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MODELLING OF THERMAL DISTURBANCES INDUCED BY DRILLING ACTIVITY: ADVANCES IN THEORY AND PRACTICE

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Recent advances in the theory and practice of correction for thermal disturbances induced by drilling activity in temperatures in the bottom parts of deep wells is reviewed. The review includes a new classification scheme of theoretical models, brief discussion of the basic assumptions made in obtaining mathematically tractable solutions, comments on practical procedures and limitations of experimental data in the definition of suitable models. The classification scheme is based primarily on principal factors that determine the drilling disturbance such as the nature of heat source during drilling, decay of disturbance after cessation of drilling and the heat transfer characteristics of the drilling fluid. A further subdivision of these factors has been made on the basis of fundamental assumptions. Combinations of these assumptions allows in principle nearly 90 theoretical models but only 13 has so far been discussed in the literature. The classification scheme has been useful not only in identifying new problem areas but also for suggesting improvements in the existing models and practical procedures. The review provides a historical perspective of the evolutionary trends in mathematical models as well as highlights some of the recent advances in theory and practice.

Neste trabalho é apresentada uma revisão sobre os progressos recentes que têm sido obtidos na teoria e na prática de remoção de perturbações térmicas induzidas pela atividade de perfuração sobre a temperatura de fundo de poço em poços profundos. Ela inclui uma discussão sobre as hipóteses e simplificações feitas para obtenção de soluções apropriadas da equação de transferência de calor, comentários sobre as limitações impostas pelos dados de campo na utilização dos modelos teóricos e, sobre o uso prático desses modelos. Além disso, é apresentada uma classificação dos modelos teóricos existentes com base nos principais fatores que determinan a extensão e a magnitude da perturbação térmica induzida pela perfuração. Entre esses fatores inclui-se o tipo de fonte de calor ativo durante a perfuração, o processo de decaimento da perturbação térmica e, o processo de transferência de calor entre o fluido e a rocha. Para uma melhor subdivisão dos modelos, o esquema de classificação inclui as várias hipóteses e simplificações usadas na formulação desses modelos. Estima-se que o número de modelos que possam ser propostos, dentro desse esquema de classificação seja da ordem de noventa. Entretanto, foram encontrados na literatura apenas treze. Portanto, a classificação proposta é útil para identificar possíveis novos modelos assim como indicar aprimoramentos possíveis nos modelos existentes. A revisão fornece uma perspectiva da evolução histórica dos modelos matemáticos e apresenta alguns resultados recentes.

INTRODUCTION

Determination of undisturbed formation temperatures made in wells soon after drilling is a problem that is of considerable interest in geothermal reservoir engineering, mine ventilation, terrestrial heat flow studies and hydrocarbon exploration. During drilling a significant amount of mud or water is circulated through the well in order to cool the drill bit, carry drill cuttings to the surface and to prevent collapse of the walls of the well. Usually, fluid from a tank at the surface is pumped down through the drill string while the return flow occurs through the annulus between the drill rod and the drill hole wall or casing. During in flow, the drilling fluid absorbs heat from the surroundings and also from the drill bit, while during return flow it loses heat to the surroundings. Fluid flow during drilling activity can therefore be considered as sometimes similar to a counter-flow heat exchanger in operation. Circulation of drilling fluid at a temperature different from that of the formation induces a thermal disturbance which propagates into the formation creating zones of anomalous temperatures around the well. When drilling activity is stopped, this disturbance decays and the well returns to thermal equilibrium with the surrounding regions.

The magnitude of the disturbance and its decay after cessation of drilling depends on several factors such as rate of drilling, drilling history, rate of fluid circulation, temperature of the fluid, formation temperature, diameter of the well, thermal properties of the drilling fluid and of the rock and, the nature of heat exchange between the wall and the drilling fluid.

There is no single theory that can account for the effects of all such disturbing factors. Models that are available in the literature are usually based on several simplifying assumptions in order to obtain mathematically tractable solutions.

Most of the simplifying assumptions that are made to render solutions of the heat transfer equation tractable are peculiar to a model or to a class of models. There are however, two basic assumptions, that the heat flow is purely horizontal and that the borehole is an infinitely long heat or temperature source, that are common to all classes of models. This means that all existing models of thermal disturbances induced by drilling neglect the effects of the existing geothermal gradient and do not consider the end effects caused by the borehole bottom.

The assumption that the borehole can be considered as an infinitely long heat or temperature source is normally made because it is common practice to make bottom hole temperature (BHT) measurements one to ten meters away from the bottom of the hole, a distance which is equivalent to some tens of well radii. It is generally accepted that at these distances the end effects of the bottom of the hole are negligible and beyond the measurement sensibility (Middleton, 1979). On the other hand, the assumption that heat flow is purely horizontal is normally done because the lateral contrast in the temperature is much greater than the local geothermal gradient, at least when the temperature of the borehole and its surroundings are far from the equilibrium temperatures.

Another assumption common to most theoretical models is that there is no contact resistance between the borehole fluid and the formation. This is probably a good approximation for the case where the drilling fluid is similar to the formation fluid. When this is not the case, a significant thermal contact resistence may occur especially for the case of gas saturated porous rocks.

One of the factors that has hindered theoretical developments in this area is the nature of experimental data. Due to technical and operational factors complete temperature logs of good quality are rarely run. Most of the available data are in the form of measurements made at the bottom of the well during short periods of interruption in drilling activity. Such data do not provide rigorous constraints on theoretical models dealing with the thermal recovery of the well.

In spite of such limiting factors steady advances in theory and practice have been made during the last few decades in dealing with BHT data. The objective of the present paper is to provide a review of the historical evolution of the models and highlight some of the recent advances in theory and practice.

REVIEW OF THEORETICAL MODELS

The earliest work on theoretical modelling of temperature disturbances induced by drilling activities seems to have been made by Bullard (1947). In this very elegant model a line source approximation was used for the thermal disturbance. Because of its simplicity this model has ever since been used widely for making corrections to temperatures measured in boreholes and wells. The next stage of advance in modelling came with the work of Jaeger (1956a) where he introduced the more realistic cylindrical source models. Though several modifications have been introduced in cylindrical source models, because of the complexity of the theory and lack of adequate field data to test the validity of the assumptions, these have found very little pratical use in the analysis of actual field data.

Instantaneous source models were introduced in the last decade as an attempt to simplify the theory and adapt the models to the limitations imposed by the nature of field data. Models proposed by Middleton (1979) and Leblanc et al. (1981) belong to this class.

The next important advance came with the recognition that heat exchange between the well and the formation takes place not only by conduction but by fluid flow as well. The problem of radial penetration of drilling fluid into the formation was modelled by Luheshi (1983) and Shen & Beck (1986). The problem of heat transfer by uniform formation fluid flow was considered by Ribeiro & Hamza (1986) and Shen & Beck (1986).

Given below is a summary of the major theoretical models, taking into account the trends in their historical evolution and limitations imposed on their mathematical formulation.

a. Line source models

In this class of models it is assumed that the thermal perturbation induced by drilling activity can be represented by a line source or sink of heat of constant strength and infinite extent. Bullard (1947) presented the solution for temperature disturbance as

$$T(z, t_d) = -\frac{Q}{4\pi K}\, Ei\,(-r^2/4\alpha\, t_d) \qquad (1)$$

where Q is the heat generation by the line source per unit length and unit time, K is thermal conductivity, α thermal diffusivity, r distance from the line source, t_d duration of drilling and Ei is the exponential integral. The decay of thermal disturbance after cessation of drilling is represented by the activation of a negative source of heat beginning at time t_s. Hence the variation of temperature during post-drilling period is given by

$$T(z,\ t) = T_f(z) - \frac{Q}{4\pi K}\ [Ei\ (-r^2/4\alpha\ t) - Ei\ (-r^2/4\alpha\ t_s)] \qquad (2)$$

where T_f is the undisturbed formation temperature and t_s is the time since stoppage of drilling. For large values of time $r^2/4\alpha t << 1$ so that (2) can be simplified as

$$T(z,t) = T_f(z) - \frac{Q}{4\pi K}\ \ell n\ (1 + t_d/t_s) \qquad (3)$$

Hence a plot of temperatures recorded (T) versus $\ell n\ (1 + t_d/t_s)$ would be a straight line whose intercept T_f is the undisturbed formation temperature. In Bullard's work T(z, t) was considered identically equal to the temperature of the borehole wall (ie: T at r = a, where 'a' is the radius of the well). A variation of the Bullard's method was presented by Timko & Fertl (1972) and is known as the 'Horner Plot', due to its similarity with a technique which describe the evolution of pressure disturbance inside the well, developed by Horner (1951). The Horner Plot is based on the empirical relation for temperature build-up in wells during shut-in times, given by

$$T_s(t) = T_f - C\ \log(\frac{t_d + t_s}{t_s}) \qquad (4)$$

Equation (4) is used widely in the oil industry to estimate T_f from BHT data. As Lachenbruch & Brewer (1959) and Shen & Beck (1986) have pointed out, the Horner Plot corresponds mathematically to solution (2) calculated at the borehole axis (r = 0). On the other hand, Bullard (1947) considered that T(z, t) given by (2) is the borehole wall temperature, which means at r = a. In this situation, solution (3) is an approximation of (2) when $a^2/4\alpha t << 1$.

