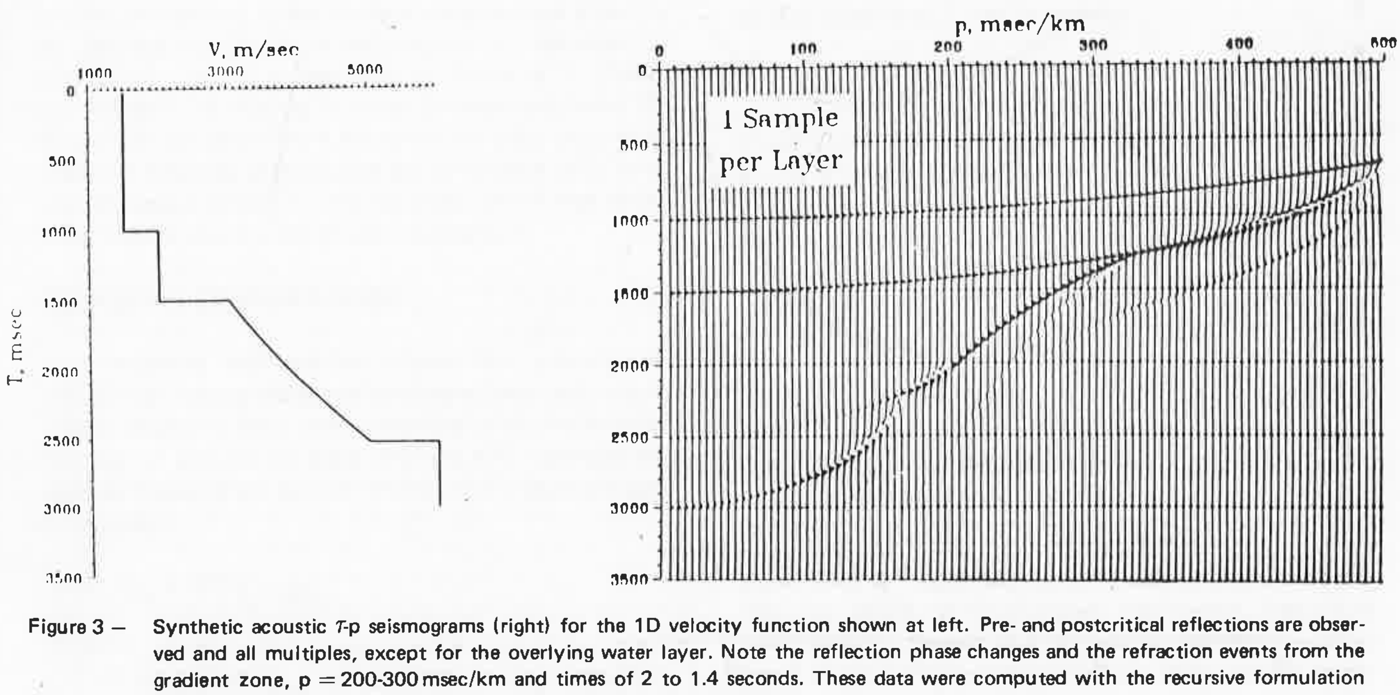


Figure 3 — Synthetic acoustic τ-p seismograms (right) for the 1D velocity function shown at left. Pre- and postcritical reflections are observed and all multiples, except for the overlying water layer. Note the reflection phase changes and the refraction events from the gradient zone, $p = 200$-300 msec/km and times of 2 to 1.4 seconds. These data were computed with the recursive formulation equation 11, in the ω-p domain. A constant density of 1. was used.

$$\Phi(i,X) = \frac{1}{\sqrt{2\pi X}} \sum_{j=0}^{NP} R'(i\text{-}j\Delta pX, j\Delta p) \frac{(ij\Delta p)^{1/2}}{\eta_1} \Delta p \qquad (13)$$

Numerical dificulties will be encountered since the discrete implementation requires limited bandwidth. Wenzel et al. (1982) considered in detail the numerical problems encountered when implementing the discrete formulation. By not artificially degrading the temporal bandwidth, including all compressional and shear conversions, and all possible multiples, they found that a dense ray parameter sampling was required.

In the integration of equation (13), one problem occurs because events which should cancel during the integration will not completely cancel in any discrete implementation. These events will not necessarily be coherent, but may have significant amplitudes. The finer the ray parameter sampling, the more likely it is that these events will add out of phase and subsequently not contribute significantly to the integral. Fig. 4 illustrates the problem by displaying the integrand for an offset of 8.5 km for a typical offshore structure. It is clear that only events that are nearly horizontally aligned should contribute significantly to the integration. These events are the points of stationary phase and correspond to the predictable geometric ray paths. Although the influence of all ray parameters will be included by the integration process, only the contributions near the points of horizontal tangency should significantly influence the results.

Rather than decrease the ray parameter sampling interval, the integral could be weighted or tapered to decrease the influence of events outside the region of interest. This can be done globally or locally by incorporating knowledge of the structure to predict the geometric ray paths. Wenzel et al. (1982) used a coherency measure, semblance, to numerically identify the points of stationary phase and then performed a weighted integration. Comparable results to the ray parameter sampling interval of 0.32 msec/km were obtained with a factor of 8 decrease in the ray parameter sampling rate (Fig. 5).

