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# SEMIEMPIRICAL MODEL FOR CALCULATING THE CRITICAL FREQUENCY OF THE F – REGION OVER CONCEPCION

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**Daily variations of the F-region critical frequency observed at Concepción (36.8°S, 73°W), representative of Winter, Equinox and Summer conditions for both low and high solar activity levels are given. A semiempirical model produced using these data is then proposed giving seasonal median critical frequencies for each hour, season, and low and high solar activity level. The model is based on a servomechanism model published in the late sixties which has recently been shown to be of value. Simple quantitative expressions for photochemical, diffusive and transport dependencies of the peak electron concentration are assumed in the proposed model. Model results are compared with both corresponding measured data and CCIR model.**

**MODELO SEMI-EMPÍRICO PARA O CÁLCULO DA FREQÜÊNCIA CRÍTICA DA REGIÃO–F SOBRE CONCEPCIÓN** – **Consideram-se as variações diárias da freqüência crítica da região–F observadas em Concepción (36,8°S, 73°W) como representativas das condições de inverno, equinócio e verão, para níveis de atividade solar baixo e alto. Utilizando esta base de dados propõe-se um modelo semi-empírico que permite calcular freqüências críticas medianas para cada hora, estação do ano, e para níveis de atividade solar baixo e alto. O modelo se fundamenta num modelo de servomecanismo proposto no fim dos anos sessenta que tem se mostrado bom nestes últimos tempos. Supõe-se no modelo proposto expressões quantitativas simples para as influências fotoquímica, difusiva e de transporte, na concentração eletrônica máxima. Os resultados do modelo se comparam com dados medidos e com dados do modelo CCIR.**

## 1. INTRODUCTION

Models to determine the peak electron concentration at a specific place (latitude and longitude) and at a particular time (hour of the day, season and solar activity level), have been known for a long time (e.g. Berkey et al., 1987).

The formulation of these models is usually based on an atmospheric model giving concentration and temperature of the neutral components and on a certain model concerning the nature of ionizing radiation. However generally the specifications needed to apply these atmospheric and radiation models are quite complex. Thus it seems reasonable to reconsider simple conceptual models to determine electron concentration, based on simple chemical and physical processes, as it has been recently suggested by Rishbeth (1986).

In this work the latter approach has been chosen to determine the peak electron concentration over Concepción, for each season and for low and high solar activity levels, taking as reference the servomechanism model originally suggested by Risbeth (1967). The criteria used to define the observational data base are shown in section 2. The proposed semiempirical model is formulated in section 3.

## 2. OBSERVATIONAL DATA BASE

In this work values of the F-region critical frequency measured in Concepción for each hour of each day from 1958 to 1980 have been used. Values were read from ionograms using international scaling and accuracy rules as described by Piggott & Rawer (1972, 1978). As for a fixed solar activity level, daily variations of the monthly median values corresponding to months of a given season show regularities in their shapes, amplitudes and times of occurrence of a minimum value, critical frequency values have been grouped for each hour and for each season. Defined groups include Winter (May, June, July and August), Equinox (March, April, September and October) and Summer (November, December, January and February) at low and high solar activity levels. According to details specified in Tab. 1, twelve months are included for each season in both solar activity levels. The median value of the corresponding

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**Table 1.** Data used to determine values of seasonal-monthly median critical frequency at a given hour.

| Low solar activity level $0 < R12 < 20$ | | | | | High solar activity level $100 < R12 < 180$ | | | | |
|---|---|---|---|---|---|---|---|---|---|
| Summer | November | 1963 | 1964 | 1965 | Summer | November | 1958 | 1969 | 1979 |
| | December | 1963 | 1964 | 1965 | | December | 1958 | 1969 | 1979 |
| | January | 1964 | 1965 | 1966 | | January | 1959 | 1970 | 1980 |
| | February | 1964 | 1965 | 1966 | | February | 1959 | 1970 | 1980 |
| Equinox | March | 1963 | 1964 | 1976 | Equinox | March | 1959 | 1970 | 1980 |
| | April | 1963 | 1964 | 1976 | | April | 1959 | 1970 | 1980 |
| | September | 1963 | 1964 | 1976 | | September | 1959 | 1970 | 1980 |
| | October | 1963 | 1964 | 1976 | | October | 1959 | 1970 | 1980 |
| Winter | May | 1963 | 1964 | 1976 | Winter | May | 1959 | 1970 | 1980 |
| | June | 1963 | 1964 | 1976 | | June | 1959 | 1970 | 1980 |
| | July | 1963 | 1964 | 1976 | | July | 1959 | 1970 | 1980 |
| | August | 1963 | 1964 | 1976 | | August | 1959 | 1970 | 1980 |

frequency distribution for each group, which includes approximately 30 x 12 = 360 values for a given hour is considered as representative of the critical frequency for that hour.