It is important to note that both procedures can lead to incorrect results in some cases. Fig. (1) shows the behaviour of BHT as given by equation (2). There is an initial decrease in temperature followed by a continuous increase. As was pointed out by Luheshi (1983) the initial decrease is due to the time lag between the cooling effect of mud circulation during drilling and warming up effect after cessation of mud flow. Such effects can cause substantial errors in the use of BHT data to estimate formation temperatures. For example a well with 22 cm diameter takes about 20 to 50 hours of shut-in time for a valid use of equation (4) when the thermal diffusivity of mud is $5 \times 10^{-7}\ m^2/s$. Such long times without mud circulation are uncommon. Since time interval for equation (4) to be valid varies with the square of the radius of the well application of Horner Plot may be impossible for wells of large radii. Drury (1984) also points out that Bullard's Method leads to reliable estimates only for small values of $r^2/4\alpha t_s$.

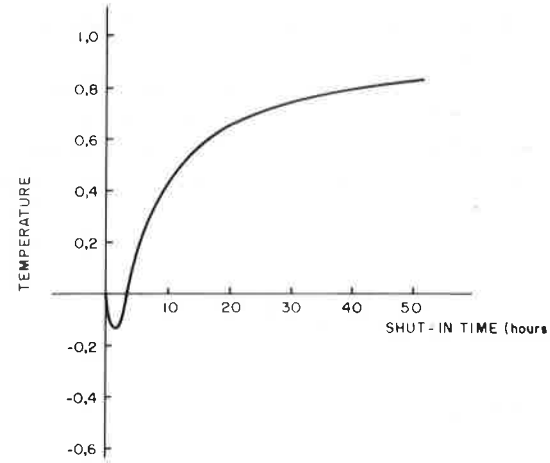

Figure 1 — Graphical representation of equation 2. The temperature is represented in a normalized scale defined by $[T(z,\ t) - T(z,\ t_d)]\ /\ [T_f - T(z,\ t_d)]$, t_d being the drilling time. (From Luheshi, 1983).

b. **Cylindrical temperature source models**

We restrict our discussion here to models where the well is treated as an infinitely long cylindrical body maintained at constant temperature during drilling. All models of this class are based on the solution of heat conduction equation in cylindrical coordinates, given by

$$\rho c\ \frac{\partial T}{\partial t} = K\ [\frac{\partial^2 T}{\partial r^2} + \frac{1}{r}\frac{\partial T}{\partial r}] \qquad (5)$$

with the initial conditions

$$T(r,0) = T_m \quad 0 \leqslant r < a \qquad (6)$$

$$T(r,t) = T_m \quad 0 \leqslant r < a \quad \text{and} \quad t < t_1 \qquad (7)$$

$$T(r,0) = T_f \quad r > a \qquad (8)$$

where ρc is the specific heat capacity, K is the thermal conductivity, T_m and T_f are the initial mud and formation temperatures and t_1 is the circulation time. The boundary conditions imposed on equation (5) are

$$T(a,t)_{\text{rock}} = T(a,t)_{\text{well}} \quad \text{for } t > t_1 \qquad (9)$$

$$K_m \left.\frac{\partial T}{\partial r}\right|_{r=a,\ \text{mud}} = K_f \left.\frac{\partial T}{\partial r}\right|_{r=a,\ \text{rock}} \quad \text{for } t > t_1 \qquad (10)$$

and

$$T(r,t) = T_f \quad \text{for} \quad r \to \infty \quad \text{and} \quad t > 0 \qquad (11)$$

The letters m and f when used as subscripts indicate mud and formation properties.

The first solution of this class of models was given by Jaeger (1956a) with the additional assumption that rock and drilling mud have identical thermal properties. The solution for temperature at the center of the borehole is

$$\frac{T - T_f}{T_m - T_f} = [1 - \exp(-\frac{a^2}{4\alpha t})] + \frac{1}{2\alpha t} \cdot \int_a^\infty \exp(-\frac{r^2}{4\alpha t})\, f(r)\, r dr \qquad (12)$$

where the function f(r) is defined as

$$f(r) = 1 - \frac{2}{\pi} \int_o^\infty \frac{J_o(ua)\, Y_o(ur) - J_o(ur)\, Y_o(ua)}{u[J_o^2(au) + Y_o^2(au)]} \cdot \exp(-\alpha u^2 t_1)\, du$$

$J_o(u)$ and $Y_o(u)$ being Bessel functions of the first and the second kind of order zero.

The first complete solution of the system of equations (9-11) was given by Luheshi (1983), who presented results obtained using numerical methods and pointed out the effect of contrast in thermal properties between the drilling mud and the formation and the effect of circulation time. In the absence of mud circulation, as Middleton (1982) has shown, the models predict shorter times for the recovery of the equilibrium formation temperature. The reason is that when there is no circulation there is no penetration of thermal disturbance into the formation. The use of identical values of thermal diffusivity for mud and rock also affect the time for return to undisturbed temperatures, as the mud generally has a lower thermal diffusivity. Middleton (1982) pointed out that for periods shorter than about ten to fifteen times the circulation time there are no significant differences between temperatures calculated by models with uniform thermal diffusivity and models with contrast in thermal properties.

Luheshi (1983) in order to evaluate the relative influence of the several parameters that affect the solution of equations 5-11 carried out a sensitivity study by comparing the variation of BHT as a function of the possible range of variations in each single parameter while fixing all other parameters at their optimum values. He found that the solutions are sensitive to variations in thermal diffusivity of the drilling mud. Variations of 20% in the drilling mud diffusivity can induce relative differences in the temperature recovery curve of the order of 10%. On the other hand, for fixed thermal diffusivities changes in the conductivity ratio (K_f/K_m) can cause relatively large differences between temperature recovery curves. Luheshi (1983) demonstrated this effect by considering an extreme case where the ratio (K_f/K_m) varied from one to nine the results obtained showed relative differences in temperatures of as much as 70%. Both of the effects described above have considerable importance in the apolication of this model. For an accurate application of Luheshi's (1983) method it is necessary to know with good precision the thermal conductivities and thermal diffusivities of both media. However, in practice, only the thermal conductivity of the rock is measured in laboratory. Rock thermal diffusivities are rarely measured while efforts for determination of thermal properties of drilling mud are seldom carried out.

Shen & Beck (1986) reconsidered the problem studied by Luheshi (1983), and presented analytical solutions. For times smaller than the circulation time (t_1) the formation temperature is given by Carslaw & Jaeger (1959).

$$T_i(r,t) = T_m - \frac{2(T_f - T_m)}{\pi} \int_o^\infty e^{-u^2\alpha_f t} \frac{J_o(ur)\, Y_o(ua) - J_o(ua)\, Y_o(ur)}{(J_o^2(ua) + Y_o^2(ua)} \frac{du}{u} \qquad (13)$$

(for $t < t_1$)

