

## THERMOSPHERIC MODELS: A SHORT REVIEW

C.J. ZAMLUTTI<sup>1</sup>

In this short review, thermospheric models are briefly discussed concerning their completeness compared to the first order Navier-Stokes equations. It is concluded that empirical models are the most reliable reference for the dynamical equilibrium behaviour of the thermosphere. The three-dimensional time dependent models still need some work before they reach their final stage.

**MODELOS TERMOSFÉRICOS: UMA CURTA REVISÃO** – Nesta curta revisão, os modelos termosféricos são questionados sumariamente no que concerne à sua inteireza quando comparado às equações de primeira ordem de Navier-Stokes. Conclui-se que os modelos empíricos são as referências mais confiáveis para o comportamento da termosfera em equilíbrio dinâmico. Os modelos tridimensionais dependentes do tempo ainda necessitam de refinamentos para que seu estágio final possa ser alcançado.

### 1. INTRODUCTION

Thermospheric models received considerable improvement, in the past years, since the remarkable work of Harris & Priester (1962). Their past history (see references therein) cannot be covered in a short review like the present one.

The basic problem is to determine the effect of solar radiation, magnetic storms and other energy mechanisms on the neutral atmosphere. The complexity is increased because part of the incident energy is not converted into heat instantaneously and may be either stored as chemical energy (in the ionized particles) or delivered elsewhere (interhemispheric trips by photoelectrons).

It is generally accepted that the hydrodynamic equations, together with an equation of state, constitute a reasonable mathematical description of the problem. The equations are those of a fluid and the neutral gas is treated as such. Atmosphere modelling involves essentially three aspects:

- a) the selection of the effective contributing terms of the hydrodynamic equations;
- b) the choice of appropriate boundary conditions;
- c) the efficiency in describing the momentum and energy sources and sinks.

In this review we take the improvement of approximated solutions to the governing equations as a guideline to discuss the first of the three aspects above. References are inserted at each stage of the development in respect to their relevant contribution. No attempt is made to give computational details or compare expected results with experimental data, since this has already been done in the included references. As a general rule, the reader can be sure that all model attempts commented in this work were successful in explaining, within a reasonable limit (say 25%

accuracy), some feature of the thermospheric behaviour. Improvements on the models became necessary over and over again as a natural consequence of the development of measurement techniques with the resulting new data bases. The reviewing does not follow a chronological order but is organized, as far as possible, based on increasing complexity. Our emphasis is placed on the hydrodynamic equations and on the importance attributed to each of their terms at the various stages of development.

### 2. THE BASIC EQUATIONS

The basic equations considered for neutral atmosphere modelling are the hydrodynamic equations: the continuity equation, the equation of motion and the energy equation, and the ideal gas law. The fluid equations (Landau & Lifchitz, 1971) are:

1. the continuity equations:

$$\partial\rho/\partial t = \delta\rho - \nabla \cdot \phi_{\text{mass}}; \quad (1)$$

2. the equation of motion:

$$\partial(\rho\mathbf{u})/\partial t = \Delta\mathbf{f} - \nabla \cdot \phi_{\text{momentum}}; \quad (2)$$

3. the energy equation:

$$\partial W/\partial t = \delta W - \nabla \cdot \phi_{\text{energy}}; \quad (3)$$

where  $\rho$  stands for mass density,  $t$  for time,  $\delta$  for local time rate variation of the parameter,  $\phi$  for flux density,  $\nabla$  for a balance or budget of parameters,  $\mathbf{u}$  for the wind velocity,  $\mathbf{f}$  for force per unit volume and  $W$  for energy density. Bold symbols stand for vectors and

<sup>1</sup> Instituto de Pesquisas Espaciais, INPE, Caixa Postal 515, 12201 São José dos Campos, SP.

those bold underlined represent matrices. Unsubscripted parameters stand for neutral bulk characteristics.

The ideal gas law is:

$$p = \rho RT/M, \quad (4)$$

where  $p$  is the pressure,  $R$  is the universal gas constant and  $M$  is the mean molecular mass in a.m.u..

In order to transform the hydrodynamic equations into forms more appropriate to computational purposes, we consider the actual constraints imposed by the thermospheric medium.

Equation (1) can be drastically simplified because the ionization-recombination, molecular dissociation, etc., occurring within the thermosphere do not significantly alter the neutral atmosphere density. This means that we can assume  $\delta\rho = 0$  in any condition. Moreover the flux of mass can be expressed by:

$$\Phi_{\text{mass}} = \rho \mathbf{u}, \quad (5)$$

then the continuity equation becomes

$$\partial\rho/\partial t + \nabla \cdot (\rho \mathbf{u}) = 0 \quad (6)$$

Equation (6), with subscripted parameters, represents the continuity equation for a given individual constituent of a gas mixture in the absence of particle production and loss. Using both, subscripted and unsubscripted, forms of eq. (6) it is possible to identify a flux of particle species  $j$  diffusing through the bulk flow of neutral particle. If we call the diffusion flux  $\Phi_{\text{dif}}$ , we have the result:

$$-\nabla \cdot \Phi_{\text{dif}} = -\nabla \cdot [\rho_j (\mathbf{u}_j - \mathbf{u})] = \rho [\partial(\rho_j/\rho)/\partial t + \mathbf{u} \cdot \nabla (\rho_j/\rho)] \quad (7)$$

(Landau & Lifchitz, 1971). The difference  $\mathbf{u}_j - \mathbf{u}$  can be computed considering the balance of forces acting on the individual species for steady state conditions and absence of a momentum flow, i.e.:

$$\Delta f = 0 \quad (8)$$

We turn now to eq. (2) for the motion of the fluid. Regarding the local forces acting on a considered unit volume we have:

$$\Delta f = -\nabla p + \rho \mathbf{g} - 2\rho \Omega \times \mathbf{u} + \sum \rho v_{ni} (\mathbf{v}_i - \mathbf{u}) \quad (9)$$

where  $\mathbf{g}$  is the gravitational acceleration,  $\Omega$  is the

earth's angular velocity;  $v_{ni}$  is a coefficient indicating the rate of transfer of momentum from particle species  $i$  (usually ions), with velocity  $\mathbf{v}_i$ , to the neutrals. The first three terms on the right hand side of eq. (9) are the usual forces considered in fluid dynamics (Landau & Lifchitz, 1971). The last term of eq. (9) is the result of particle-particle interaction (Rishbeth & Garriott, 1969). The small centripetal acceleration  $\Omega \times (\Omega \times \mathbf{r})$  may be subtracted from  $\mathbf{g}$  ( $\mathbf{r}$  being the geocentric distance).