When computing seismic models, we can choose the required sampling rates and/or implement approximations as required by economic considerations. For real seismic data, we are often not able to choose the spatial sampling required to accurately implement a discrete form of equation (10). Sampled data are rarely aliased in time, but to avoid aliasing in space requires a spatial sample interval ΔX, less than $V/2f_{max}$, where f_{max} is the highest frequency. Thus, the Nyquist ray parameter is equal to $1/(2\Delta Xf_{max})$. For seismic data with a maximum frequency of 125 Hz,

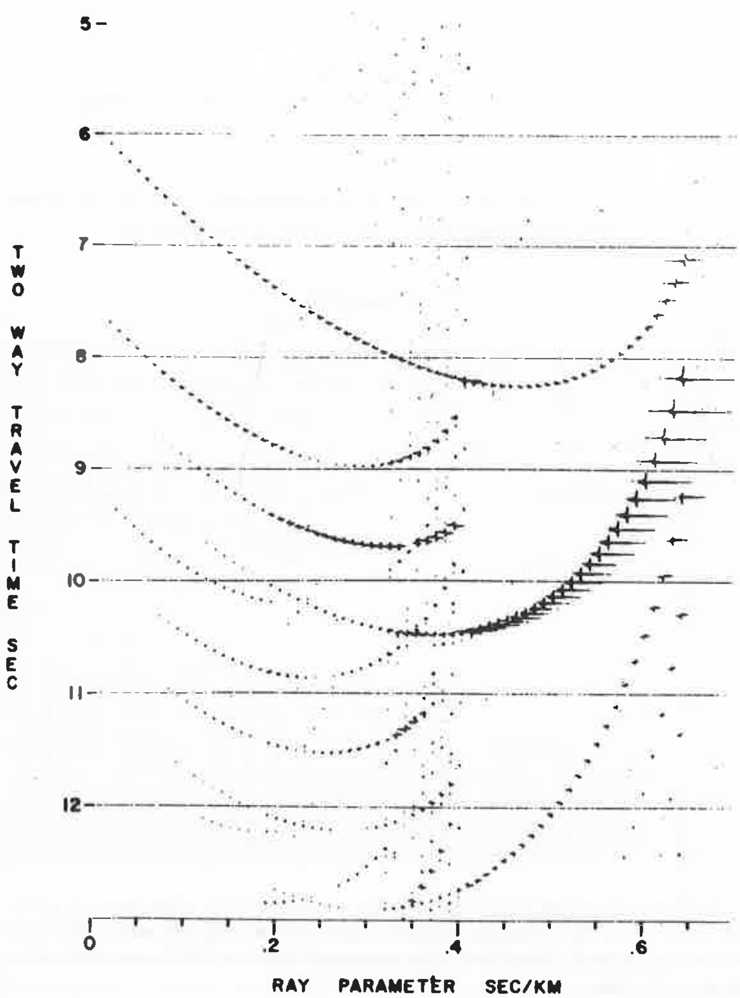


Figure 4 — The integrand of equation 8 for elastic Model C of Wenzel et al. 1982 for an offset of 8.5 km. After the integration over ray parameter, only the horizontally aligned (stationary phase) events should contribute significantly to the final seismogram at this offset.

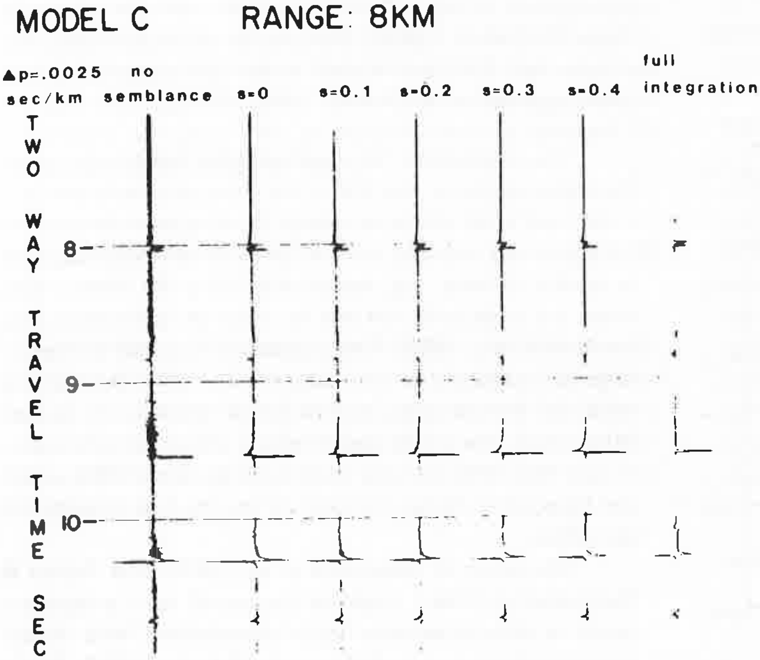


Figure 5 – Stationary phase seismograms computed by weighted integration for elastic Model C of Wenzel et al. 1982 at 8km. The seismogram on the far left was computed by direct integration over 256 ray parameters with a ray parameter sampling interval of 0.0025s/km. On the far right, the seismogram was computed by a direct integration of 2048 ray parameter traces with a ray parameter sampling interval of 3.125 x x 10^{-4} s/km. This fine ray parameter sampling interval is required to approximate the analytic solution. The seismograms in the center were computed by a weighted integration. In this example, the semblance values above the indicated minimum values were used as weights, and zero was used as the weight for values of semblance below the minimum. Semblance was computed over ray parameter intervals of 0.02 s/km as a measure of coherent to total energy in the interval. A limit of 0.1 indicates that only arrivals with a 10 percent coherent to total energy ratio over each subinterval are included in the seismogram. The nonstationary refraction arrival just below 8s of travel time is, however, not properly recovered.

the required spatial sampling is 6m. Thus, the discrete form of equation (10) will usually be aliased above 30Hz since in seismic acquisition a spatial sampling of 25m is typically employed.