The solution of equation (5) for $t > t_1$ with conditions 6-11 is, for the formation temperature:

$$T(r,t) = T_i(r,t) + \frac{2a(T_f - T_m)}{\pi} \int_o^\infty J_o(aw\sqrt{\frac{\alpha f}{\alpha_m}}) \cdot \frac{J_o(wr)\, \Phi(wa) - Y_o(wr)\, \Psi(wa)}{\Phi^2(wa) + \Psi^2(wa)} G(w,t,t_1)\, dw \qquad (14)$$

where $T_i(r, t)$ is given by (13) and

$$G(w,t,t_1) = \frac{4}{\pi^2 a^2} \int_o^\infty \frac{e^{-w^2\alpha_f(t-t_1)} - e^{-u^2\alpha_f(t-t_1)}}{(u^2 - w^2)} \cdot \frac{e^{-u^2\alpha_f t_1}}{(J_o^2(au) + Y_o^2(au))} \frac{du}{u} \qquad (15)$$

$$\Phi(w) = \frac{K_m}{K_f}\sqrt{\frac{\alpha_f}{\alpha_m}}\, Y_o(w)\, J_1(w\sqrt{\frac{\alpha_f}{\alpha_m}}) - Y_1(w)\, J_o(w\sqrt{\frac{\alpha_f}{\alpha_m}}) \qquad (16)$$

and

$$\Psi(w) = \frac{K_m}{K_f}\sqrt{\frac{\alpha_f}{\alpha_m}}\, J_o(w)\, J_1(w\sqrt{\frac{\alpha_f}{\alpha_m}}) -$$

$$- J_1(w)\, J_o\left(w\sqrt{\frac{\alpha_f}{\alpha_m}}\right) \tag{17}$$

For drilling mud, the temperature is given by

$$T(r,t) = T_m + $$

$$+ \frac{4(T_f - T_m)}{\pi^2} \int_o^\infty \frac{J_o\left(aw\sqrt{\frac{\alpha_f}{\alpha_m}}\right) J_o\left(rw\sqrt{\frac{\alpha_f}{\alpha_m}}\right)}{\Phi^2(aw) + \Psi^2(aw)} \cdot$$

$$\cdot\; G(w, t, t_1)\, \frac{dw}{w} \tag{18}$$

For low viscosity muds lateral mixing results in substantial increase in the effective values of its thermal conductivity and thermal diffusivity. Such a situation can be envisaged by considering the limiting case where K_m (and consequently α_m) is infinite. This corresponds to treating the drilling fluid as a perfect conductor. As a result the functions $\Phi(w)$ and $\Psi(w)$ reduce to

$$\Phi_\infty(w) = \frac{1}{2}\,\frac{\rho_m c_m}{\rho_f c_f}\, w\, Y_o(w) - Y_1(w) \tag{19}$$

$$\Psi_\infty(w) = \frac{1}{2}\,\frac{\rho_m c_m}{\rho_f c_f}\, w\, J_o(w) - J_1(w) \tag{20}$$

Shen & Beck (1986) show that there are no great differences in the solutions with finite and infinite drilling mud conductivities, except for very short times.

The above mentioned model can be used for determining the equilibrium temperature and the initial thermal disturbance by fitting BHT data to equation (18). The fitting procedure consists of a linear regression of the type

$$BHT_i = T_m - (T_f - T_m)\,\theta_i$$

where θ_i represent the integral involved in equation (18).

c. **Instantaneous source models**

BHT measurements are usually carried out in the final section of the well, which has been drilled in a sufficiently short time, that is, the drilling disturbance can be considered as having been introduced "instantaneously" into the space occupied by the well. This assumption allows a substantial simplification in that it is necessary to consider only the disturbing effects arising from the decay of the instantaneous source, the perturbations due to drilling or mud circulation being negligible. The instantaneous source models are thus simplified version of cylindrical source models of counter-flow heat exchangers. The initial conditions applied to the heat conduction equation are that at, the instant at which the borehole is emplaced, the wall is at the formation temperature (T_f) and the drilling mud has an initial temperature (T_m).

The first model of this class was proposed by Jaeger (1956b) in which he assumed that the drilling fluid is a perfect conductor. Cooper & Jones (1959) used this model for the determination of thermal conditions in deep coal mines from temperature measurements made in drill holes. The solution for heat conduction equation proposed by Jaeger (1956b), is

$$\frac{T - T_f}{T_m - T_f} = \frac{4\delta}{\pi} \int_o^\infty \exp(-\tau u^2) \cdot \frac{du}{u\{[uJ_o(u) - (\delta - hu^2)J_1(u)]^2 + [uY_o(u) - (\delta - hu^2)Y_1(u)]^2\}} \tag{21}$$

where

$$\tau = \frac{\alpha t}{a^2}$$

α being the thermal diffusivity of the formation, a radius of the drillhole, δ twice the ratio between the specific heat capacities of the formation ($\rho_f c_f$) and the drilling fluid ($\rho_m c_m$) and,

$$h = \frac{K}{aH}$$

with K the thermal conductivity and H the reciprocal of thermal contact resistance. When the cylinder is filled with a fluid with good thermal contact with the wall, h can be considered zero.

To check the validity of this model, Cooper & Jones (1959) set up a laboratory scale model with a sandstone block having a central hole of about four centimeter diameter which stops just after passing the block center. The uniformity of block temperature was verified and cold water was introduced in the central hole. Temperatures measured at distances of about eight hole diameters were used to study the decay of disturbance introduced by the instantaneous source. The model studies showed that equation (21) gives reasonably good fit to experimental data acquired half an hour or more after the introduction of the instantaneous source.

Middleton (1979) proposed a method for determination of equilibrium temperatures from bottom hole measurements in oil and gas wells where no information about circulation time of the drilling fluids is available. This model is also based on the assumption that the final section of the well is drilled in a time interval sufficiently small, that it can be considered as having been introduced instantaneously into the rock formation. Futhermore, Middleton (1979) assumed that the temperature of the drilling mud is constant at distances of about five to ten meters from the bottom of the hole and that the heat flow is essentially radial.

Table 1 — Comparison between the results obtained by the methods of Horner and Middleton with different values of thermal diffusivity for oil wells in the Paraná Basin. The values between parentheses are the initial thermal disturbance ($T_f - T_m$).

WELL LOCATION	WELL DEPTH (m)	CIRCULATION TIME (h : min)	SHUT-IN TIME (h : min)	T_f (HORNER) (°C)	T_f (MIDDLETON) (°C)			
					$\alpha = 2.5 \times 10^{-7}$	$\alpha = 3.0 \times 10^{-7}$	$\alpha = 5.0 \times 10^{-7}$	$\alpha = 1 \times 10^{-6}$ (m^2/s)
Ronda Alta	1504	2 : 30 3 : 35 3 : 35 3 : 35	4 : 00 11 : 30 14 : 30 21 : 30	50.4	52.2 (21.9)	52.0 (24.6)	51.7 (35.8)	51.5 (64.5)
Esmeralda	1524	2 : 30 2 : 30 2 : 30	6 : 30 9 : 15 14 : 15	54.3	54.5 (6.5)	54.4 (7.4)	54.3 (11.1)	54.3 (20.5)
	2424	1 : 00 1 : 00 1 : 00	7 : 00 12 : 00 21 : 33	71.7	71.6 (18.4)	71.5 (21.1)	71.4 (32.2)	71.3 (60.3)
	2310	5 : 30 5 : 30 5 : 30	5 : 15 10 : 30 14 : 00	71.4	70.9 (18.9)	70.7 (21.3)	70.4 (31.4)	70.2 (57.2)
	1300	1 : 00 1 : 00 1 : 00	14 : 15 15 : 30 20 : 30	46.1	46.0 (20.4)	45.9 (23.9)	45.8 (37.6)	45.8 (72.2)
	2465	1 : 30 1 : 30 1 : 30 1 : 30 1 : 30	5 : 00 8 : 40 14 : 00 21 : 15 25 : 30	65.5	65.6 (5.5)	65.5 (6.3)	65.4 (8.5)	65.4 (17.2)
Paranapanema	158	8 : 00 8 : 00 8 : 00	2 : 15 4 : 15 7 : 00	32.1	32.0 (1.5)	32.0 (1.5)	31.9 (1.8)	31.8 (3.0)
	776	1 : 30 1 : 30 2 : 00	5 : 00 11 : 15 22 : 00	39.8	39.7 (1.7)	39.7 (2.0)	39.7 (3.0)	39.6 (5.5)
Rio Aporé	3476	3 : 45 3 : 00 3 : 00	7 : 30 21 : 20 16 : 50	72.5	73.1 (14.1)	73.1 (10.2)	72.9 (24.8)	72.9 (46.3)

Mathematically the Middleton (1979) model can be represented by the solution of heat flow equation in two dimensions with the initial conditions that at the end of mud circulation the mud temperature (T_m) is constant and the formation temperature has not been affected. The correct solution to this problem at the center of the borehole was given by Leblanc et al. (1981)

$$BHT(t) = T_f - (T_f - T_m)\ [1 - \exp(-\frac{a^2}{4\alpha t})] \quad (22)$$

or

$$BHT(t) = T_m + (T_f - T_m) \exp(-\frac{a^2}{4\alpha t}) \quad (23)$$

where t is the time elapsed after the end of mud circulation.

Although Leblanc et al. (1981) have obtained the solution (22) directly from heat conduction equation and the above stated initial conditions, it can be seen that this solution corresponds to equation (12) when t_1 is zero. Thus Leblanc et al. (1981) model is a special case of the model proposed by Jaeger (1956a).