The flux of momentum is composed of:

$$\Phi_2 = \rho \mathbf{u} \mathbf{u}, \quad (10)$$

where  $\mathbf{u} \mathbf{u}$  stands for the dyadic product, and a viscosity flux,  $\Phi_{\text{vis}}$ , expressed by

$$\Phi_{\text{vis}} = -\eta [\nabla \mathbf{u}^t + (\nabla \mathbf{u})^t - (2/3) (\nabla \cdot \mathbf{u}) \mathbf{I}], \quad (11)$$

where  $\eta$  stands for a viscosity coefficient (Landau & Lifchitz, 1971; Schunk, 1975) and the superscript  $t$  denotes transposal.  $\mathbf{I}$  stands for the identity matrix.

The total flux of momentum, to be used in eq. (2), is then given by:

$$\Phi_{\text{momentum}} = \Phi_2 + \Phi_{\text{vis}} \quad (12)$$

Equation 2, with subscripted parameters, represents the equation of motion for an individual constituent in a gas mixture. In this case the summation of the collision term of eq. (9) must include the collisions of the considered neutral species with the other neutrals.

Finally, we ought to consider the thermospheric constraints to be imposed on eq. (3). To start with the energy density,  $W$  is composed of an internal energy,  $\rho \epsilon$ , stored as random motion within the unit volume and an organized kinetic energy,  $\rho \mathbf{u}^2/2$ . The time variation of the internal energy is expressed by:

$$\partial(\rho \epsilon)/\partial t = (Q - L) - \nabla \cdot (\rho \epsilon \mathbf{u}) + (p/\rho) (dp/dt) \quad (13)$$

where  $Q$  and  $L$  stand for local heating and cooling rates per unit volume, respectively, from energy exchange with the surroundings;  $\nabla \cdot (\rho \epsilon \mathbf{u})$  is the transported energy; and the last term on the right hand side of eq. (13) is the energy variation from adiabatic motion. Also  $\epsilon = C_v T$ , where  $C_v$  is the specific heat at constant volume.

Using the continuity equation, eq. (13) is converted into:

$$\partial(\rho \epsilon)/\partial t = (Q - L) - \nabla \cdot (\rho \epsilon \mathbf{u}) - p \nabla \cdot \mathbf{u} \quad (14)$$

The kinetic energy component was developed using the continuity and momentum equations in the same manner as in Landau & Lifchitz (1971) and we

obtained:

$$\partial(\rho u^2/2)/\partial t = (u^2/2) \nabla \cdot \phi_{\text{mass}} + \mathbf{u} \cdot \Delta \mathbf{f} -$$

$$\mathbf{u} \cdot \nabla \cdot \phi_{\text{momentum}} \quad (15)$$

The kinetic energy density is irrelevant in the context of thermospheric modelling. It may become important when considering the effect of nuclear explosions, provided that the appropriate collision terms are considered. Without this component the energy equation becomes the thermodynamic equation.

Adding up eq. (14) to eq. (15) and comparing the result with eq. (3) gives:

$$\delta W = (Q - L) + (u^2/2) \nabla \cdot \phi_{\text{mass}} + \mathbf{u} \cdot \Delta \mathbf{f} \quad (16)$$

$$\phi_{\text{energy}} = [\rho \epsilon + p] \mathbf{u} + \phi_{\text{momentum}} \cdot \mathbf{u} \quad (17)$$

where  $\Delta \mathbf{f} = \Delta f + \nabla p$ . This result is the same as that of Gleeson & Axford (1967) when no thermal flux is considered. With  $\delta W = 0$ , we recover the equation of conservation of energy of fluid dynamics (Landau & Lifchitz, 1971; Grad, 1958). The effect of a heat flow is computed by adding one more term to the right hand side of eq. (17), namely  $-(\lambda \nabla T)$  (see Schunk, 1975). Finally, the interaction between neutral and ionized particles requires the addition of one more term to the right hand side of eq. (16), namely

$$\rho \nu_{ni} (m_n + m_i)^{-1} 3k (T_i - T_n)$$

Equation 3 with subscripted parameters, represents the energy equation for an individual constituent in a gas mixture. Besides the considerations imposed on eqs. (1) and (2), the heat flux must consider the interaction between the considered neutral species with the other neutrals.

The above system of equations, with its inherent limitations, constitute a simplified version to what are called first-order equations or Navier-Stokes equations (Schunk, 1975). They do not constitute an accurate description for the thermosphere. Among their shortcomings, one can mention:

- a) viscosity and heat effects were considered separated here, whereas in nature they are intimately connected;
- b) the interaction between different particle species was considered here in a drastically simplified

form.

These aspects were discussed in Schunk (1975). Nevertheless the Navier-Stokes equations, or even simplified versions of them, are currently used for thermospheric modelling with satisfactory results.

### 3. THE HYDROSTATIC EQUILIBRIUM

Hydrostatic equilibrium implies a null velocity and no time variation to be imposed on the basic equations. This constitutes the most drastic simplification that one expects the system of the basic equations to bear. Nevertheless, it is a very significant zero-order approximation for neutral atmosphere modelling and is very often used to infer first order parameters (e.g. wind velocity).