In the case of forward modelling, the problem of aliasing can be minimized by additional computational effort. For real data, it can only be solved by considerable additional cost during data acquisiton. For a given spatial aperture and bandwidth the number of channels required is often prohibitive. If the τ-p data are to be used primarily for deriving velocity-depth information (as described below) or for imaging as described by Treitel et al. (1982), the problem of aliasing is often unimportant since the aliases will not be coherent along the τ-p trajectories of interest. If, however, the τ-p data are to be used in an inversion procedure which requires estimates of the true reflectivity, aliasing may be a more serious problem.

GEOMETRIC CONSIDERATION

Originally, multichannel seismic data primarily recorded near vertical incidence reflections, because the array lengths employed were small compared to the depth of the horizons of interest. In these data the X-T traveltime trajectories followed by seismic reflections are approximately hyperbolic:

$$T^2 = T_{0_n} + X^2/V^2_{RMS_n}$$

where T_{0_n} is the total two-way normal time to the base of the n^{th} layer and V_{RMS} is the RMS velocity defined by Dix (1955). Although only reflections from the first interface and its multiples follow truly hyperbolic X-T trajectories, for limited aperture seismic data, reflections from deeper horizons can also be approximated by this assumption on hyperbolic traveltimes. If all horizons are horizontal, the source-receiver configuration employed is not of consequence. In the case of dipping horizons, the common source/receiver geometry will cause traveltime differences from predicted by the hyperbolic assumption.

Consider a fixed source or receiver at location A (Fig. 6). The traveltime, T, can be written:

$$T = p_b X + \sum_{j=1}^{N} \Delta z_{a_j}(q_{a_j} + q_{b_j}) \quad (14)$$

where X is the source-receiver offset, Δz_{a_j} is the thickness of the j^{th} layer at location A, q_{a_j}, q_{b_j} are the vertical slowness in each layer at location A and B, and p_b is the horizontal ray parameter observed at the surface at location B. If the fixed source or receiver is at B,

$$T = p_a X + \sum_{j=1}^{N} \Delta z_{b_j}(q_{a_j} + q_{b_j}) \quad (15)$$

where Δz_{b_j} is the thickness of the j^{th} layer at location B, and p_a is the horizontal ray parameter observed at the surface at location A, see Diebold & Stoffa (1981) for a complete derivation and discussion.

With the advent of common midpoint data acquisition (Mayne, 1962) the effect of dip on seismic traveltimes was found to be of minor significance. Diebold & Stoffa (1981), showed that the exact traveltime equation for a common midpoint profile can be found by simply averaging the fixed source-receiver traveltime equations:

$$T_{CMP} = \frac{(p_a + p_b)}{2} X + \sum_{j=i}^{N} \Delta z_j (q_{a_j} + q_{b_j}) \quad (16)$$

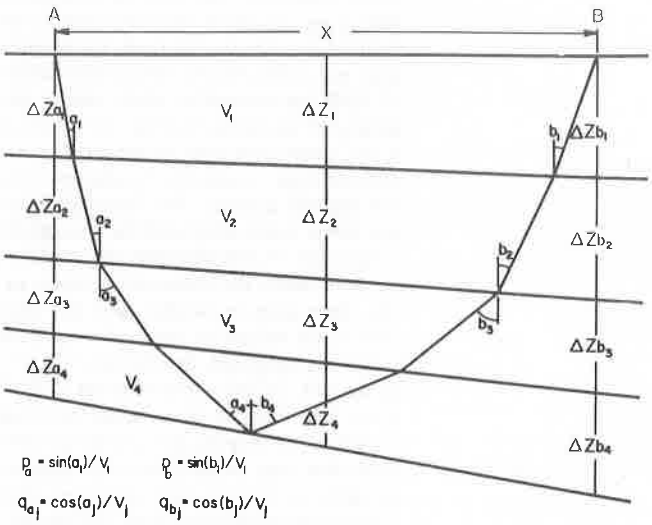


Figure 6 — Definition of coordinates for a layred earth with source and receiver at surface locations A or B. For a fixed source or receiver at location A or B, the vertical delay times will be referred to the fixed location, A or B, respectively. For common midpoint data, the vertical delay times are referenced to the midpoint of the profile.

Where Δz_i is the layer thickness at the midpoint of the profile. The small effect of dip for CMP traveltimes and the symmetry of CMP traveltime trajectories is a result of the horizontal ray parameter averaging inherent CMP data.

As shown earlier, the τ-p parameterization of seismic data appears naturally and the exact traveltime equations are all of the form $T = \tau + px$. But, the interpretation of both τ and p depends on the actual geometry employed. For common source or receiver profiles the ray parameter observed is the true ray parameter and the delay time is referenced to the fixed source or receiver location. For common midpoint profiles the ray parameter is an average of the ray parameters observed on reversed profiles and the vertical delay time is referenced to the midpoint of the profile.

VELOCITY ANALYSIS IN THE τ-p DOMAIN

One of the prime motivations for seismic acquisitior and analysis is the derivation of information about the subsurface velocity-depth structure. Previously, all analysis methods were based on the observational parameters of source-receiver offset and traveltime since seismic reflection/refraction events form X-T trajectories that can be used do derive velocity-depth information. Even for the case of one-dimensional structures, the true X-T trajectories for reflections are often approximated by an assumption of hyperbolic traveltime, see for example, Taner & Koehler (1969). But, as the offset increases, this assumption fails and higher order approximations have been suggested by Al-Chalabi (1973, 1974).