It is easy to note from equation (23) that for given values of ($T_f - T_m$) and 'a' the nature of decay of the thermal disturbance is controlled by the thermal diffusivity of the drilling mud. The value of thermal diffusivity of the drilling mud can be calculated from the values of conductivity (K) specific heat capacity (ρc) given by:

$$K = K_1 \frac{2(1-f) K_\ell + (1+f) K_s}{(2+f) K_\ell + (1-f) K_s} \quad (24)$$

$$\rho c = (\rho c)_\ell \ (1-f) + (\rho c)_s f \quad (25)$$

where the subscripts ℓ and s refer to liquid and solid components of the drilling mud respectively while f is the volume fraction of the solid phase. Using tabulated values of thermal diffusivities of common rocks and water Leblanc et al. (1981) estimated a value of $2.7 \times 10^{-7}\ m^2/s$ as representative of the drilling mud. The effect of low thermal diffusivity of the drilling mud relative to that of the wall rocks is to enlarge the thermal inertia of the system and to retard the penetration of the disturbance. On the other hand it forces the well to take longer time to reach equilibrium temperatures.

An independent check on the effective value of thermal diffusivity to be used can be obtained from a comparative study of the Horner Plot and the technique proposed by Middleton (1979). The results of such a study is presented in Table (1) for BHT data from the Paraná basin. Here the BHT data were adjusted to values of thermal diffusivity of $0.20 \times 10^{-6}\ m^2/s$, $0.3 \times 10^{-6}\ m^2/s$, $0.5 \times 10^{-6}\ m^2/s$ and $1.0 \times 10^{-6}\ m^2/s$. The results obtained show that higher values of thermal diffusivity lead to unacceptable values of initial thermal disturbance. On the other hand for lower values of thermal diffusivity the equilibrium temperatures obtained by the Horner Plot and the Middleton technique are very similar. Similar conclusions were also reached by Leblanc et al. (1982).

d. Cylindrical heat source models

Some models try to represent the thermal disturbance induced by drilling activity assuming that the borehole behaves as a heat source rather than a temperature source during drilling. Middleton (1982), in order to consider the effect of mud circulation over the formation temperature, proposed a model that represents the temperature distribution in a borehole and around it as a superposition of the temperature distribution due to instantaneous emplacement of the well (Leblanc et al., 1981), plus the temperature distribution due to an exponentially decaying heat sink of the type

$$a = A_o \exp(-bt) \quad (26)$$

In this equation A_o is the initial strength of the heat sink and b is its decay constant.

In the center of the well, the temperature predicted by this model is (Middleton, 1982)

$$T(0,t) = [T_m - (T_f - T_m) \exp(-\frac{a^2}{4\alpha t})] -$$

$$- T_s [1 - \exp(-bt)] + bT_s \exp(-bt) \int_o^t \exp(bu - \frac{\beta}{u})\, du \quad (27)$$

where T_s is an equivalent sink temperature defined as

$$T_s = \frac{A_o}{b\rho c}$$

and β is defined as

$$\beta = \frac{a^2}{4\alpha}$$

Numerical evaluation of equation (27) shows that the initial temperature contrast ($T_f - T_m$) is a good approximation of the equivalent sink temperature (T_s). When T_s is made much lower than the initial temperature contrast the thermal sink has no effect. On the other hand, the assumption that T_s is much higher than ($T_f - T_m$) leads to unacceptable results such as lowering of the temperature in the center of the well below the initial mud temperature T_m.

Shen & Beck (1986) developed a model for correcting bottom hole temperatures which consists of treating the drilling activity as a cylindrical heat source or sink generating or absorbing heat at a constant rate Q. According to Birch (1947) and Beck & Shen (1985) this assumption is valid for the case where circulation of drilling mud is maintained at slow rates allowing the mud to attain thermal equilibrium with the borehole wall. Shen & Beck (1986) however argue that this is also a good approximation for the case where the drilling mud circulates rapidly and is maintained at a practically constant temperature, as for example when rotatory drilling is employed.

In their model, Shen & Beck (1986) considered that the thermal properties of the drilling mud and wall rock are different. The boundary conditions imposed on the heat conduction equation are the same as those considered for the case where the drilling mud temperature is maintained constant during drilling. Initial conditions however have to be modified.

The initial condition to this problem is obtained assuming that during drilling and circulation the borehole is filled by a perfectly conducting fluid to which heat is furnished at a constant rate Q, per unit time and unit length. The temperature distribution in the rock is, for $r \geqslant a$ and $t < t_1$, given by (Carslaw & Jaeger. 1959).

$$T_i(r,t) = T_f - \frac{Q}{\pi^2 a K_f} \int_0^\infty \frac{1 - e^{-\alpha_f u^2 t}}{u^2} \, \frac{J_o(ur)\,\Phi_\infty(ua) - Y_o(ur)\,\Psi_\infty(ua)}{\Phi_\infty^2(ua) + \Psi_\infty^2(ua)} \, du \tag{28}$$

with $\Phi_\infty(w)$ and $\Psi_\infty(w)$ given by (19) and (20). The initial condition for the temperature of the fluid in the well is given by

$$T(r, t_1) = T_m \quad \text{for } r < a$$

Temperature distributions for this model given by Shen & Beck (1986) are:

1) for the rock

$$T(r,t) = T_f - T_i(r, t - t_1) + T_i(r,t) + \frac{aQ}{\pi^2 K_f} \int_0^\infty \left[\frac{1}{2} \left(\frac{\rho_m c_m}{\rho_f c_f} \right)_c J_o\left(aw\sqrt{\frac{\alpha_f}{\alpha_m}}\right) - \frac{1}{aw} \frac{K_m}{K_f} \sqrt{\frac{\alpha_f}{\alpha_m}} \cdot J_1\left(aw\sqrt{\frac{\alpha_f}{\alpha_m}}\right)\right] \frac{J_o(wr)\,\Phi(wa) - Y_o(wr)\,\Psi(wa)}{\Phi^2(aw) + \Psi^2(aw)} \cdot D(w, t, t_1)\, dw \tag{29}$$

The function $D(w, t, t_1)$ is defined by

$$D(w,t,t_1) = \frac{4}{a^2 \pi^2} \cdot \int_0^\infty \frac{\left(e^{-w^2 \alpha_f (t - t_1)} - e^{-u^2 \alpha_f (t - t_1)}\right)}{u^2 - w^2} \cdot \frac{(1 - e^{-u^2 \alpha_f t_1})}{\Phi_\infty^2(au) + \Psi_\infty^2(au)} \, \frac{du}{u} \tag{30}$$

2) for the drilling mud

$$T(r,t) = T_f - T_i(r, t - t_1) + T_i(r,t) + \frac{2Q}{\pi^3 K_f} \int_0^\infty \left[\frac{1}{2} \left(\frac{\rho_m c_m}{\rho_f c_f} \right)_c J_o\left(aw\sqrt{\frac{\alpha_f}{\alpha_m}}\right) - \frac{1}{aw} \frac{K_m}{K_f} \sqrt{\frac{\alpha_f}{\alpha_m}} \, J_1\left(aw\sqrt{\frac{\alpha_f}{\alpha_m}}\right)\right] \cdot \frac{J_o\left(rw\sqrt{\frac{\alpha_f}{\alpha_m}}\right)}{\Phi^2(aw) + \Psi^2(aw)} \, D(w, t, t_1) \, \frac{dw}{w} \tag{31}$$

The subscript c in the ratio of thermal capacities indicates that this value is representative of the conditions during circulation.

The solution for the case when the drilling mud remains as a perfect condutor after the end of circulation can be obtained taking the limits

$$J_o\left(aw\sqrt{\frac{\alpha_f}{\alpha_m}}\right) \to 1$$

and

$$\frac{K_m}{K_f}\sqrt{\frac{\alpha_f}{\alpha_m}}\, J_1\left(aw\sqrt{\frac{\alpha_f}{\alpha_m}}\right) \to \frac{K_m}{K_f}\sqrt{\frac{\alpha_f}{\alpha_m}}\, \frac{aw}{2}\sqrt{\frac{\alpha_f}{\alpha_m}} = \frac{aw}{2}\left(\frac{\rho_m c_m}{\rho_f c_f}\right)_s$$

when

$$\alpha_m \to \infty$$

The flow of drilling mud in the borehole during circulation removes heat from the borehole enhancing its capacity for absorption of heat. Hence, the heat capacities before and after circulation are not equal. The subscript "s" implies that the specific heat capacity ratio is calculated after the end of mud circulation.