Under hydrostatic equilibrium conditions, the continuity equation states that the density is a function of space only. The equation of motion reduces to:

$$\partial p / \partial z = - \rho g \quad (18)$$

$$\partial p / (r \partial \theta) = 0 \quad (19)$$

$$\partial p / (r \sin \theta \partial \phi) = 0 \quad (20)$$

where  $r$  is the distance between the considered point and the center of the earth,  $\theta$  stands for colatitude and  $\phi$  represents longitude. This is the same as saying that the pressure varies only in the radial direction. The energy equation yields:

$$-\nabla \cdot (\lambda \nabla T) = \langle (Q - L) \rangle \quad (21)$$

where the angular brackets denote daily time average. Equation (21) states that the heat flow tends to restore the thermal equilibrium broken by the presence of the daytime solar energy.

Equations (18) - (21), together with equation of state (4), were used at the very early stage of modelling (see Nicolet, 1960a). Equation (18), together with eq. (4), yields the pressure independently of the density:

$$p = p_0 \exp(-g M R^{-1} \int_0^z T^{-1} ds) \quad (22)$$

where  $p_0$  is the pressure at ground level. The parameters  $g$  and  $M$  may also vary in eq. (22).

Though the hydrostatic equilibrium approach is a very drastic simplification to the equations, Nicolet (1960a) was able to associate it with the dependence of atmospheric parameters on the solar activity, latitude and season, which modify the actual  $(Q - L)$  daily average.

The importance of the hydrostatic model, besides historical, is that eq. (22) constitutes the basis for more sophisticated models. The reason is that, where the

wind velocities are large the density is small, and thus eq. (18) continues being a reasonable approximation to eq. (9). The other terms are all second order terms for eq. (2). Therefore, the hydrostatic equilibrium may only be broken if either large acceleration of neutral particles occurs or else extremely high ion velocities steady flow affects it. Both situations do not occur regularly in the upper atmosphere.

#### 4. TIME DEPENDENT MODELS

To understand the model attempts of this and the next sections it is important to give to the reader some preceding information about the atmosphere research in the early 60's. At those days temperature data were obtained through eq. (21) and density data started being derived from satellite drag. Therefore, researchers were worried to produce temperature and density models which conformed to a local hydrostatic equilibrium and were consistent with the existing data.

Densities computed with the temperatures obtained from eq. (21) were not consistent with those obtained from satellite drag data. Moreover, data exhibited a sinusoidal diurnal variation. To solve these problems the models discussed next were proposed.

The earliest time dependent model considered a time variation allowance for eq. (21) only and assumed the thermal variation to drive perturbations in the other parameters through eq. (22) and the equation of state (4). Nicolet (1960b) proposed that eq. (21) be extended to:

$$\partial W/\partial t = (Q - L) + \nabla \cdot (\lambda \nabla T) \quad (23)$$

and, moreover, that  $W = \rho \epsilon$ . Thus, eq. (23) discards the irrelevant contribution of the kinetic part of the energy. Harris & Priester (1962) extended further eq. (23) by considering the term  $\mathbf{u} \cdot \Delta \mathbf{f}$  from eq. (16) and the terms  $\rho \mathbf{u} \cdot \mathbf{u}$  and  $\mathbf{p} \cdot \mathbf{u}$  from eq. (17). Additionally they imposed the condition  $\Delta \mathbf{f} = \mathbf{0}$  and restricted the velocity to the component along the vertical. The energy equation becomes then:

$$\begin{aligned} \partial(\rho \epsilon)/\partial t + u_z \partial(\rho \epsilon)/\partial z + (\rho \epsilon + p) \partial u_z/\partial z = \\ (Q - L) + \nabla \cdot (\lambda \nabla T) \end{aligned} \quad (24)$$

The original version assumes no time variation of density.

To complete the system of equations composed of eqs. (4), (22) and (24), Harris & Priester (1962) introduced the idea of a "breathing" atmosphere, which is otherwise expressed by the restriction:

$$dp/dt = 0$$

None of these models could reproduce the amplitude and phase of the diurnal density variation derived from satellite drag data and the problem claimed to have its origin in the net local heat balance ( $Q - L$ ). Grounded on the existing evidence of correlation of the density behaviour with the magnetic activity, Harris & Priester (1962) suggested the existence of a corpuscular source of energy in addition to the solar radiation energy source. The local characteristics of the "second heat source" were proposed by Harris & Priester (1962), although without a consistent data or physical support.

The problematic "second heat source" of Harris & Priester led an enthusiastic research on the energy equation which influenced the models to be commented in the next two sections. It also draw attention to the need of improved measurements of the solar EUV fluxes. It is now recognized that it reflects more the difficulties in a one-dimensional theory than actually the influence or auroral sources of energy on the low and middle latitudes.

#### 5. EMPIRICAL MODELS

Satellite drag data provided enough information about the space-time behaviour of the bulk neutral densities, and consequently their corresponding temperatures, before any reliable model could be derived from the fundamental equations. Researchers then thought to resort to mathematical techniques, like least squares fit, to adjust analytical expressions to describe the observed features, within the context of simplified physical concepts.

The basic idea to build up thermospheric empirical models is to overcome the problem created by the energy equation, which involves a lot of variables and whose results are not satisfactory. On these grounds, Jacchia proposed in 1964 to give up completely this equation and substitute it for the analytical expression:

$$T = T_\infty - (T_\infty - T_{120}) \exp[-s(z - 120)] \quad (25)$$

where 120 km was the chosen lower boundary altitude,  $T_{120}$  the temperature at this altitude,  $T_\infty$  the asymptotic (exospheric) temperature,  $s$  a height-independent parameter expressed as a function of  $T_\infty$  and  $z$  the considered altitude expressed in kilometers (see Jacchia, 1964).

The history of empirical models, however, precedes the considerations by Jacchia, since in 1951 Bates proposed a set of linearized temperature models (Bates, 1951), and the expression given by eq. (25) had already been proposed in Bates (1959). The Jacchia (1964) model was inspired by a preceding model due to Nicolet (1961), who derived the thermosphere temperatures from their effects on the

atmosphere density, deduced from the rate of change of the periods of the motion of satellites.

Static models based on eqs. (4), (22) and (25) proved to be very useful because of their easier computer processing.