The motivation for employing the hyperbolic traveltime assumption is two-fold. First, it is possible to perform a trial and error computer search for all reasonable stacking velocities and two-way normal times. A coherency measure is usually defined, e.g., semblance, using the seismic data along the trajectories defined by these two parameters (Taner & Koehler, 1969). This coherency function is then interpreted manually or automatically to define the stacking velocities and two-way normal times. Second, for limited offset data, the interpreted stacking velocities will approximate the RMS velocity as defined by Dix (1955) which can be used to derive estimates of the the true propagation velocities.

The work of Bessonova et al. (1976) and Gerver & Markushevich (1967) explored the use of the τ-p representation in deriving velocity-depth information. These theoretical descriptions were followed by Schultz (1976), Stoffa et al. (1981), Phinney et al. (1981), and Clayton & McMechan (1982) who exploited various methods of deriving velocity-depth information from seismic profiles after their direct transformation to τ-p. For one-dimensional structures, all source-receiver offsets can be used in an analysis of seismic trajectories in the τ-p domain without any limiting assumptions or approximations. Precritical reflections, postcritical reflections and refractions can be simultaneously incorporated in the traveltime inversion. Finally, the hyperbolic traveltime assumption can be readily derived and its validity shown to be limited to angles of incidence, θ, where $\sin^4 \theta$ is neglibible.

We illustrate the derivation of velocity-depth information from seismic data transformed to the τ-p domain. We will consider one-dimensional structures (no dip) or assume that we are analyzing CMP data where the effect of dip is minimal. We consider first the analysis of postcritical reflections and refractions, then precritical reflections and finally both types of arrivals simultaneously.

In the case of no dip, the traveltime equation:

$$T = px + \tau$$

has a τ contribution related to the sum of the layer thicknesses and vertical slownesses:

$$\tau_n = 2 \sum_{j=1}^{n} q_j \Delta z_j \quad (17)$$

Since the vertical slowness, q_j, can be written in terms of the media velocity, v_j, and the horizontal ray parameter, p, as $q_j = (1 - p^2 v_j)^{1/2}/v_j$ and the two-way normal time, $\Delta\tau_j(0)$, is $2\Delta z_j/v_j$, the τ-p "traveltime" trajectories can be written as:

$$\tau_n(p) = \sum_{j=1}^{n} \Delta\tau_j(0)(1 - p^2 v_j^2)^{1/2} \quad (18)$$

The τ-sum (Diebold & Stoffa, 1981) inversion procedure makes use of only critical and postcritical seismic arrivals. It is equivalent to Slotnick's (1936) slope intercept method except that both postcritical reflections and refractions are used. The inversion formula is obtained by solving equation (20) for $\Delta\tau_n(0)$:

$$\Delta\tau_n(0) = \frac{\tau_n(p) - \tau_{n-1}(p)}{(1 - p^2 v_n^2)^{1/2}} \qquad (19)$$

the two-way normal time in the n^{th} layer. In this equation, $\tau_n(p)$ is observed, and $\tau_{n-1}(p)$ is predicted from knowledge of the overlying structure using equation (18). In this formulation, we implicitly assume that every ray parameter sampled corresponds to a layer whithin the earth of corresponding interval slowness ($u_n = 1/v_n$). We use equation (19) simply to predict the two-way normal time of this layer. If a new layer is not present and we are following a postcritical reflection, equation (19) will predict zero two way normal time.

Thus, Diebold & Stoffa (1981) showed that this inversion method can be used for both reflected and refracted arrivals. If p is densely sampled, it can also be used for velocity gradients. The main requirements are that critical reflection/refraction events are observed and that no low velocity zones are present. In the example of Fig. 7, 20km of source-receiver offset data were transformed to τ-p and band-pass filtered (0-20Hz). The envelope function of the data was computed and the critical and postcritical arrivals were automatically picked (because of their high amplitudes). The τ-p picks were then used in a τ-sum in-