Substituting the above limits and assuming, as Shen & Beck (1986) did, that the specific heat capacity ratios before and after the drilling mud circulation are equal, equation (32) becomes

$$T(r,t) = T_f + T_i(r,t) - T_i(r, t - t_1)$$

representing a superposition of two heat sources in the same way that Bullard's (1947) model does. Taking now the limit of the well radius tending to zero equation (2), which is the mathematical representation of Bullard's (1947) model, is obtained. Thus, the solution obtained by Shen & Beck (1986) can be understood as a generalization of Bullard's method.

e. Radial heat exchange models

All models discussed up to now assume that conduction is the only mode of heat transfer between the borehole and the rock. There are situations, bowever, where advective transport of heat is important. One clear example of such a situation is the case when "thief zones" are present in boreholes through which considerable loss of circulating fluid may occur. In fact, advective transport may have an important role in the total heat transfer in all cases where the drill hole cuts across permeable layers, due to the common practice of maintaining high fluid pressures in the well relative to the formation fluid pressures.

Propagation of excess pressure into the formation obeys the diffusion equation. In the case where the properties of the fluids in the borehole and the permeable layer are nearly identical the diffusion equation can be written as

$$\frac{\partial^2 P}{\partial r^2} + \frac{1}{r}\frac{\partial P}{\partial r} = \frac{1}{\alpha_h}\frac{\partial P}{\partial t} \qquad (32)$$

where the hydraulic diffusivity α_h is

$$\alpha_h = \frac{K_h}{\Phi \mu C_o}$$

K_h being the permeability of the rock, Φ its porosity, μ the viscosity of the fluid and, C_o its compressibility. Fig. 2 shows the nature of decay of pressure excess into a formation as a function of the radial distance from the center of the well, for the case that the borehole is maintained at a constant pressure P_o. The fluid flow into the formation is given by Darcy law, which for the case of cylindrical symmetry becomes

$$U = -\frac{K_h}{\mu}\frac{\partial P}{\partial r} \qquad (33)$$

Luheshi (1983) has considered the effect of radial fluid flow on the thermal disturbance induced by drilling. His model incorporates the assumption that the drilling fluid and the formation fluid have identical properties and considers that the fluid flow is in steady state. This means that the velocity of fluid flow U(r) is given by

$$U(r) = \frac{aU_o}{r}$$

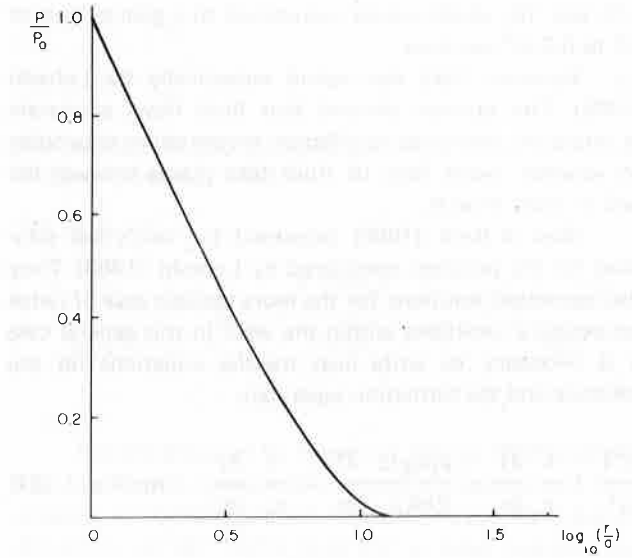

Figure 2 — Decay of pressure as a function of the radial distance from the center of the well maintained at constant pressure P_o. It is assumed that the well fluid and the formation fluid have identical properties.

where U_o is related to the loss (or gain) Q of fluid per unit length of the borehole by the relation

$$U_o = \frac{Q}{2\pi a}$$

The heat transfer equation in this case reduces to

$$\rho_f c_f \frac{\partial T}{\partial t} = K_f \left[\frac{\partial^2 T}{\partial r^2} + \frac{1}{r}\frac{\partial T}{\partial r}\right] - \frac{\rho_\ell c_\ell a U_o}{r}\frac{\partial T}{\partial r} \qquad (34)$$

To complete his model, Luheshi (1983) assumed that radial stirring motion of fluid in the well would make the temperatures constant after the end of circulation. Under this assumption the borehole temperature is given by the solution of the equation

$$\frac{(\rho c)_\ell a}{2}\frac{\partial T}{\partial t}\bigg|_{\text{borehole}} = K_f \frac{\partial T}{\partial r}\bigg|_{r=a} \qquad (35)$$

The relative influence of these two heat transfer mechanisms can be estimated by the Peclet number, defined as the ratio between advected heat to that transferred by conduction

$$2\nu = \frac{a\rho_\ell c_\ell \dfrac{U_o}{r}}{K_f \dfrac{\partial T}{\partial r}} = \frac{a\rho_\ell c_\ell U_o}{K_f} \qquad (36)$$

Convective heat transfer is dominant for $|2\nu| > 1$. In practical situations, the Peclet's number can be between

−10 and 10, which would correspond to a gain or loss of 0.1 to 0.3 m^3 per hour.

Equation (34) was solved numerically by Luheshi (1983). The solution showed that fluid flows accelerate or retard the return to equilibrium temperatures depending on whether radial flow of fluid take places towards the well or away from it.

Shen & Beck (1986) presented full analytical solutions for the problem considered by Luheshi (1983). They also presented solutions for the more realistic case of radial temperature variations within the well. In this general case it is necessary to write heat transfer equations for the borehole and the formation separately

$$\frac{\partial^2 T}{\partial r^2} + \frac{1}{r}\frac{\partial T}{\partial r} - \frac{\rho_\ell c_\ell Q}{2\pi K_f r}\frac{\partial T}{\partial r} = \frac{1}{\alpha_f}\frac{\partial T}{\partial t} \quad \text{for} > a \qquad (37)$$

$$\frac{\partial^2 T}{\partial r^2} + \frac{1}{r}\frac{\partial T}{\partial r} - \frac{\rho_\ell c_\ell Q}{2\pi K_m r}\frac{\partial T}{\partial r} = \frac{1}{\alpha_m}\frac{\partial T}{\partial t} \quad \text{for } r \leqslant a \qquad (38)$$

The boundary conditions imposed on these two equations are represented by equations (9), (10) and (11). It is not necessary to modify equation (10) for heat transport since it is assumed that formation and borehole fluids have identical properties.

The initial conditions imposed on equations (37) and (38) are represented by equations (6) and (7) as well as the temperature distribution predicted by equation (37) for the case where the cylinder is maintained at constant temperature. Thus, for times smaller than the circulation time (t_1) the temperature in the formation is given by,

$$T_i(r,t) = T_m - (T_f - T_m)\left(\frac{r}{a}\right)^{(\nu-|\nu|)} - \frac{2}{\pi}(T_f - T_m)\left(\frac{r}{a}\right)^{\nu} \int_0^\infty e^{-\alpha_f u^2 t}\, \frac{[J_{|\nu|}(ur)\,Y_{|\nu|}(ua) - J_{|\nu|}(ua)\,Y_{|\nu|}(ur)]}{[J_{|\nu|}^2(ua) + Y_{|\nu|}^2(ua)]}\,\frac{du}{u} \qquad (39)$$

Equation (39) was developed by Carslaw & Jaeger (1959) for $\nu > 0$. Shen & Beck (1986) have extended this solution for $\nu < 0$.

