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ON THE STABILITY OF THE PACIFIC OCEAN SUBARCTIC FRONT

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It is analitically shown that in the case of the Pacific Ocean Subarctic fronts in the upper layer, instability due to horizontal shear may only be possible on very small horizontal scales. Therefore, barotropic instability should not occur.

É mostrado analiticamente que no caso de frentes oceânicas na camada superior, a instabilidade devida ao cisalhamento horizontal pode ser possível somente para escalas horizontais muito pequenas. Portanto, a instabilidade barotrópica não deve ocorrer.

INTRODUCTION

Camerlengo (1982) studied the large scale response of the Pacific Ocean Subarctic Front to momentum transfer. In that numerical work there were no instabilities. It should be noted that in the upper layer of the Subarctic Front the temperature and salinity fronts compensate each other (Roden, 1972). The aim of the present work is to derive a stability criterion for an oceanic front in the upper layer, in general, and for the Subarctic Front, in particular.

The instability of parallel inviscid flows was first addressed by Lord Rayleigh (1880). The main conclusion of his theorem, valid for nonrotating systems, states that if the velocity profile does not have an inflection point, the inviscid flow should be stable. A direct result of the Rayleigh's theorem was observed by Hoiland (1953). Denoting the velocity profile and the inflection point by U and U_o, respectively, Hoiland concluded that d^2U/dy^2 should negatively covariate with $U - U_o$, between the meridional walls, as a condition for horizontal shear instability. It should be noted that if the rotational effects are included in Hoiland analysis, it is immediately concluded that the β – plane approximation should introduce a stabilizing effect.

Eddies may be generated by the horizontal shear of the mean flows. In such a case, eddies extract energy from the mean flow kinetic energy. However, eddies may extract additional energy from the mean available potential energy field through baroclinic processes. More generally, energy may be supplied to the eddies by both the horizontal shear of the mean flow and the mean available potential energy field.

MODEL FORMULATIONS

Statement of the Problem

A two-layer model is considered. In order to filter out the barotropic mode, the lower layer is chosen to be motionless (Fig. 1). Furthermore, it is assumed that the height of the upper layer, $\bar{h}$, has a hyperbolic profile of the form

$$\bar{h} = \bar{h}_{oo} - \Delta h \tanh (y/L_y) \qquad (1)$$

where the mean value, $\bar{h}_{oo}$, and the amplitude, Δh, of the upper layer are given arbitrary values of 100 and 10 m, respectively. The meridional length scale of the oceanic front, L_y is set to be equal to 10 km.

To simplify the equations to be used, the following hypotheses are made

1) the oceanic front is geostrophically balanced, i.e.

$$- f\bar{u} = g^* \frac{\partial \bar{h}}{\partial y}, \quad \bar{v} = 0 \qquad (2)$$

where g^* represents the reduced gravity and the over-bar ($^-$) quantities represent the mean state;

2) since the meridional length scale of the oceanic front in the upper layer, L_y, has an order of magnitude of ten kilometers, the f-plane approximation is used;

3) the zonal wave number, k, is much smaller than $(L_y)^{-1}$; therefore only long waves are considered; and

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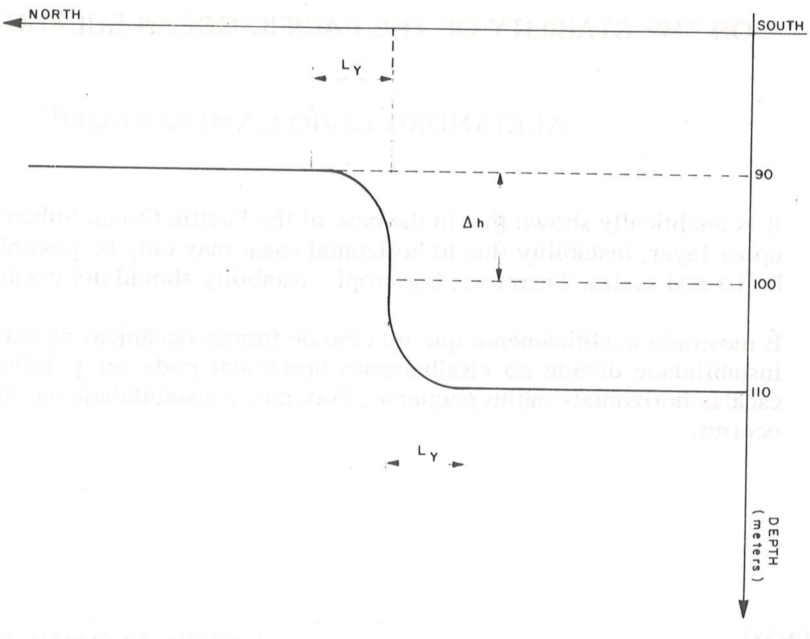