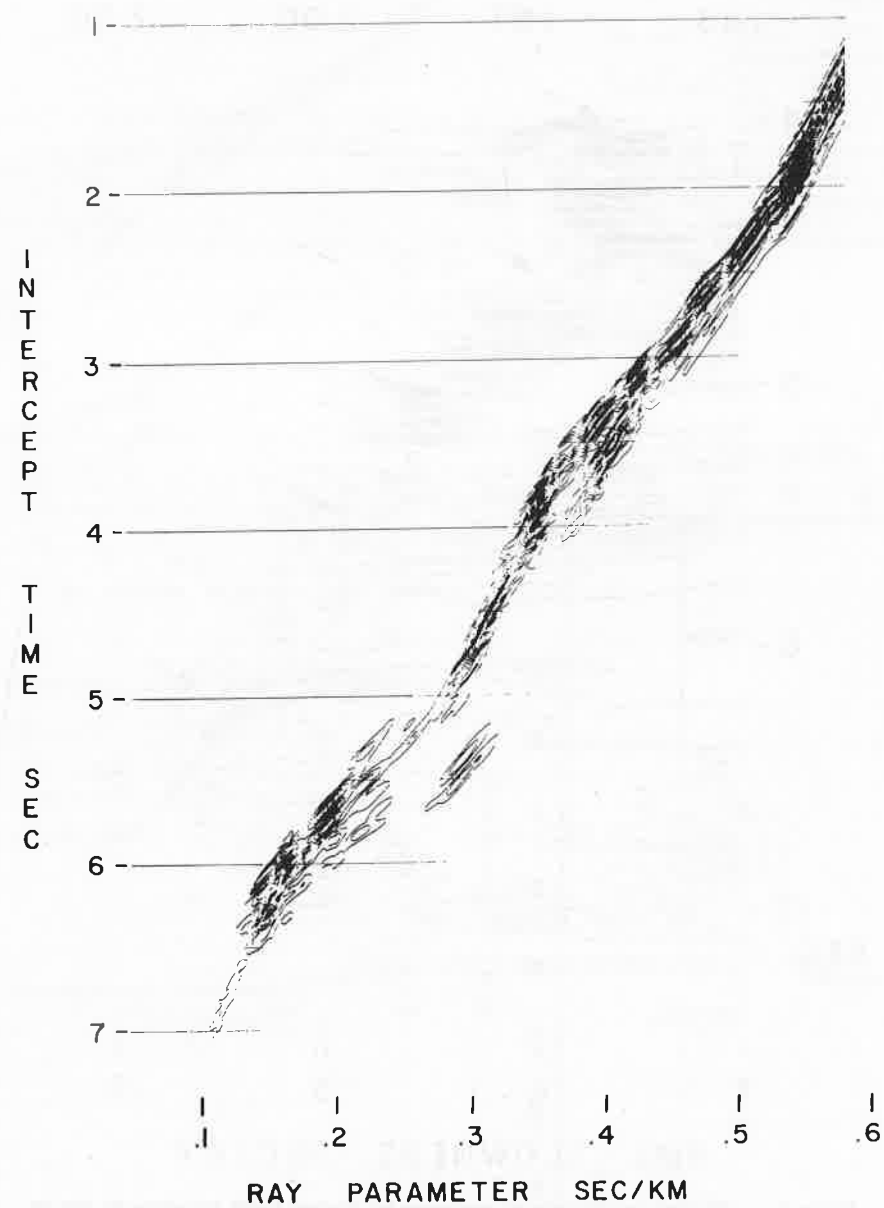


Figure 7 — 10-level contour plot of the envelope of the τ-p data of Fig. 9 after the application of a zero phase 0-20Hz bandpass filter.

version to derive the velocity and interval two-way normal time. (Fig. 9, velocity function 1).

For precritical reflections Dix's two-term traveltime approximation can be derived in terms of the parameters τ and p. A binomial expansion of equation (18) results in:

$$\tau_n(p) = \sum_{j-1}^{n} \Delta\tau_j(0) \left(1 - \frac{p^2}{2} v_j^2 + \frac{p^4}{8} v_j^4 + \ldots\right) \quad (20)$$

where the pv_j terms we recognize as the sines of the vertical ray angles. Normalizing by the total two-way normal traveltime,

$$\tau_n(p) \cong T_{0_n} \left(1 - \frac{p^2}{2} \frac{\sum_{j=1}^{n} \Delta\tau(0) v_j^2}{T_{0_n}} + \frac{p^4}{8} \frac{\sum_{j=1}^{n} \Delta\tau_j(0) v_j^4}{T_{0_n}} + \ldots\right) \quad (21)$$

the RMS velocity can be recognized in the second term of this series. Substituting for $V^2_{RMS_n}$ we have:

$$\tau_n(p) \cong T_{0_n} \left(1 - \frac{p^2}{2} V^2_{RMS_n} + \ldots\right) \quad (22)$$

Next, dropping all terms greater than second order we recognize the resulting two-term series as the expansion of:

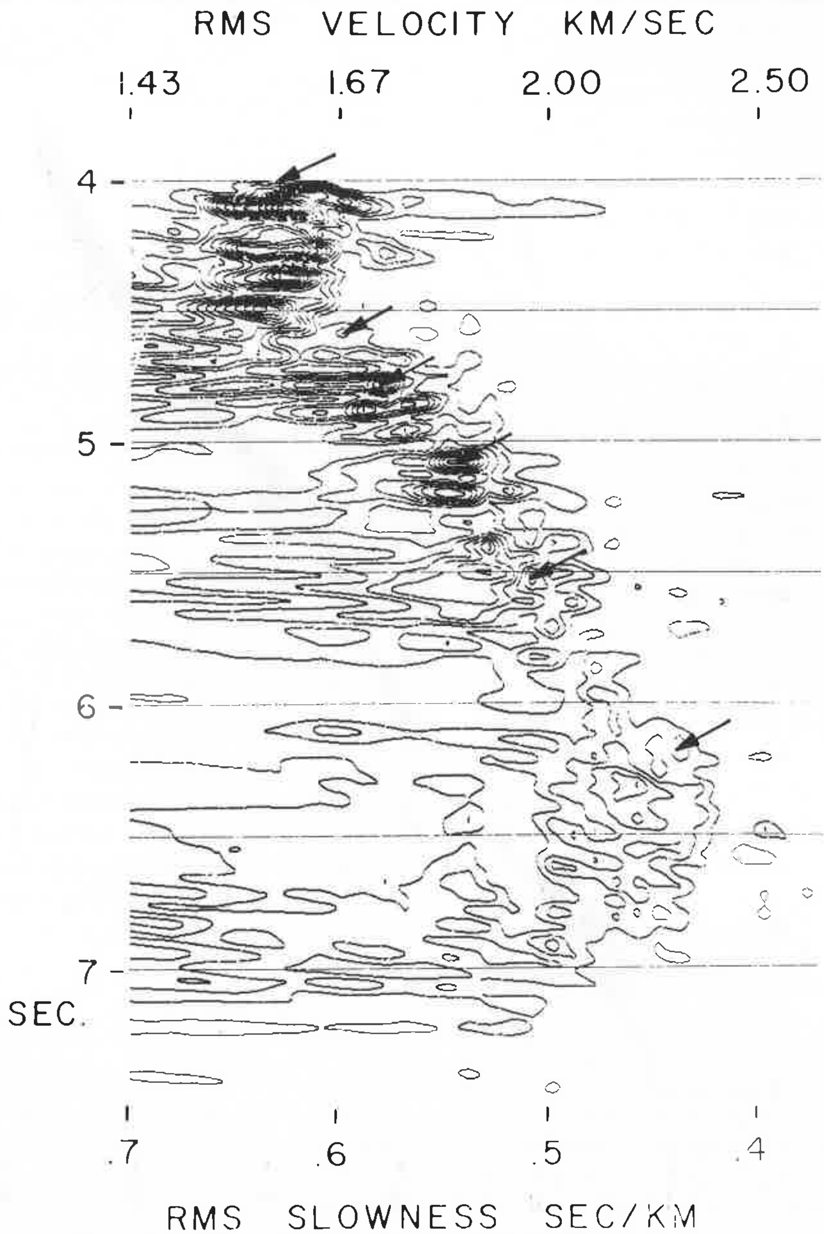


Figure 8 — 10-level contour plot of semblance for the best single τ-p ellipse for the τ-p data of Fig. 9. The interpreted two-way normal times and RMS velocities are indicated.

$$\tau_n(p) \cong T_{0_n} \left(1 - p^2 V^2_{RMS_n}\right)^{1/2} \qquad (23)$$

Using the definitions of $X = -d\tau/dp$ and $T = \tau + pX$, this τ-p ellipse can be recognized as the two-term hyperbolic traveltime equation of Dix (1955):

$$T^2 \cong T_{0_n} + X^2/V^2_{RMS_n}$$

Cutler & Love (1980) first suggested using this single ellipse approximation in a computer trial and error search to find the total two-way normal time and the RMS velocity in a manner analogous to that employed in X–T. Fig. 8 is an example of how the best single τ-p ellipse method using semblance can be used to define the RMS velocities and two-way normal times. The RMS velocities were then converted to thickness and interval velocities using Dix's (1955) formula.

This method offers little advantage over performing the corresponding hyperbolic traveltime analysis in the original X-T domain. But for a given seismic τ-p profile the deepest events will, because of the limits on source-receiver offset, often not contain postcritical reflections and/or refractions. In this case, the shallower events can be analyzed using the τ-sum method and the deeper events using the single ellipse method in the τ-p domain and conventional methods in the X-τ domain.

Stoffa et al. (1981, 1982) proposed an analysis method that includes all τ-p primary arrivals (Fig. 9). They considered both the τ-sum and best fit ellipse methods of

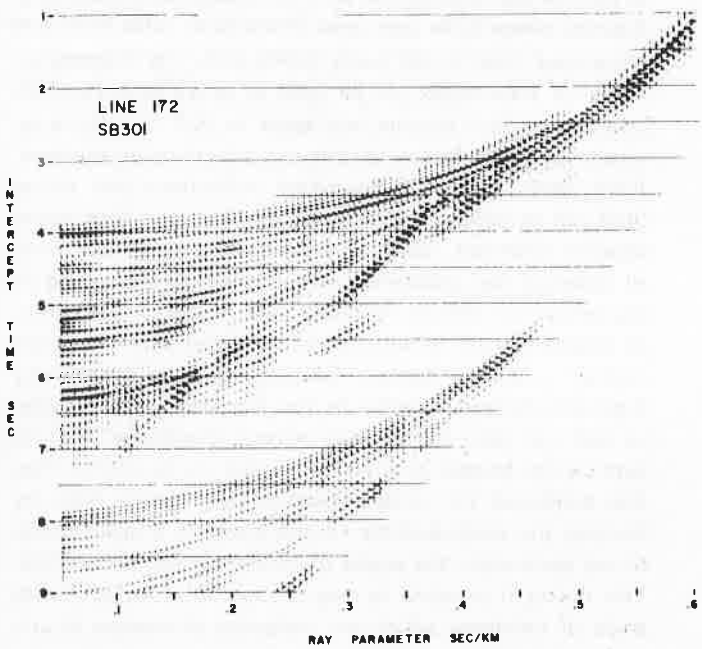
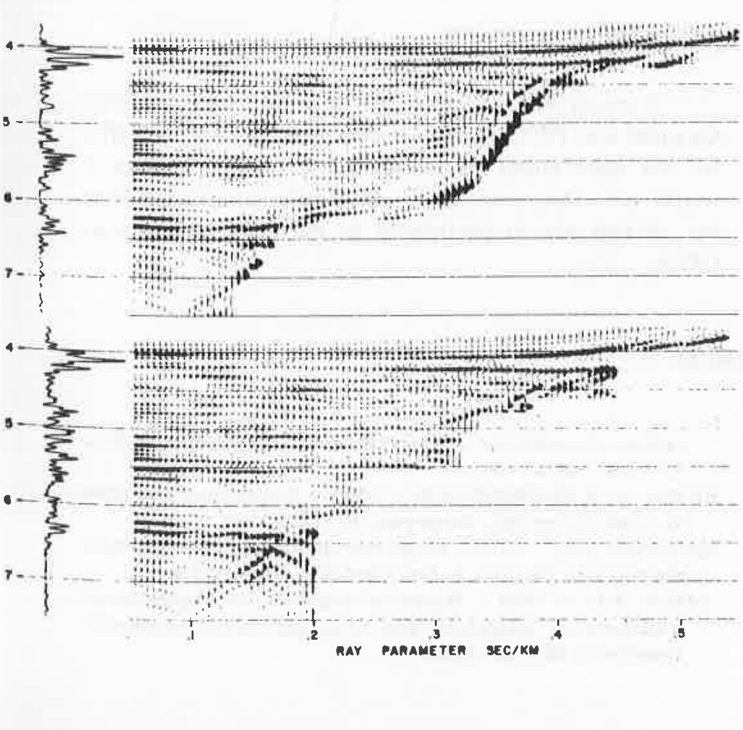