The importance of the Nicolet Model (Nicolet, 1961) was the introduction of a diffusion flux to explain the distribution of minor constituents of the upper atmosphere. This flux was assumed to be:

$$\Phi_{\text{dif}} = \rho_m \mathbf{u}_d \quad (26)$$

where the subscript m refers to minor constituent and  $\mathbf{u}_d$  is the diffusion velocity relative to a reference frame moving with the air bulk velocity. That velocity was computed with the aid of eq. (9), which yields:

$$\mathbf{u}_d = (\rho_m \nu_{mn})^{-1} [\nabla p_m - \rho_m \mathbf{g}] \quad (27)$$

where  $\nu_{mn}$  is the "collision frequency" between the minor constituent and the background gas.

The Jacchia (1964) temperature model was consistent with both the Nicolet model and the Harris & Priester model. This enabled Jacchia to avoid the restriction imposed by eqs. (19) and (20) and determine a set of empirical formulas to describe the upper atmosphere variations which affects only the variable  $T_\infty$ . These expressions account for the influence of solar cycle, solar rotation, latitude, local time, season and geomagnetic activity on the thermospheric parameters.

## 6. PROGRESS IN MODELLING

Looking for the causes responsible for not obtaining agreement between the solutions of the fundamental equations and data, researchers analysed the results obtained from the Jacchia Model and concluded that eqs. (19) and (20) were not satisfied in the thermosphere. This led to an improvement of the equations actually used in thermosphere modelling with the inclusion of other important terms. The transition from elementary modelling to modern modelling is commented in this section.

The CIRA (1965) model incorporated the Harris & Priester (1962) theory and some of the Jacchia (1964) empirical relations. This model improved the description of the  $(Q - L)$  term relative to the Harris & Priester model.

Lagos & Mahoney (1967) contested the validity of neglecting horizontal transport. Dickinson & Geisler (1968) proposed meridional velocities which did not affect the continuity equation. Their complementary wind  $\mathbf{u}_c$  satisfies the equation:

$$\nabla \cdot (\rho \mathbf{u}_c) = 0 \quad (28)$$

which, neglecting higher order velocity terms, produces from eqs. (3), (16) and (17) the thermodynamic equation:

$$\partial W / \partial t = (Q - L) + (p/\rho) \mathbf{u}_c \cdot \nabla \rho + \nabla \cdot (\lambda \nabla T) \quad (29)$$

To compute the vertical velocity from eq. (28), the meridional velocities were determined from the solution of a form of eq. (2) proposed by Geisler (1967). This approach considers the earth's rotation and ion drag in eq. (9), as well as a viscosity term given by:

$$\nabla \cdot \underline{\phi}_{\text{vis}} = \eta (\partial^2 \mathbf{u}_h / \partial z^2) \quad (30)$$

where  $\mathbf{u}_h$  is the horizontal component vector of  $\mathbf{u}_c$ . The required meridional pressure gradients were computed using eq. (22) in connection with the temperature model expressed by eq. (25) and an exospheric worldwide temperature map published by Jacchia & Slowey (1967).

Equation (30) implies that the horizontal velocity be much larger than the vertical velocity and also that the vertical derivative be much larger than the horizontal derivatives. Equation (28) expresses the conservation of mass above a constant pressure surface if variations of gravity and of the spherical coordinate system metrics are neglected (Dickinson & Geisler, 1968).

The results obtained by Dickinson & Geisler (1968) did not confirm their expectation of replacing the "second heat" source of the Harris & Priester (1962) model by the simple inclusion of the adiabatic term contribution to the energy equation.

So far the proposed approaches had the inconvenience of requiring at least eight boundary conditions. This was pointed out by Volland & Mayr (1970) as the responsible source of errors. These authors proposed instead the use of a model based on an oscillatory medium where several different waves interfere. The approach was developed by Volland and co-workers in a series of papers (Volland, 1966; 1969a, b; Volland & Mayr, 1970). The wavelike approach implies a dynamical equilibrium assumption, for which a prescribed time dependence can be used.

The oscillatory approach amounts to replacing as much as possible differential terms by algebraic ones. This is possible by means of a space-time Fourier decomposition of the dependence of the parameters on longitude and time. The resulting system of differential equations depends only on the altitude and latitude.

The original two dimensional model of Volland (1966) still preserved the traditional boundary conditions of the Jacchia (1964) model. This model was modified (Volland, 1969a) such that the only

boundary parameters became two heat sources:

- a) the solar EUV heating input;
- b) the heating input produced by dissipation of tidal waves propagating from the lower atmosphere.

Further theoretical considerations were incorporated in the model by Volland (1969b) and Volland & Mayr (1970). While these models circumvented the source of errors from the boundary conditions they had the shortcoming of linearizing the hydrodynamic equations.

The simplification intrinsic to the Volland series is that the fluid is considered incompressible for the hydrodynamic equations and all terms in  $u^2$  are discarded in the energy equation. Moreover in eq. (16) only pressure gradients and ion drag are considered to affect the term  $\mathbf{u} \cdot \Delta \mathbf{r}$ .

Volland (1969b) identified four pairs of plane characteristics waves:

- a) acoustic-gravity waves;
- b) heat conduction waves;
- c) ordinary viscosity waves;
- d) extraordinary viscosity waves.

which can subsist in the thermosphere. Of these, four are upgoing waves and the other four are downgoing waves. Moreover, Volland (1969b) showed that the ion-drag and Coriolis force have the only role of driving the medium anisotropic. Acoustic gravity waves are propagating waves, the other types are evanescent waves.

The two-dimensional part of the Volland series is important because it was successful to obtaining satisfactory quantitative agreement between theoretical results and measurements. The "second heat source" of Harris & Priestner could be eliminated by the introduction of horizontal longitudinal winds.

Friedman (1967) proposed one of the preliminary approaches to a three-dimensional model. His equation of motion was simplified by neglecting friction forces and interaction with charged components of the atmosphere. He also assumed a constant boundary at 120 km and neglected any horizontal or vertical bulk velocity. These last assumptions constitute weak points in his approach. The interesting aspect of this model was its coordinate system, which is fixed with respect to the sun.