The solutions of equations (37) and (38) are (Shen & Beck, 1986) : 1) for the rock

$$T(r,t) = T_i(r,t) + \frac{2a(T_f - T_m)}{\pi}\left(\frac{r}{a}\right)^{\nu} \int_0^\infty J_{|\mu|}\left(aw\sqrt{\frac{\alpha_f}{\alpha_m}}\right) \frac{J_{|\nu|}(wr)\,\Phi_\pm(aw) - Y_{|\nu|}(wr)\,\Phi_\pm(aw)}{\Phi_\pm^2(aw) + \Psi_\pm^2(aw)} \cdot G(w,t,t_1)\,dw \qquad (40)$$

2) for the drilling mud

$$T(r,t) = T_m + \frac{4(T_f - T_m)}{\pi^2}\left(\frac{r}{a}\right)^{\nu} \int_0^\infty \frac{J_{|\mu|}\left(aw\sqrt{\frac{\alpha_f}{\alpha_m}}\right) J_{|\mu|}\left(rw\sqrt{\frac{\alpha_f}{\alpha_m}}\right)}{\Phi_\pm^2(aw) + \Psi_\pm^2(aw)}\, G(w,t,t_1)\,\frac{dw}{w} \qquad (41)$$

with $T_i(r,t)$ given by equation (39) and $G(w,t,t_1)$, $\Phi_\pm(w)$ and $\Psi_\pm(w)$ defined as

$$G(w,t,t_1) = (|\nu| - \nu)\,\frac{1 - e^{-w^2\alpha_f(t-t_1)}}{a^2 w^2} + \frac{4}{\pi^2 a^2}\int_0^\infty \frac{e^{-w^2\alpha_f(t-t_1)} - e^{-u^2\alpha_f(t-t_1)}}{(u^2 - w^2)} \cdot \frac{e^{-u^2\alpha_f t_1}}{J_{|\nu|}^2(au) + Y_{|\nu|}^2(au)}\,\frac{du}{u} \qquad (42)$$

$$\Phi_\pm(w) = \mp\frac{K_m}{K_f}\sqrt{\frac{\alpha_f}{\alpha_m}}\, Y_{|\nu|}(w)\, J_{|\mu|\mp 1}\left(w\sqrt{\frac{\alpha_f}{\alpha_m}}\right) \pm Y_{|\nu|\mp 1}(w)\, J_{|\mu|}\left(w\sqrt{\frac{\alpha_f}{\alpha_m}}\right) \qquad (43)$$

$$\Psi_\pm(w) = \mp\frac{K_m}{K_f}\sqrt{\frac{\alpha_f}{\alpha_m}}\, J_{|\nu|}(w)\, J_{|\mu|\mp 1}\left(w\sqrt{\frac{\alpha_f}{\alpha_m}}\right) \pm J_{|\nu|\mp 1}(w)\, J_{|\mu|}\left(w\sqrt{\frac{\alpha_f}{\alpha_m}}\right) \qquad (44)$$

The parameter μ is defined in the same way as ν, using the mud thermal conductivity K_m.

In the case where the drilling mud behaves as a perfect thermal conductor after the end of mud circulation, the solution for $r > a$ is

$$T(r,t) = T_i(r,t) + \frac{(\nu - 1) + (|\nu| - 1)}{(\rho_\ell c_\ell/\rho_f c_f)(|\nu| - 1) + 1}(T_f - T_m)\,G(t,t_1,0) + \frac{2a(T_f - T_m)}{\pi}\left(\frac{r}{a}\right)^{\nu}\int_0^\infty J_{|\mu|}\left(aw\sqrt{\frac{\alpha_f}{\alpha_m}}\right)$$

$$\frac{J_{|\nu|}(wr)\,\Phi_{\pm\infty}(aw) - Y_{|\nu|}(wr)\,\Psi_{\pm\infty}(aw)}{\Phi^2_{\pm\infty}(aw) + \Psi^2_{\pm\infty}(aw)} \cdot G(t, t_1, w)\, dw \quad (45)$$

where

$$\Phi_{\pm\infty}(w) = \frac{\rho_\ell c_\ell}{\rho_f c_f}\frac{w}{2}\, Y_{|\nu|}(w) + Y_{|\nu| \mp 1}\left(w \sqrt{\frac{\alpha_f}{\alpha_m}}\right) \quad (46)$$

$$\Psi_{\pm\infty}(w) = \frac{\rho_\ell c_\ell}{\rho_f c_f}\frac{w}{2}\, J_{|\nu|}(w) + J_{|\nu| \mp 1}\left(w \sqrt{\frac{\alpha_f}{\alpha_m}}\right) \quad (47)$$

This solution corresponds to the numerical model of Luheshi (1983).

f. Advective heat transport models

Fluid movement in subsurface formations is, in general, a rule rather than an exception. The velocities of fluid movements are in general low but their effects on heat transfer are significant. Oil wells are usually drilled to intercept potential reservoir type rocks, which are highly porous and permeable. We may therefore expect significant formation fluid movements in the bottom parts of the well where BHT measurements are made. Ribeiro & Hamza (1986) presented a simple method to study the decay to thermal disturbances in the presence of formation fluid flows. They considered the case of a circular borehole cutting a permeable and homogeneous layer in which there is an homogeneous and horizontal flow of water (Darcy's velocity $\vec{v}$ is taken to be constant and not affected by drilling). Ribeiro & Hamza (1986) further assumed that heat flow from the formation to the drill hole is exclusively horizontal and that there is no contrast between the thermal properties of the rock and drilling mud. The heat transfer equation, subject to the simplifications described above, reduces to

$$\frac{\partial T}{\partial t} = \alpha \left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2}\right) - \beta v_x \frac{\partial T}{\partial x} \quad (48)$$

where

$$\beta = \frac{\rho_\ell c_\ell}{\rho_f c_f}$$

Cartesian coordinates were chosen since the presence of unidimensional fluid flow breaks the cylindrical symmetry.

Initial conditions imposed on equation (48) are identical to those of Leblanc et al. (1981). At the time the drill hole cuts the permeable layer, the drilling mud temperature is T_m and the rock temperature is T_f. Circulation effects were not considered. Mathematically, these conditions are represented by

$$T(x, y, 0) = T_m \qquad 0 < x^2 + y^2 < a^2 \quad (49)$$

$$T(x, y, 0) = T_f \qquad x^2 + y^2 \geqslant a^2 \quad (50)$$

The solution of this problem is, for the center of the well (Ribeiro & Hamza, 1985)

$$T(0, 0, t) = T_f - \frac{T_f - T_m}{\sqrt{4\pi k t}} \int_{-a}^{a} dx'\, e^{-\left(\frac{x' + \beta v_x t}{2\sqrt{kt}}\right)^2} \operatorname{erf}\left(\sqrt{\frac{a^2 - x'^2}{4kt}}\right) \quad (51)$$

Results obtained by Ribeiro & Hamza (1986) indicate that equilibrium temperature recovery is accelerated in the presence of formation fluid flows. For example a thermal diffusivity of 10^{-6} m²/s, β of 1.7 (a value representative of sandstones) and an initial temperature difference of 50°C, would mean that more than fifty hours are necessary to attain 99% of the equilibrium temperature if the heat transfer is purely by conduction. In the presence of fluid flow with Darcy velocity of 5×10^{-6} m/s, 99% of equilibrium temperature is attained in about 25 hours. A set of theoretical thermal recovery curves are shown in Fig. (3) for different fluid flow velocities. Note that for fluid flow with velocities lower than 10^{-6} m/s causes no perceptive effects in the rate of return to thermal equilibrium.

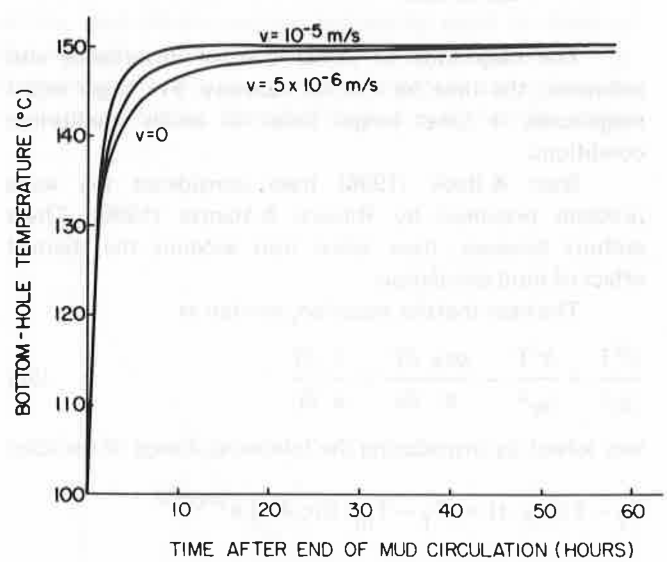

Figure 3 — Theoretical thermal recovery curves for different fluid flow velocities obtained from equation (51). The thermal diffusivity used is 10^{-6} m²/s.

At this point it is worth pointing out the contrasting roles of thermal diffusivity and formation fluid flow in the dissipation of thermal disturbances created by drilling activity. In the absence of fluid flow the dissipation of thermal disturbance is controlled by the thermal diffusivity and hence in the case of low diffusivities even though the area affected is small it takes longer time for the complete decay of the disturbance. In the presence of fluid flows the situation is just contrary. The disturbing heat contained within a relatively small volume can easily be ''washed off''. Thus low thermal diffusivity favors quicker return to equilibrium conditions. The theoretical thermal recovery curves shown in Fig. (4) illustrates the above reasoning.