Figure 1. Initial position of the Subarctic Front, modelled via an hyperbolic tangent profile.

4) the vorticity at the front, represented by $-\partial\bar{u}/\partial y$, is much less than the planetary vorticity, f.

Equations of Motion

With the above assumptions, the linear inviscid perturbation equations may be written as

$$\frac{\partial u'}{\partial t} + \bar{u}\,\frac{\partial u'}{\partial x} + v'\,\frac{\partial \bar{u}}{\partial y} - fv' = -\,g^*\,\frac{\partial h'}{\partial x}\,, \qquad (3)$$

$$\frac{\partial v'}{\partial t} + \bar{u}\,\frac{\partial v'}{\partial x} + fu' = -\,g^*\,\frac{\partial h'}{\partial y}\,, \qquad (4)$$

$$\frac{\partial h'}{\partial t} + \bar{h}\,\frac{\partial u'}{\partial x} + \bar{u}\,\frac{\partial h'}{\partial x} + v'\,\frac{\partial \bar{h}}{\partial y} + \bar{h}\,\frac{\partial v'}{\partial y} = 0 \qquad (5)$$

where the prime quantities represent the perturbation. In equation (4) the geostrophic balance term is substracted.

Assuming that the perturbation quantities have a wavelike form

$$\begin{Bmatrix} u' \\ v' \\ h' \end{Bmatrix} = \begin{Bmatrix} U\,(y) \\ V\,(y) \\ H\,(y) \end{Bmatrix} \exp\,[\,ik\,(\,x - ct\,)\,] \qquad (6)$$

where U (y), V (y) and H (y) are respectively the amplitudes of u', v' and h', the above set of equations has the form

$$ik\,(\,\bar{u} - c\,)\,U + (\frac{d\bar{u}}{dy} - f\,)\,V = -\,g^*\,ikH, \qquad (7)$$

$$fU + ik\,(\,\bar{u} - c\,)\,V = -\,g^*\,\frac{dH}{dy} \qquad (8)$$

$$ik\bar{h}U + \frac{d}{dy}\,(\,\bar{h}V\,) + ik\,(\,\bar{u} - c\,)\,H = 0 \qquad (9)$$

This system of equations may be reduced to the single equation[1]

$$(\,\bar{h}H'\,)' - \frac{f\bar{h}'}{\bar{u} - c} + \frac{(\,\bar{h}\,\bar{u}'\,)'}{\bar{u} - c} + \alpha^2\,\bar{h}\,H = 0 \qquad (10)$$

where $(\ )' = d(\ )/dy$ and $\alpha^2 = f^2(g^*\bar{h})^{-1}$ is the inverse of the Rossby radius of deformation squared.

Since the meridional length scale of interest is of the order of ten kilometers, it may be assumed that $H(\infty) = H(-\infty) = 0$

STABILITY CRITERION

For the perturbation to be unstable, the phase speed, $c\ (= c_r + ic_i)$, must be complex. The amplitude functions must also be complex.

For the sake of simplicity, the variable $Z(= \bar{u} - c)$ is introduced, and the following change of variables is made (Howard, 1961): $H = Z^{1/2}\,Q$. With these considerations, equation (10), may be expressed as[2]

1 See Appendix A for derivation
2 See Appendix B for derivation

$$(\bar{h}ZQ')' - [f\bar{h}' + \frac{1}{2}(\bar{h}\bar{u}')' + \alpha^2\bar{h}Z + \frac{\bar{h}\bar{u}'^2}{4Z}]Q = 0 \quad (11)$$

Multiplying equation (11) by Q* (complex conjugate of Q) and upon integration by parts yields

$$\int \{ \bar{h}Z|Q'|^2 + [f\bar{h}' + \frac{1}{2}(\bar{h}\bar{u}')' + \alpha^2\bar{h}Z + \frac{\bar{h}\bar{u}'^2}{4Z}]|Q|^2\} dy = 0 \quad (12)$$

where the limits are omitted.