In the earlier 70's three-dimensional models received substantial improvement. However, authors improved their earlier works without changing their current trends. Thus Friedman (1970) included bulk motions neglected in his preceding formulation (Friedman, 1967). Jacchia (1971) changed the boundary altitude from 120 km to 90 km, the temperature profile from exponential to inverse tangent, and made additional refinements in both composition and the empirical formulas for the description of individual types of atmospheric variations (with solar activity, geomagnetic activity and so on). Volland & Mayr (1972a, b, c) extended

their two-dimensional theory to three dimensions, using a spherical harmonic development, and included corpuscular heating as a third boundary value. This time, however, they introduced further assumptions by replacing the differential forms of the conduction and viscosity by algebraic ones. This severely restricts the validity of their results.

A new oscillatory approach which appeared also in the early 70's was proposed by Lindzen and Blake (Lindzen, 1970; Lindzen & Blake, 1970). They introduced the concept of "equivalent gravity waves" to simulate the vertical structure of tidal modes. Their simplifications included the use of a hydrostatic pressure relation to replace the vertical momentum equation; the neglect of earth's rotation; the neglect of the kinetic energy in the energy equation. With these restriction they were able to separate the vertical from the horizontal dependence, obtaining two systems of uncoupled differential equations. They set the lower boundary condition at ground level and restricted the solutions to be bounded as  $z \rightarrow \infty$  as an upper boundary. The restriction to this approach is that it is only valid at the equator.

To some extent the works of Volland & Mayr (1972a, b, c) and Lindzen (1970) are complementary to each other. Whereas the spherical harmonic approach (Volland & Mayr, 1972a) is valid where the ion-neutral drag and the viscosity forces exceeds the Coriolis force (above 140 km), an improved version of the "equivalent gravity waves" approach (Forbes & Hagan, 1979) constitutes a satisfactory approximation for the lower thermosphere (100-140 km).

CIRA (1972) expressed the trust of the scientific community still in the empirical formulas revised by Jacchia (1971). No theoretical model contributed to this CIRA version. Nevertheless, the theoretical works mentioned in this section are important to the extent they established a satisfactory system of equations to be used in modelling and, moreover, showed the importance of spherical harmonic development to describe the latitudinal behaviour of the thermosphere.

## 7. PROGRESS OF RECENT MODELS

The availability of large data bases, resulting from in situ satellite measurements of various thermospheric parameters and remote probing to the thermosphere by incoherent scatter technique, led to the development of a new generation of thermospheric models. We call them here modern models and comment on their basic trends. Their common aspect is the emphasis on a detailed study of the thermospheric dynamics in an attempt to explain through composition-dynamics interaction what could not be explained by bulk thermospheric characteristics.

Hedin et al. (1974) started a new series of empirical models following the same trend of Volland & Mayr (1972a, b, c). They assumed that the solution

of the hydrodynamics equations can be divided into two parts: static and dynamical. The static part consists of Bates temperature profile model to substitute the energy equation (like in Jacchia series) and the solution of the barometric equation (eq. (22)) to determine the corresponding pressures and densities. The dynamical part consists in expressing  $T_\infty$ ,  $T_{120}$  and  $s$  by: a spherical harmonic development to represent the latitudinal variations, a sinusoidal series to describe temporal and longitudinal variations and a polynomial series to account for solar/magnetic activity variations. The coefficients of the resulting expansion functions are determined by least squares fitting with experimental data (Hedin, 1983). Diffusive equilibrium was assumed to describe atmospheric composition.

Although spherical harmonics are not the actual solution of the hydrodynamic equations in the range 100–400 km, one can show that Hough functions can be developed in spherical harmonic components (Siebert, 1961). Therefore, the final solution of the hydrodynamic equations can be expressed by a spherical harmonic series without any intrinsic theoretical inconsistency.

The static solution of the Jacchia (1977) model uses a  $\tan^{-1}$  temperature profile instead of the Bates profile employed by Hedin et al. (1977). The J77 version uses still the same empirical formulas of his earlier models to describe the dynamical behaviour of the thermosphere.

An interesting fully analytical static model was proposed by Alcayde (1981). It accounts also for the mesopause temperature variation as a function of both thermospheric and mesospheric heat input. It is intended to provide an alternative model to be used in problem where both thermospheric and mesospheric parameters are necessary (for instance, tidal studies).

In 1975 Dickinson and co-workers started a new series of papers based on the numerical integration of the complete set of the basic equations (Dickinson et al., 1975, 1977, 1981). Numerical integration of these equations has also been pursued more recently by Fuller-Rowell and co-workers in another series (Fuller-Rowell & Rees, 1980, 1981, 1983; Fuller-Rowell, 1984; Smith et al., 1982). In both series the intrinsic simplifications were the use of an incompressible fluid formulation and the neglect of the  $u^2$  terms in the energy equation.

The Dickinson series constitutes an alternative approach to simulate dynamics using the basic equations. The approach is also based on the perturbation theory. These models do not use a prescribed time dependence, like the Volland models, but instead determine the time variation of the atmospheric parameters from the solution of the governing equations subjected to the known driving energy and momentum sources. The static reference is taken from the Hedin models. The considered energy

sources are the solar EUV and UV radiation and particle precipitation. The considered momentum source is ion drag. The boundary conditions assumed are vanishing perturbations at mesopause altitudes and vanishing vertical gradient of the perturbations at exospheric altitudes.

Two shortcomings of Dickinson initial works were:

- a) poor account of auroral energy sources;
- b) no account for the energy source resulting from atmospheric tides propagating from the lower atmosphere.

The first problem was reconsidered in the most recent papers of the series (Roble et al., 1982, 1983).

To compute the thermospheric composition, three major constituents: atomic oxygen, molecular oxygen and molecular nitrogen are considered in the Dickinson series. These species are assumed to be in thermal equilibrium, which reduces the problem to the simultaneous solution of the continuity equation and the momentum equation for all three constituents, with the need of matrix methods to determine the final solution (Dickinson et al., 1972, 1984).