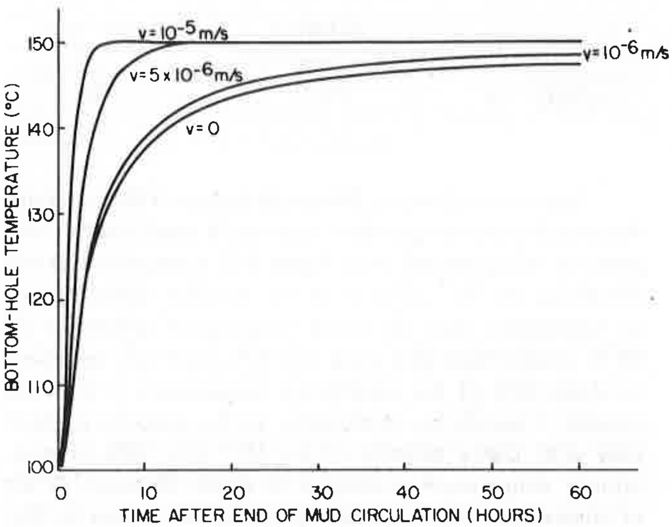

Figure 4 — Theoretical thermal recovery curves for different fluid flow velocities for a thermal diffusivity of 2.5×10^{-7} m^2/s. Comparison with Fig. 3 shows that low thermal diffusivity favors rapid return to equilibrium.

The magnitude of initial thermal disturbance also influences the time for thermal recovery. For larger initial magnitudes it takes longer times to attain equilibrium conditions.

Shen & Beck (1986) have considered the same problem proposed by Ribeiro & Hamza (1986). These authors however, have taken into account the thermal effect of mud circulation.

The heat transfer equation, written as

$$\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} - \frac{\rho c v}{K}\frac{\partial T}{\partial x} = \frac{1}{\alpha}\frac{\partial T}{\partial t} \qquad (52)$$

was solved by introducing the following change of variables

$$T_f - T(x, y, t) = (T_f - T_m)\, S(r, \theta, t)\, e^{\nu r \cos\theta}$$

where

$$\nu = \frac{1}{2}\frac{\rho c v}{K}$$

With this transformation, equation (48) is written as

$$\frac{\partial^2 S}{\partial r^2} + \frac{1}{r}\frac{\partial S}{\partial r} + \frac{1}{r^2}\frac{\partial^2 S}{\partial \theta^2} - \nu^2 S = \frac{1}{\alpha}\frac{\partial S}{\partial t} \qquad (53)$$

The boundary conditions imposed on equation (53) are that after circulation ($t > t_1$) S and $\frac{\partial S}{\partial r}$ are continuous through the borehole wall ($r = a$). The temperature distribution of the formation during drilling is obtained solving equation (53) with the condition that the drilling mud temperature is constant for $t < t_1$. This solution is (Carslaw & Jaeger, 1959)

$$S_i(r, \theta, t) = e^{-\nu r \cos\theta} \qquad \text{for } r < a \qquad (54)$$

and

$$S_i(r, \theta, t) = \sum_{n=0}^{\infty} \epsilon_n I_n(a\nu) \cos n\theta \cdot \left[\frac{K_m(\nu r)}{K_n(\nu a)} + \frac{2}{\pi} \int_0^{\infty} \frac{e^{-(\nu^2 + u^2)\alpha t}}{(\nu^2 + u^2)} \frac{J_n(ur) \cdot Y_n(ua) - Y_n(ur) J_n(ua)}{J_n^2(ua) + Y_n^2(ua)} udu\right] \qquad (55)$$

for $r \geqslant a$. The parameter ϵ_n is defined as

$$\epsilon_o = 1$$

and

$$\epsilon_n = 2(-1)^n \quad \text{with} \quad u = 1, 2, 3, \ldots$$

The initial condition for equation (53) is then given by equations (49) and (50) for $t = t_1$.

The solution of equation (53) for times larger than t_1 is given by (Shen & Beck, 1986)

$$S(r, \theta, t) = S_i(r, \theta, t) - \sum_{n=0}^{\infty} \epsilon_n \cos n\theta \cdot \int_0^{\infty} J_n(aw) J_n(rw) \left[\frac{1 - e^{-(\nu^2 + w^2)\alpha(t - t_1)}}{(\nu^2 + w^2) K_n(a\nu)} + a^2 I_n(a\nu) H(w, t, t_1)\right] wdw \qquad (56)$$

where the function $H(w, t, t_1)$ is given by

$$H(w, t, t_1) = \frac{4}{a^2\pi^2} e^{-\nu^2 \alpha t} \int_0^{\infty} \frac{e^{-w^2 \alpha(t - t_1)} - e^{-u^2 \alpha(t - t_1)}}{u^2 - w^2} \cdot \frac{e^{-(\nu^2 + u^2)\alpha t_1}}{J_n^2(au) + Y_n^2(au)} \frac{udu}{\nu^2 + u^2} \qquad (57)$$

For zero circulation time, the model proposed by Shen & Beck (1986) gives the temperature in the center of the well as

$$T(0, 0, t) = T_f - a(T_f - T_m) \cdot \int_0^{\infty} [w J_o(a\nu) J_1(aw) +$$

$$+ \nu J_1(a\nu) J_o(aw)] \frac{e^{-(\nu^2 + w^2)\alpha t}}{\nu^2 + w^2} \, wdw \qquad (58)$$

This equation corresponds to the solution given by Ribeiro & Hamza (1986).

DATA RESOLUTION AND MODEL LIMITATIONS

Accuracy of bottom-hole temperature measurements in oil wells are no better than a few °C. Leblanc et al. (1982) point out that data from Alberta sedimentary basin have an error of the order of 5°C. In addition measurements are usually carried out during periods of interruptions in drilling activity, which is determined by the logistics of drilling operations. Also measurements do not always cover a sufficient time span after the end of mud circulation. It is common to find sets of BHT measurements made less than tens of hours after mud circulation. Thus acquisition of BHT data do not follow any systematic plan. The accuracy of BHT measurements is also not good being usually about a few °C. Because of these difficulties available data are not of considerable help in checking the validity of basic assumptions made in theoretical models as well as in comparing the usefulness of more sophisticated models.

The models proposed also have limitations, because of simplifications made in the mathematical formulation of the problem. For example the most widely used Horner Plot has severe limitations in its basic formulation. Luheshi (1983) points out that Horner Plot based on measurements made soon after cessation of mud circulation can lead to substantial errors. Drury (1984) reached the conclusion that errors of the order of 10 K may occur in the estimation of formation temperatures, if Horner Plot is used without proper caution. Theoretical model of Bullard (1947) which is the basis of Horner Plot does not allow for contrasts in thermal properties between the drilling mud and the wall rocks. Middleton (1982) concludes that results of models with single diffusivity value approaches consistently those of models with thermal property contrast for times lower than fifteen hours after the end of mud circulation when an appropriate value of bulk diffusivity is used. However thermal diffusivity measurements are rarely carried out for drilling muds.

Instantaneous source models provide methods that are simple to use in practice. The main limitation arises from the basic assumption that the rock temperature is not affected by drilling. This is not strictly true since drilling and circulation times are in general of the order of few hours at the bottom parts of the well. The presence of a cylindrical cavity with temperatures lower than the rock temperatures for several hours can induce a thermal perturbation which can penetrate distances of some well radii. In the method of Middleton (1979) there is no contrast between thermal properties of drilling mud and the rock. Leblanc et al. (1982) pointed out that a knowledge of the thermal diffusivity of the drilling mud allows better fit of theoretical curves to experimental data.

Cylindrical temperature source models represent the closest approximation to the situation where conductive heat transfer is the dominant mode of decay of thermal disturbance. Jaeger's (1956a) original work, Luheshi's (1983) numerical formulation and Shen & Beck's (1986) analytical models belong to this class. However the complexity of the theory and lack of experimental data on the basic parameters put severe restrictions on the practical use of such complete models.

Models that take into account both conductive and advective transport of heat are the ones that come close to representing the real situation. Theoretical developments in this class are much more recent but are based on extreme simplifying assumptions. Luheshi's (1983) model for example assumes that radial fluid flow induced by drilling is in steady state at the time when BHT measurements are made. There is no verification of the validity of this assumption. Also the nature of mud flow from borehole into the formation considered in Luheshi's (1983) model is an extremely simple approximation of the real process.