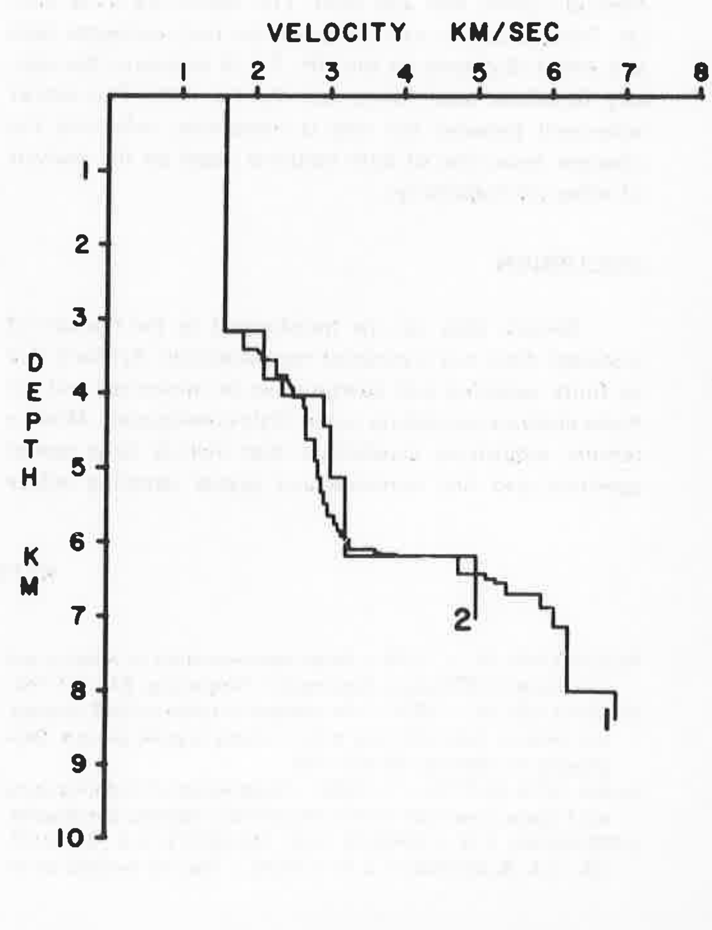
Figure 9

TOP LEFT: THE TAU(P) MAPPING OF DIGITALLY RECORDED SONOBUOY DATA FROM THE NORWEGIAN SEA.

BOTTOM RIGHT: VELOCITY-DEPTH FUNCTIONS OBTAINED BY THE TAU-SUM METHOD (1), AND BY PICKING SEMBLANCE MAXIMA ON ELLIPTICAL VELOCITY SCANS (2).

BOTTOM LEFT: THE TAU(P) DATA CORRECTED TO TWO-WAY NORMAL TIME USING THE TAU-SUM VELOCITY FUNCTION (UPPER), AND THE ELLIPTICAL VELOCITY FUNCTION (LOWER). SEMBLANCE VALUES CORRESPONDING TO A HORIZONTAL STACK ACROSS RAY PARAMETER ARE SHOWN AT THE LEFT.





analysis as the initial steps in deriving velocity-depth functions. These estimates can be refined by using by the τ-p data simultaneously. For example, equation (18) can also be used to predict the time correction required to move a point on a τ-p reflection trajectory to two-way normal reflection time. By analogy with the two-term hyperbolic traveltime assumption which predicts the "normal moveout" of a reflection event, equation (18) predicts the decrease in intercept time with increasing ray parameter. Rewriting equation (18):

$$\tau_n(p) = \tau_{n-1}(p) + \Delta\tau_n(0)\,(1 - p^2 v_n^2)^{1/2} \quad (24)$$

where $\tau_{n-1}(p)$ is the intercept time for the reflection for layer n-1 at ray parameter p. The time correction for the n^{th} reflector at ray parameter p is:

$$T_{0n} - \tau_n(p) = T_{0n} - \tau_{n-1}(p) - \Delta\tau_n(0)\,(1 - p^2/v_n^2)^{1/2} \quad (25)$$

Using these corrections the entire suite of τ-p arrivals can be downward continued to their two-way normal time. This method is exact and is the equivalent of a ray-traced normal moveout correction (tomography method) in X-T or a downward continuation performed via phase shifts applied in the frequency domain (Clayton & McMechan, 1982).

Fig. 9 illustrates this analysis method. The velocity function used to correct the τ-p data (upper) was derived using the τ-sum method of inversion. A refined version of the velocity function obtained from the best single ellipse method (lower) was also used. The results are quite similar. Semblance was also computed for both corrected data sets and is displayed on the left. Fig. 9 compares the velocity functions used to correct the τ-p data. The overall agreement between the two is reasonable, reflecting the inherent resolution of both methods based on the analysis of reflection trajectories.

DISCUSSION

Seismic data can be transformed to the domain of intercept time and horizontal ray parameter. Artifacts due to finite sampling and aperture can be minimized and for many analyses procedures are of little consequence. Modern seismic acquisition capabilities that include large spatial apertures and fine temporal and spatial sampling reduce these problems significantly. The application of appropriate windows during the transformation can further reduce these effects. Although the effects of aliasing and truncation may be significant for studies that use reflection amplitude, they are less important for velocity analyses and imaging applications.