The Fuller-Rowell series deals also with numerical simulation of the dynamical behaviour of the thermosphere. It is similar to the Dickinson approach as far as the time dependence is concerned. The statical reference is taken from the Jacchia models. The energy sources considered are solar UV and EUV radiation and particle precipitation. The momentum source considered is ion drag. The boundary conditions are vanishing perturbations at 80 km altitude and vanishing vertical gradient of the perturbations at exospheric altitude.

The Fuller-Rowell series also did not account for the energy delivered by atmospheric oscillations propagating from the lower atmosphere.

In the Fuller-Rowell series all neutral species are assumed to be in thermal equilibrium. They are divided into two categories: light and heavy species. This reduce the problem to the solution of one conservation equation for the mean molecular mass and two equations of motion to determine the two constituents velocities (Fuller-Rowell & Rees, 1983).

Forbes (1982a, b) undertook the matter of continuing and improving the Volland approach. His method uses the complete set of hydrodynamic equations for an incompressible fluid with no  $u^2$  terms in the energy equation. The driving mechanisms considered for the tidal oscillations were the solar UV and EUV radiation. Integration was carried out from ground level up to 400 km altitude. The perturbations on the atmospheric parameters were assumed to vanish at the lower boundary and the vertical gradient of these perturbations were considered to vanish at the upper boundary. Mean zonal winds as well as meridional temperature gradients and a detailed account for damping mechanisms were also

considered. No account was made of high latitude energy sources. The static solution considered was that of Hedlin models.

A shortcoming of Forbes series is the neglect of appropriate auroral sources. This may represent a considerable factor during geomagnetic storms, as emphasized by the works of Straus (Straus & Schulz, 1976; Straus, 1978).

The Forbes series considers the minor constituent diffusion model of Nicolet (see eqs. (26) and (27)) to explain the composition of all neutral species, except molecular nitrogen which is the major specie.

Another attempt to model the thermospheric dynamics came from Creekmore et al. (1975) and was pursued more recently by Fontanari et al. (1982). This is an improved version of the Blum & Harris (1975) approach and determines the time variation of density and winds for a prescribed temperature field. The shortcoming of this approach, as discussed by Fuller-Rowell & Rees (1980), is the use of an empirical temperature field which is unable to reproduce a transient situation which occurs, for instance, during a magnetic storm.

In this section, the three basic options, available for users of thermospheric models, were well identified. Thus:

- a) Empirical models may be used when one seeks the average value of the thermospheric parameters expected for the actually existing thermospheric conditions.
- b) Spectral analysis models can be chosen when the regular wavelike behaviour of the thermosphere is under consideration.
- c) Theoretical models are preferred when transient responses, of the thermosphere, to unpredicted excitation actually occurred.

## 8. DISCUSSION

If people once thought about a model to predict the thermospheric behaviour exactly, they now see how far we are from this expectation. The reasons are manifold, starting with the heating efficiency of the solar UV and EUV radiation and going to the statistical nature of particle precipitation and their subsequent behaviour.

The thermospheric models proposed so far are either incomplete as far as the equations are concerned, or else incomplete in the consideration of all important sources of momentum and energy. Empirical models, which are very reliable, are only valid for steady state situations.

Three questions arise relative to thermospheric modelling:

- a) Which system of equations is an actually appropriate system to describe the thermospheric behaviour?
- b) What sources and sinks need to be considered as

effective in the thermosphere?

- c) What type of solution is more convenient to describe thermospheric dynamics?

Regarding the first question, a significant theoretical work was presented by Schunk (1975). In practice, two basic guidelines have been attempted:

- I) to treat the coupled system of equations for both neutral and ionized particles (Stubbe, 1970);
- II) to consider the system of equations for neutral particles only and assume ionized particle models.

The first approach implies a considerable computational work and was not pursued further on. The second needs the definition of a "heating efficiency" which was considered in great detail recently by Torr et al. (1980). Ionized particle models, however, do not reproduce transient situations.

As far as the second question is concerned, considerable work was done by Torr et al. (1980) relative to solar UV and EUV, by Fuller-Rowell and co-workers relative to high latitude energy sources and by Groves (1983a, b) and Groves & Forbes (1984) relative to the energy delivered by tidal oscillations.

The third question is also an open question. Two basic approaches survived in recent modelling:

- 1) spectral analysis to determine the effective oscillatory modes in the atmospheric space time dynamics;
- 2) a time stepping procedure solution to account for any arbitrary time dependence of the sources of energy and momentum.

A recent effort on the first approach was done by Forbes (1982a, b). The second approach received substantial improvements with the works of Roble et al. (1982, 1983). Although the first approach is more appropriate to dynamical equilibrium situations, it cannot be ruled out since the regular input of energy is periodic in time anyway.

Empirical models have long been considered as more reliable than theoretical models to describe the regular behaviour of the thermosphere. The reason is that they overcome, at least, the problems of our second question.

A back and forth interactive procedure to treat the coupled system of equations for ionized and neutral particles is now possible using parallel type computation. This constitutes an alternative to extend the idea proposed by Stubbe (1970) which has not been explored so far.

## 9. CONCLUSION

In this work we briefly reviewed the most significant guidelines that thermosphere modelling have followed since the remarkable work of Harris & Priester (1962). We focussed on the building up procedure on which more and more terms were included until the model equations approach the theoretical basic equations.



Although considerable progress has been made, there is not still a complete model including all term of the theoretical equations as well as all source mechanisms. Empirical models are to date the most reliable reference for the dynamical equilibrium behaviour of the thermosphere since the proposed three-dimensional time dependent models are still incomplete as far as momentum and energy balances are concerned.

Theoretical models continue being improved and

soon will be complete. Nevertheless, they are impractical to most of the users since they require huge computer facilities and large computer time to be used. At present we classify them as "state of art" models.

#### ACKNOWLEDGEMENT

This research was supported by FINEP at the Instituto de Pesquisas Espaciais - INPE.