The imaginary part of Z is $-c_i$. It follows immediately that the imaginary part of Z^{-1} is $-c_i|Z|^{-2}$. Therefore, the imaginary part of equation (12) is

$$c_i \int [\bar{h}|Q'|^2 + \{\alpha^2\bar{h} - \frac{\bar{h}\bar{u}'^2}{4Z^2}\}|Q|^2] dy = 0 \quad (13)$$

If

$$|Z|^2 = |\bar{u} - c|^2 > R^2 = \frac{g^*\bar{h}\bar{u}'^2}{4f^2} \quad (14)$$

through the entire domain, the integrand of equation (13) has a positive defnite form. In such a case, equation (13) can hold only if $c_i = 0$. Therefore, the condition (14) guarantees that the front is stable with regard to the perturbations.

Looking at the problem from a mathematical point of view, criterion (14) is met outside the circle of radius R centered at $(\bar{u}, 0)$ along the c_r axis in the complex C plane. For a given oceanic front, both $\bar{h}$ and $\bar{u}$ are functions of the latitude, y. Therefore, to every latitudinal point, y, across the front there correspond one interval and one radius.

The two extremes – maximum and minimum – of this family of intervals delimit the region of stability. In other words, any point outside that region corresponds to a stable fluctuation of the front. This constitutes a sufficient condition of stability.

Let us nondimensionalize the governing equations according to:

$$\eta = y/L_y \qquad \chi = k/|\alpha|$$

$$\gamma = c/|\bar{u}|_m \qquad \lambda = \bar{h}/\bar{h}_{oo}$$

$$\mu = \bar{u}/|\bar{u}|_m \qquad \Delta\lambda = \Delta\bar{h}/\bar{h}_{oo}$$

$$\kappa = \omega/|f| \quad (15)$$

where η, γ, μ, κ, χ, λ and $\Delta\lambda$ are the dimensionless variables. The expression for the zonal velocity at the front, $|u|_m$, which is maximum, has the form

$$|u|_m = \frac{g^*\Delta h}{f L_y} \quad (16)$$

The basic state may then be rewritten as

$$\lambda = 1 - \Delta\lambda \tanh\eta \quad (17)$$

$$\mu = \text{sech}^2\eta \quad (18)$$

Thus, for the basic state (17 and 18) the stability criterion (14) becomes

$$|\mu - \gamma|^2 \geq \frac{g^*\bar{h}_{oo}}{f^2 L_y^2}(1 - \Delta\lambda\tanh\eta)\,\text{sech}^4\eta\,\tanh^2\eta = \phi^2 \quad (19)$$

For the purpose of clarity, a family of circles of radius ϕ, each one centered at μ, is constructed in the complex γ plane (Fig. 2). The envelope defined by the family of circles delimits the outer region of stability. Because of the positive contribution of the term $\bar{h}|Q'|^2$ in equation (13), it would be inappropriate to state that the region delimited by the points inside the envelope corresponds to the unstable case.

Let γ_o and γ_1 be the minimum and maximum real values of the points along the envelope, respectively. Three cases are possible:

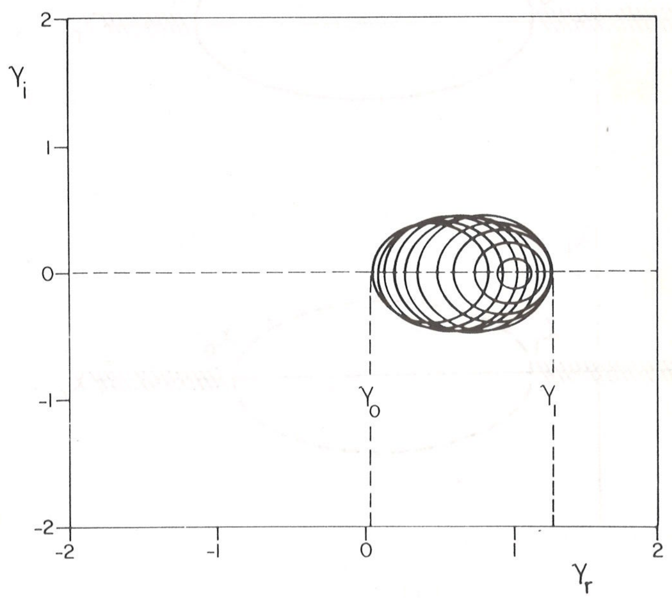


Figure 2. Envelope showing outer region of stability of equation $|\gamma_r - \mu|^2 + \gamma_i^2 = \phi^2$ in the complex γ plane for $|\eta| \leq 1$.

(i) $0 < \gamma_o < \gamma_1,$ (20a)

(ii) $\gamma_o < \gamma_1 < 0,$ (20b)

(iii) $\gamma_o < 0 < \gamma_1,$ (20c)

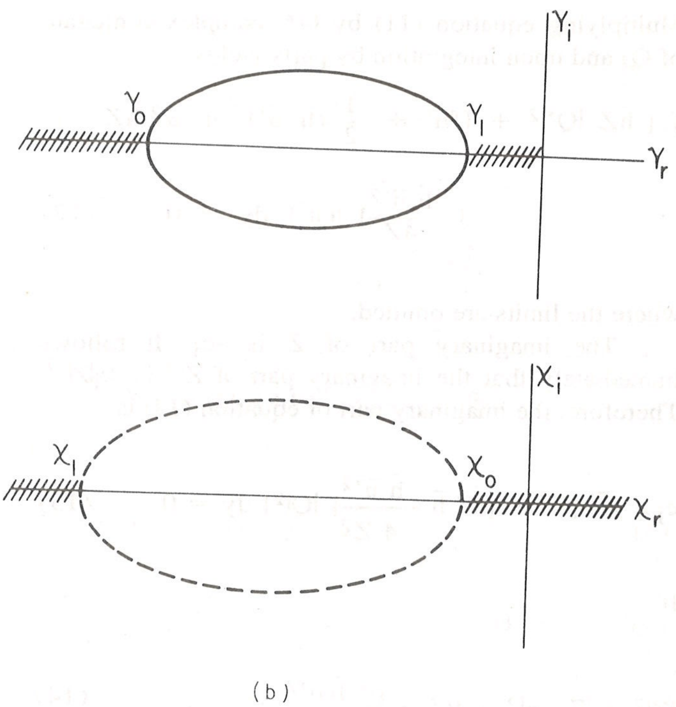
(i) If both γ_o and γ_1 are positive, the stability in the χ complex plane corresponds to the points outside the interval (χ_1, χ_o) since the dimensionless frequency κ is positive and real (Fig. 3a).

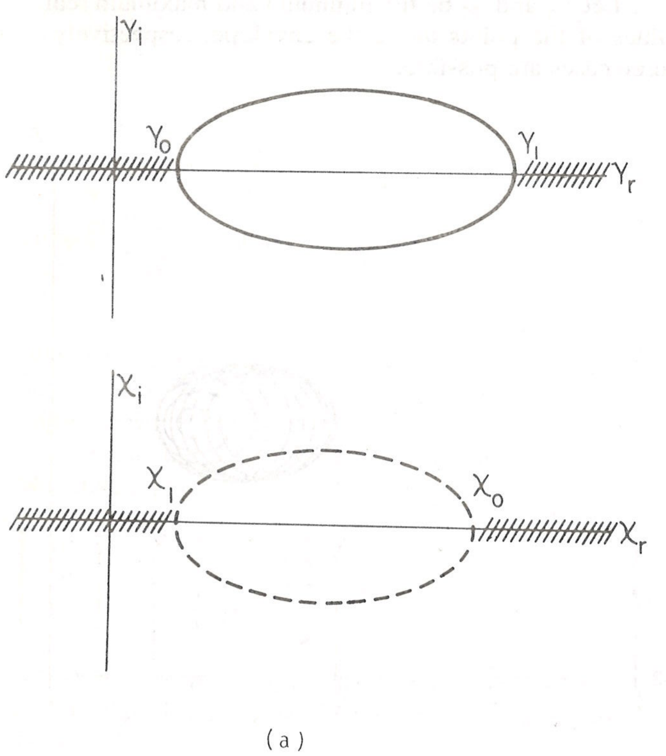
(ii) The same result holds if both extremes, γ_o and γ_1, are negative (Fig. 3b).