Analysis of seismic data in the τ-p domain has several practical advantages. The seismic arrivals are reorganized into a more interpretable form with no crossovers and postcritical reflections and refractions are easily identified. Second, spherical spreading loss is correctly taken into account by the plane wave decomposition. Third, as shown by equation (11) all multiple contributions to the plane wave response are periodic. This means that predictive deconvolution based on plane wave assumptions to remove multiples can be accomplished more successfully. Fourth, since both pre- and postcritical reflections and refractions align along easily predictable τ-p trajectories, these τ-p trajectories can be used to derive velocity-depth information in a manner analogous to X-T traveltime inversion methods, but with little or no limiting assumptions. Fifth, both pre- and postcritical reflections and refractions can be used simultaneously to derive and then refine velocity estimates. Sixth, the incorporation of data from all observed ray parameters is equivalent to employing all source-receiver offsets recorded. Thus, seismic data from all source-receiver offsets can be analyzed in a consistant manner in the τ-p domain. Seventh, by using the velocity functions derived directly in the τ-p domain to exactly correct τ-p data to two-way normal traveltime, the τ-p data can be imaged in a manner similar to, but better than that employed for limited aperture seismic data. Also, by limiting the contributions to the stack to a select group of ray parameters the angles of incidences are also limited. This makes it possible to map changes in reflectivity with angle of incidence which are indicative of changes in subsurface lithology.

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REFERENCES

AL-CHALABI, M. – 1973 – Series approximation in velocity and traveltime computation. Geophysical Prospecting, **21**: 783-795.

AL-CHALABI, M. – 1974 – An analysis of stacking RMS, average, and interval velocities over a horizontally layered ground. Geophysical Prospecting, **22**: 458-475.

ALAM, M. & AUSTIN, J. – 1981 – Suppression of multiples using slant stacks, presented at 51st Annual SEG meeting, Los Angeles.

BESSONOVA, E.N., FISHMAN, V.M., RYABOYI, V.Z., SITNIKOVA, G.A. & JOHNSON, L.R. – 1976 – The tau method for inversion of traveltimes – II, Earthquake Data. Geophys. J. Roy. Astr. Soc., **46**: 87-108.

BRYSK, H. & McCOWAN, D.W. – 1986 – A slant-stack procedure for point-source data. Geophysics, **51**: 1370-1386.

CHAPMAN, C.H. – 1978 – A new method for computing synthetic seismograms. Geophys. J. Roy. Astr. Soc., **54**: 481-518.

CHAVE, A.D. – 1983 – Numerical integration of related Hankel transforms by quadrature and continued fraction expansion. Geophysics, **48**: 1671-1686.

CLAYTON, R.W. & McMECHAN, G.A. – 1981 – Inversion of refraction data by wave field continuation. Geophysics, **46**: 860-868.

CUTLER, R.T. & LOVE, P.L. – 1980 – Elliptical velocity analysis. Geophysics, **45**: 540.

DIEBOLD, J.B. & STOFFA, P.L. – 1981 – The traveltime equation, tau-p mapping, and inversion of common midpoint data. Geophysics, **46**: 238-254.

DIX, C.W. – 1955 – Seismic velocities from surface measurements. Geophysics, **20**: 68-86.

DUNKIN, J.W. – 1965 – Computation of model solutions in layered elastic media at high frequencies. Bull. Seismol. Soc. Amer., **55**: 335-358.

EWING, W.M., JARDETZKY, W.S. & PRESS, F. – 1957 – Elastic waves in layered media. McGraw-Hill.

FRASIER, C.W. – 1970 – Discrete time solution of plane P-SV waves in a plane layered medium. Geophysics, **35**: 197-219.

GERVER, M. & MARKUSHEVICH, V. – 1967 – On the characteristic properties of traveltime curves. Geophys. Roy. Astr. Soc., **13**: 241-246.

HASKELL, N.A. – 1953 – The dispersion of surface waves in multilayered media. Bull. Seismol. Soc. Amer., **43**: 17-34.

KIND, R. – 1976 – Computation of reflection coefficients for layered media. J. Geophys. Res., **42**: 191-200.

MAYNE, W.H. – 1962 – Common reflection point horizontal data stacking techniques. Geophysics, 28 (Part III): 927-938.

PHINNEY, R.A., CHOWDHURY, K.R. & FRAZER, L.N. – 1981 – Transformation and analysis of record sections. J. Geophys. Res., **86**: 359-377.

SHULTZ, P.S. – 1976 – Velocity estimation by wave front synthesis. Ph.D. Thesis, Stanford University, Stanford, CA.

SLOTNICK, M.M. – 1936 – On seismic computations with applications. Geophysics, **1**: 22.

STOFFA, P.L., BUHL, P., DIEBOLD, J.B. & WENZEL, F. – 1981 – Direct mapping of seismic data to the domain of intercept time and ray parameter – A plane-wave decomposition. Geophysics, **46**: 255-267.

STOFFA, P.L., DIEBOLD, J.B. & BUHL, P. – 1982 – Velocity analysis for wide aperture common midpoint data. Geophysical Prospecting, **30**: 25-27.

TANER, M.T. & KOELER, F. – 1969 – Velocity spectra – Digital derivation and application of velocity functions. Geophysics, **34**: 859-881.

THOMSON, W.T. – 1950 – Transmission of elastic waves through a stratified medium. J. Appl. Phys., **21**: 89-93.

TOLSTOY, I. & CLAY, C. – 1966 – Ocean acoustics: Theory and experiments in underwater sound. Mc-Graw-Hill.

TREITEL, S., GUTOWSKY, P. & WAGNER, D. – 1982 – Plane wave decomposition of seismograms. Geophysics, **47**: 1375.

WENZEL, F., STOFFA, P.L. & BUHL, P. – 1982 – Seismic modeling in the domain of intercept time and ray parameter. IEEE Transactions on Acoustics, Speech, and Signal Processing. ASSP, **30**: 406-422.