The drilling mud cnsists normally of a filtrate mixture where the fluid phase is a colloidal suspension or a real solution, or both. When the drilling mud is forced to penetrate a permeable layer, the suspended particles are retained at the borehole wall, forming a "mud cake", while the fluid phase of filtrate penetrates the formation. The quantity of fluid transferred to the formation does depend not only on its permeability but also on the physical characteristics of the drilling mud. Furthermore, the thermal properties of the mud filtrate are not necessarily equal to those of the formation fluid and its flow is not homogeneous. Fig. 5 shows schematically pressure distribution during invasion of the mud filtrate. Near the borehole wall there is a region

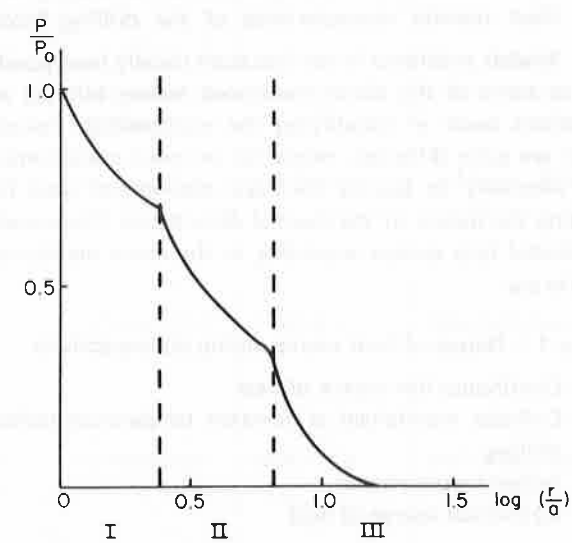

Figure 5 — Schematic representation of the pressure distribution during the mud filtrate invasion in the case where mud filtrate and formation fluid have different properties. The maximum removal of formation fluid occurs in zone I. Zone III is not reached by the mud filtrate and zone II is a transition zone.

where the formation fluid displacement is maximum. If the formation fluid is oil, for instance, the fluid removal is not complete since oil wets preferentially the sand grains. Between the region where the filtrate have not reached and the first region there is a transition zone. The radial drilling mud penetration is therefore, much more complex than that described by equation (32).

Models incorporating advective transport of heat due to formation fluid flows in addition to conduction are representative of the real situations when the well is terminated in reservoir type rocks and the mud invasion is not significant. Pioneering work in this area was done by Ribeiro & Hamza (1986). An extension of their model incorporating effects of mud circulation times has been done by Shen & Beck (1986). Limitations of both these models are the necessity to know the formation fluid flow velocity as well as, access to experimental data on thermophysical properties of drilling mud and the rock formation.

CLASSIFICATION OF THERMAL MODELS OF DRILLING DISTURBANCE

It is convenient to classify the theoretical models presented above into groups on the basis of assumptions used in developing the models and the factors affecting the nature of drilling disturbance. Such a classification is useful not only for presenting an overall view of past developments but also in identifying new problems and suggesting improvements in the existing models.

In the present work we have adopted a classification that takes into account the following factors that control the drilling disturbances:

1. The nature of heat source during drilling activity;
2. Decay of disturbance after cessation of drilling; and
3. Heat transfer characteristics of the drilling fluids.

Models published in the literature usually incorporate one or more of the above mentioned factors but the assumptions made in simplifying the mathematical formulation are quite different. Hence for an exact classification it is necessary to specify the basic assumptions used for defining the nature of the thermal disturbance. The models subdivided into groups according to the above mentioned criteria are:

Group 1 – Nature of heat source during drilling activity

I. Continuous line source of heat
II. Cylinder maintained at constant temperature during drilling
III. Instantaneous source
IV. Cylindrical source of heat

Group 2 – Decay of disturbance after cessation of drilling

A. Conductive dissipation of heat
B. Finite duration of drilling or mud circulation
C. Zero drilling or mud circulation time
D. Effect of fluid movements induced by drilling activity
E. Effect of formation fluid movements
F. Exponentially decreasing heat source strength

Group 3 – Heat transfer characteristics of the drilling fluid

1. Drilling fluid properties ignored
2. Drilling fluid as a perfect conductor
3. Drilling fluid with thermal properties equal to those of the rock
4. Drilling fluid with thermal properties different to those of the rock

A further subdivision is made to classify the kind of mathematical solution arrived at in theoretical models

a. Full analytical solution
b. Assymptotic solutions or expansions
c. Numerical solutions

For convenience in the classification of models we have used Roman numeral for the first group, Latin capital letters for the second group and Arabic numerals for the third group. The kind of solution in each model is identified by small Latin letters. The models published in the literature can be specified by a convenient combination of those letters and numbers. For example, the earliest and the widely used Bullard's (1947) model is defined as model (I-A/B-1-a)whereas the latest Ribeiro & Hamza (1986) is defined a (III-C/F-3-a). Table 2 provides a list of the models in accordance with the above mentioned classification.

As could be noted easily, convenient combinations of the assumptions or conditions of the three groups can produce more than 90 theoretical models out of which only 13 or so have so far been considered in the literature. Also it is important to point out there are only a few models based on asymptotic or approximate solutions and expansions. This kind of solutions may be encouraged since it can give more tractable expressions for practical work even when more complicated models are being used.

CONCLUDING REMARKS

The review of existing models for correcting bottom hole temperature measurements for drilling disturbances, presented above is by no means complete. The problem is really much more complex than has been believed so far. However an overall view of the progress in theoretical modelling achieved shows that there are ample opportunities for further research and development work in this fascinating field.

Future theoretical developments could be accelerated considerably if further improvements in experimental work can be achieved. This includes better accuracy and precision in the measurement of bottom-hole temperatures and additional efforts to measure thermophysical properties (such as conductivity, diffusivity, specific heat and density) of not only the formation but also of the drilling fluid.

Better and more complete models have potential applications not only in Geophysics (Geothermal Energy

Table 2 — Classification of theoretical models for correcting BHT data.

Nature of heat source during drilling activity	Decay of disturbance after cessation of drilling	Heat transfer characteristics of the drilling fluid	Type of solution	Model classification	Reference
Line source of heat	Conductive dissipation and finite circulation time	Fluid properties ignored	Full analytical solution	I-A/B-1-a	Bullard, 1947
Line source of heat	Conductive dissipation and finite circulation time	Fluid properties ignored	Assymptotic expansion	I-A/B-1-b	Dowdle & Cobb, 1975
Cylinder maintained at constant temperature during drilling	Finite duration of mud circulation and conductive dissipation	Drilling fluid with thermal properties equal to those of the rock	Full analytical solution	II-A/B-3-a	Jaeger, 1956b
Cylinder maintained at constant temperature during drilling	Conductive dissipation of heat and finite duration of mud circulation	Drilling fluid with thermal properties different from those of the rock	Numerical solution	II-A/B-4-c	Luheshi, 1983
Cylinder maintained at constant temperature during drilling	Finite duration of mud circulation and fluid movements induced by drilling	Drilling fluid as a perfect conductor	Numerical solution	II-B/D-2-c	Luheshi, 1983
Cylinder maintained at constant temperature during drilling	Conductive dissipation of heat and finite duration of mud circulation	Drilling fluid with thermal properties different from those of the rock	Full analytical solution	II-A/B-4-a	Shen & Beck, 1986
Cylinder maintained at constant temperature during drilling	Finite duration of mud circulation and fluid movements induced by drilling	Drilling fluid with thermal properties different from those of the rock	Full analytical solution	II-B/D-4-a	Shen & Beck, 1986
Cylinder maintained at constant temperature during drilling	Finite duration of mud circulation and formation fluid movements	Drilling fluid with thermal properties equal to those of the rock	Full analytical solution	II-B/E-3-a	Shen & Beck, 1986
Instantaneous source	Conductive dissipation and zero mud circulation time	Drilling fluid as a perfect conductor	Full analytical solution	II-A/C-2-a	Jaeger, 1956a
Instantaneous source	Conductive dissipation and zero mud circulation	Drilling fluid with thermal properties equal to those of the rock	Full analytical solution	II-A/C-3-a	Middleton, 1979; Leblanc et al., 1981
Instantaneous source	Zero circulation and formation fluid movements	Drilling fluid with thermal properties equal to those of the rock	Full analytical solution	II-E/C-3-a	Ribeiro & Hamza, 1986
Cylindrical source of heat	Conductive dissipation and exponential decrease of heat source strength	Drilling fluid with thermal properties equal to those of the rock	Full analytical solution	IV-A/F-3-a	Middleton, 1982
Cylindrical source of heat	Conductive dissipation of heat and finite duration of mud circulation	Drilling fluid with thermal properties different from those of the rock	Full analytical solution	IV-A/B-4-a	Shen & Beck, 1986

and Petroleum Exploration) but also in Engineering and Industrial sectors as well.

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