(iii) If γ_o is negative and γ_1 is positive, the stability region is determined by the points inside the interval (χ_o, χ_1) (Fig. 3c).

CONCLUSION

It has been observed that the time scale of the oceanic mixed layer response to atmospheric wind forcing is on the order of a couple of days, at most (Garwood, 1977). For h_{oo} = 100 m, L_y = 10 km, γ_o and γ_1 are found to be 0 and 1.25, respectively. Using a two-day period, the above values correspond to a wavelength less than 21.6 km. Thus a sufficient condition of stability for a Pacific Ocean Subarctic Front acted upon by wind forcing in the upper layer is established. This criterion guarantees that waves whose wavelengths are larger than 22 km may be barotropically stable. Therefore, instability due to horizontal shear is only possible on very small scales (i.e., < 21.6 km) and should not occur in upper layer Pacific Ocean Subarctic Front.





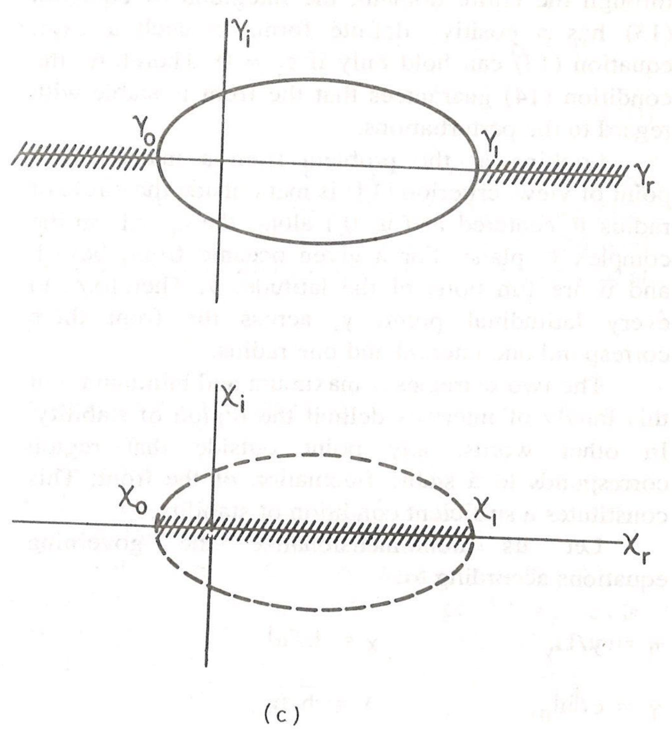


Figure 3. Region of stability in the χ complex plane whenever a) γ_o and γ_1 are positive; b) γ_o and γ_1 are negative; c) γ_o is negative and γ_1 is positive.

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APPENDIX A

Derivation of Equation 10

In subtracting the product of equation (7) and ik $(\bar{u} - c)$ from the product of equation (8) and $(\bar{u}' - f)$ (primed quantities represent derivatives with respect to y) we will have an expression for U of the form:

$$U = \frac{- g^{*} [k^2 H (\bar{u} - c) + H' (\bar{u}' - f)]}{k^2 (\bar{u} - c)^2 + f\bar{u}' + f^2} \qquad (A.1)$$