#### REFERENCES

- ALCAYDE, D. - 1981 - Analytical static model of temperature and composition from 20 to 2000 km altitude. *Ann. Geophys.*, **37**: 515-528.
- BATES, D.R. - 1951 - The temperature of the upper atmosphere. *Proc. Phys. Soc.*, **B64**: 805-821.
- BATES, D.R. - 1959 - Some problems concerning the terrestrial atmosphere above 100 km level. *Proc. Roy. Soc. London*, **A253**: 451-462.
- BLUM, P.W. & HARRIS, I. - 1975 - Full non-linear treatment of the global thermospheric wind system. 1. Mathematical method and analysis of forces. *J. Atmos. Terr. Phys.*, **37**: 193-212.
- CIRA - 1965 - *Cospar International Reference Atmosphere*. North Holland Pub. Co., Amsterdam.
- CIRA - 1972 - *Cospar International Reference Atmosphere*. Akademik Verlag-Berlin.
- CREEKMORE, S.P.; STRAUS, J.M.; HARRIS, R.M.; CHING, B.K. & CHIU, Y.T. - 1975 - A global model of thermospheric dynamic. I. Wind and density field derived from phenomenological temperature. *J. Atmos. Terr. Phys.*, **37**: 491-515.
- DICKINSON, R.E. & GEISLER, J.E. - 1968 - Vertical motion field in the middle thermosphere from satellite drag densities. *Mon. Wea. Rev.*, **96**: 606-616.
- DICKINSON, R.E. & RIDLEY, R.E. - 1972 - Numerical solution for the composition of a thermosphere in the presence of a steady subsolar-to-antisolar circulation with application to Venus. *J. Atmos. Sci.*, **29**: 1557-1570.
- DICKINSON, R.E.; RIDLEY, E.C. & ROBLE, R.G. - 1975 - Meridional circulation in the thermosphere. I. Equinox conditions. *J. Atmos. Terr. Phys.*, **32**: 1737-1754.
- DICKINSON, R.E.; RIDLEY, E.C. & ROBLE, R.G. - 1977 - Meridional circulation in the thermosphere. II. Solstice conditions. *J. Atmos. Sci.*, **34**: 178-192.
- DICKINSON, R.E.; RIDLEY, E.C. & ROBLE, R.G. - 1981 - A three-dimensional general circulation model of the thermosphere. *J. Geophys. Res.*, **86**: 1499-1512.
- DICKINSON, R.E.; RIDLEY, E.C. & ROBLE, R.G. - 1984 - Thermosphere general circulation with coupled dynamics and composition. *J. Atmos. Terr. Phys.*, **41**: 205-219.
- FONTANARI, J.D.; ALCAYDE, D.; AMAYENC, P. & KOCKARTS, G. - 1982 - Simulations numeriques tridimensionnelles de la circulation à grande echelle induite par des modeles globaux de thermosphere. *Ann. Geophys.*, **38**: 815-840.
- FORBES, J.M. & HAGAN, M.E. - 1979 - Tides in the joint presence of friction and rotation: an f plane approximation. *J. Geophys. Res.*, **84**: 803-810.
- FORBES, J.M. - 1982a - Atmospheric tides. 1. Model description and results for the solar diurnal component. *J. Geophys. Res.*, **87**: 5222-5240.
- FORBES, J.M. - 1982b - Atmospheric tides. 2. The solar and lunar semidiurnal components. *J. Geophys. Res.*, **87**: 5241-5252.
- FRIEDMAN, M.P. - 1967 - A three-dimensional model of the upper atmosphere. *SAO, Special Report*, 250.
- FRIEDMAN, M.P. - 1970 - Upper atmosphere dynamics. *SAO, Special Report*, 316.
- FULLER-ROWELL, T.J. & REES, D. - 1980 - A three-dimensional time-dependent global model of the thermosphere. *J. Atmos. Sci.*, **37**: 2545-2567.
- FULLER-ROWELL, T.J. & REES, D. - 1981 - A three-dimensional time-dependent simulation of the global dynamical response of the thermosphere to a geomagnetic substorm. *J. Atmos. Terr. Phys.*, **43**: 701-721.
- FULLER-ROWELL, T.J. & REES, D. - 1983 - Derivation of a conservative equation for mean molecular weight for a two constituent gas within a three-dimensional time-dependent model of the thermosphere. *Planet. Space Sci.*, **31**: 1209-1222.
- FULLER-ROWELL, T.J.; REES, D.; QUEGAN, S.; BAILEY, G.J. & MOFFETT, R.J. - 1984 - The effect of realistic conductivities on the high-latitude thermospheric circulation. *Planet. Space Sci.*, **32**: 469-480.
- GEISLER, J.E. - 1967 - A numerical study of the wind system in the middle thermosphere. *J. Atmos. Terr. Phys.*, **29**: 1469-1482.
- GLEESON, L.J. & AXFORD, W.I. - 1967 - Electron and ion temperature variations in temperature zone sporadic E layer. *Planet. Space Sci.*, **15**: 123-136.
- GRAD, H. - 1958 - Principle of the kinetic theory of gases. *Hand. Phys.*, **XII**, 205-294.
- GROVES, G.V. - 1983a - Energy fluxes of the (1, 1, 1) atmospheric oscillation. *Planet. Space Sci.*, **31**: 67-71.
- GROVES, G.V. - 1983b - Thermospheric energy flux of the semidiurnal tide. *Planet. Space Sci.*, **31**: 1183-1186.
- GROVES, G.V. & FORBES, J. - 1984 - Equinox tidal heating of the upper atmosphere. *Planet. Space Sci.*, **32**: 447-456.
- HARRIS, I. & PRIESTER, W. - 1962 - Time-dependent structure of the upper atmosphere. *J. Atmos. Sci.*, **9**: 286-301.
- HEDIN, I. - 1983 - A revised thermospheric model based on mass spectrometer and incoherent scatter data: MSIS-83. *J. Geophys. Res.*, **88**: 10170-10188.
- HEDIN, A.E.; MAYR, H.G.; REBER, C.A.; SPENCER, N.W. & CARIGNAN, G.R. - 1974 - Empirical model of global thermospheric temperature and composition based