The appropriate physical scale is: $L_x = 10^6$ m, H = 100 m, $(\bar{u} - c) = 10^{-1}$ m.s^{-1} f = 10^{-4} s^{-1} and $L_y = 10^4$ m. In the ocean typical values of u and v are in the order 10^{-1} m.s^{-1}. Therefore $\bar{u}' = du/dy$ can be scaled as $\bar{u}/L_y$. Thus, $\bar{u}' << 10^{-4}$ s^{-1}. Using these scaling terms we will have:

$$k^2 H(\bar{u} - c) = \frac{(2\pi)^2}{L_x^2} H(\bar{u} - c) = 10^{-9} \, s^{-1} \qquad (A.2)$$

$$H (\bar{u}' - f) = 10^{-6} \, s^{-1} \qquad (A.3)$$

Therefore:

$$k^2 H (\bar{u} - c) << f' (\bar{u}' - f) \qquad (A.4)$$

and

$$k^2 (\bar{u} - c)^2 << f(f - \bar{u}') \qquad (A.5)$$

Then equation (A.1) will have the final (geostrophic) form:

$$U = -g^{*} H'/f \qquad (A.6)$$

By subtracting the product of equation (7) and f from the product of equation (8) and ik $(\bar{u} - c)$ we wind up with an expression for V of the form:

$$V = \frac{ikg^{*}}{f^2} [fH - H' (\bar{u} - c)] (1 + \bar{u}'/f) \qquad (A.7)$$

The product of $H'(\bar{u} - c)$ by $\bar{u}'/f$ is disregarded because it is at least one order of magnitude less than the rest of the products. Therefore, a final expression for V will be:

$$V = \frac{ikg^{*}}{f} [H - \frac{(\bar{u} - c)}{f} H' + \frac{\bar{u}' H}{f}] \qquad (A.8)$$

Introducing equations (A.6 and A.8) into equation (9), and after some algebraic manipulations yields:

$$\frac{-g^{*}(\bar{u} - c)}{f^2} \{\bar{h} H'' + \bar{h}' H' - [\frac{f \bar{h}'}{(\bar{u} - c)} (1 + \bar{u}'/f) + \alpha^2 \bar{h} + \frac{\bar{u}'' \bar{h}}{\bar{u} - c}] H\} = 0$$

or

$$(\bar{h} H')' - [\frac{f \bar{h}'}{\bar{u} - c} + \frac{(\bar{h} \bar{u}')'}{\bar{u} - c} + \alpha^2 \bar{h}] H = 0,$$

Which is the final expression we want (10).

APPENDIX B

Derivation of equation (11)

Let $Z = \bar{u} - c$ and

$$H = Z^{1/2} Q \qquad (B.1)$$

An expression for H' will have the form:

$$H' = Z^{1/2} Q' + \frac{\bar{u}'}{2} Q Z^{-1/2} \qquad (B.2)$$

Thus, the product $(\bar{h} H')'$ will be:

$$(\bar{h} H')' = \bar{h} Z^{1/2} Q'' + (\bar{h} \bar{u}' Z^{-1/2}) Q' + \bar{h}' Z^{1/2} Q' + [\frac{\bar{h}' \bar{u}'}{2} Z^{-1/2} + \frac{\bar{h} \bar{u}''}{2} Z^{-1/2} - (\frac{\bar{u}'}{4})^2 \bar{h} Z^{-3/2}] Q \qquad (B.3)$$

We will have also

$$(\bar{h} Z Q')' = \bar{h} Z Q'' + \bar{h} Q' \bar{u}' + \bar{h}' Q' Z \qquad (B.4)$$

Introducing (B.4) into the product of equation (B.3) by $Z^{1/2}$ yields:

$$Z^{1/2} (\bar{h} H')' = (\bar{h} Z Q')' + [(\frac{\bar{h} \bar{u}'}{2})' - \frac{\bar{h} \bar{u}'^2}{4Z}] Q \qquad (B.5)$$

The product of equation (10) by $Z^{1/2}$ will be:

$$Z^{1/2} (\bar{h} H')' = [f \bar{h}' + (\bar{h} \bar{u}')' + \alpha^2 \bar{h} Z] Q \qquad (B.6)$$

Thus comparing equations (B.5 and B.6) yields

$$(\bar{h} Z Q')' - [f\bar{h}' + (\frac{\bar{h} \bar{u}'}{2})' + \frac{\bar{h} \bar{u}'^2}{4Z} + \alpha^2 \bar{h} Z] Q = 0 \qquad (B.7)$$

Which is the final expression we want (11).