- on data from the OGO6 quadrupole mass spectrometer. *J. Geophys. Res.*, **79**: 215-225.
- HEDIN, A.E.; SALAH, J.E.; EVANS, J.V.; REBER, C.A.; NEWTON, G.P.; SPENCER, N.W.; KAYSER, D.C.; ALCAYDE, D.; BANER, P.; COGGER, L. & McCLURE, J.P. - 1977 - A global thermospheric model based on mass spectrometer and incoherent scatter data: MSIS,IN density and temperature. *J. Geophys. Res.*, **82**: 2139-2147.
- JACCHIA, L.G. - 1964 - Static diffusion models of the upper atmosphere with empirical temperature, profiles. SAO, Special Report, 170.
- JACCHIA, L.G. - 1971 - Revised static models of the thermosphere and exosphere with empirical temperature profiles. SAO, Special Report, 332.
- JACCHIA, L.G. - 1977 - Thermospheric temperature density ad composition: new models. SAO, Special Report, 375.
- JACCHIA, L.G. & SLOWEY, J. - 1967 - The shape and location of the diurnal bulge in the upper atmosphere. Space Research VII, North Holland, Amsterdam.
- LAGOS, C.P. & MAHONEY, J.R. - 1967 - Numerical studies of seasonal and latitudinal variability in a model thermosphere. *J. Atmos. Sci.*, **24**: 88-94.
- LANDAU, L.; LIFCHITZ, E. - 1971 - *Mécanique des fluides*. Mir, Moscou.
- LINDZEN, R.S. - 1970 - Internal gravity waves in atmosphere with realistic dissipation and temperature. I. Mathematical development and propagation of waves into the thermosphere. *Geophys. Fluid Dyn.*, **1**: 303-355.
- LINDZEN, R.S. & BLAKE, D. - 1970 - Mean heating of the thermosphere by tides. *J. Geophys. Res.*, **75**: 6868-6871.
- NICOLET, M. - 1960a - The properties and constitution of the upper atmosphere. *Physics of the upper atmosphere*. Academic Press, New York, 17-71.
- NICOLET, M. - 1960b - Les variations de la densité et du transport de chaleur par conduction dans l'atmosphère supérieure. Space Research 1, Amsterdam, North Holland, Publ. Co., 46-89.
- NICOLET, M. - 1961 - Density of the heterosphere related to temperature. SAO, Special Report, 75.
- RISHBETH, H. & GARRIOTT, O.K. - 1969 - Introduction to ionospheric physics. Academic Press, New York.
- ROBLE, R.G.; DICKINSON, R.E. & RIDLEY, E.C. - 1982 - The global circulation and temperature structure of thermosphere with high-latitude plasma convection. *J. Geophys. Res.*, **87**: 1599-1614.
- ROBLE, R.G.; DICKINSON, R.E.; RIDLEY, E.C. & EMERY, B.A.; HAYS, P.B.; KILLEEN, T.L. & SPENCER, N.W. - 1983 - The high-latitude circulation and temperature structure of the thermosphere near solstice. *Planet. Space Sci.*, **31**: 1479-1499.
- SCHUNK, R.W. - 1975 - Transport equations for aeronomy. *Planet. Space Sci.*, **23**: 437-485.
- SIEBERT, M. - 1961 - Atmospheric tides. *Advan. Geophys.*, **7**: 105-187.
- SMITH, M.F.; REES, D. & FULLER-ROWELL, T.J. - 1982 - The consequence of high-latitude particle precipitation on global thermospheric dynamics. *Planet. Space Sci.*, **30**: 1259-1267.
- STRAUS, J.M. & SCHULTZ, M. - 1976 - Magnetospheric convection and upper atmospheric dynamics. *J. Geophys. Res.*, **81**: 5822-5832.
- STRAUS, J.M. - 1978 - Dynamics of the thermosphere at high latitudes. *Rev. Geophys. Space Phys.*, **16**: 183-194.
- STUBBE, P. - 1970 - Simultaneous solution of the time dependent coupled continuity equations heat conduction equations, and equations of motion for a system consisting of a neutral gas, and electron gas, and a four ion gas. *J. Atmos. Terr. Phys.*, **32**: 865-903.
- TORR, M.R.; TORR, D.G. & RICHARDS, P.G. - 1980 - The solar ultraviolet heating efficiency of the midlatitude thermosphere. *Geophys. Res. Lett.*, **7**: 373-376.
- VOLLAND, H. - 1966 - A two-dimensional dynamic model of the diurnal variation of the thermosphere. Part I: theory. *J. Atmos. Sci.*, **23**: 799-807.
- VOLLAND, H. - 1969a - A theory of thermospheric dynamics. I. Diurnal and solar cycle variations. *Planet. Space Sci.*, **17**: 1581-1597.
- VOLLAND, H. - 1969b - The upper atmosphere as a multiply refractive medium for neutral air motions. *J. Atmos. Terr. Phys.*, **31**: 491-514.
- VOLLAND, H. & MAYR, H. - 1970 - A theory of the diurnal variations of the thermosphere. *Ann. Geophys.*, **26**: 907-919.
- VOLLAND, H. & MAYR, H. - 1972a - A three-dimensional model of thermosphere dynamics. I. Heat input and eigenfunctions. *J. Atmos. Terr. Phys.*, **34**: 1745-1768.
- VOLLAND, H. & MAYR, H. - 1972b - A three-dimensional model of thermosphere dynamics. II. Tidal waves. *J. Atmos. Terr. Phys.*, **34**: 1769-1799.
- VOLLAND, H. & MAYR, H. - 1972c - A three-dimensional model of thermosphere dynamics. III. Planetary waves. *J. Atmos. Terr. Phys.*, **34**: 1797-1816.

Versão recebida em: 30/08/89

Versão revista e aceita em: 22/03/90

Editor Associado: M.A.F.S. Dias