The daily variations of these median values are adopted as the data base to model the peak electron concentration. The daily variations for Winter, Equinox and Summer at low and high solar activity levels are shown in Figs. 5 and 6. All variations have a minimum value at dawn and a peak between 12:00 and 15:00 hours. In particular, the hour of the minimum values changes in phase with the time of sunrise in all the cases.

A study of the standard deviation of the median values indicates that they are larger for Equinox both at low and high solar activity levels. These tend to occur in the afternoon and evening hours, respectively. Vertical bars representing the largest standard deviation for each season and activity level are also given in the Figs. 5 and 6.

## 3. MODEL

Peak electron concentration at a given hour is assumed to be the addition of two terms: one related to photochemical and diffusive processes ($N_m$), modelled from first principles, and the other associated to collisional drag transport ($\Delta N_m$), here considered as a perturbation to the first term, and determined empirically by comparison of $N_m$ to appropiate values of the observational data base. The first term is derived by numerical solution of the continuity equation with photochemical and diffusive contributions, valued at the height of the peak electron concentration:

$$(dN_m/dt) = q - L_Q - L_d)$$

where the rate of change of the peak electron concentration, $dN_m/dt$, depends on:

q = rate of production, assumed to be the Chapman production function. This function corresponds to a spherically stratified isothermic atmosphere, with a single ionizable component, whose concentration decreases exponentially with the height. The atmosphere behaves as an ideal gas and is ionized with a monochromatic solar radiation.

$L_Q$ = rate of change of chemical loss, assumed to depend on recombination and charge transference reactions only. This rate is supposed to be proportional to the existing electron concentration, that is, $L_Q = \beta N_m$, in an hydrostatic equilibrium atmosphere, formed by atomic oxygen and molecular nitrogen. $\beta$ varies with height as $\beta = \beta_o \exp(-Kz_m)$, where K is fixed. K specifies the vertical gradient of $\beta$, its value for the case of $\beta$ being proportional to the $N_2$ concentration is 1.75 (Rishbeth, 1967). $z_m$ is the reduced height according to the atomic oxygen scale height, at the peak electron concentration.

$L_d$ = rate of change of the electron concentration by diffusive transport, related to pressure vertical gradients which is supposed to be also proportional to the existing electron concentration. Thus $L_d = d N_m$, in which d can be interpreted as an equivalent coefficient of diffusive loss and varies exponentially with the height according to $d = d_o \exp(z_m)$. At the height $z_m$, the rate of change $L_d$ is usually comparable to the rate of chemical loss $L_Q$, so that both can be included in a single loss term, $L_{Q,d} = c_N \beta N_m$, where $c_N$ is constant.

Specifically, the equation to be solved is:

$$(dN_m/dt) = q_o \exp (1 - z_m - \exp (- z_m) \mathrm{Ch} (X)) - c_N N_m \beta_o \exp (- K z_m)$$

where $q_o$ is the rate of production for vertical incidence of the ionizing radiation at the height of the peak production, and Ch (X) is the grazing Chapman function. The height $z_m$ varies during the 24 hours, in three periods (Rishbeth, 1967):

1. Sunrise period $z_m = \ln \mathrm{Ch} (X) + c_{zl}$
2. Diurnal period $z_m = (1/(K + 1)) \ln (\beta_o/(d_o/L_e))$
3. Night period $z_m = (1/(K + 1)) \ln (\beta_o/(d_o/L_s))$

**Table 2.** Proposed model constants used for every season and solar activity levels.

| K | $\beta_o$ ($10^{-3}$ s$^{-1}$) | $d_o$ ($10^{-6}$ s$^{-1}$) | $L_e$ | $L_s$ | Sunrise $c_N$ | Day $c_N$ | Night $c_N$ | $c_{z1}$ |
|---|---|---|---|---|---|---|---|---|
| 1.75 | 9.0 | 9.0 | 0.80 | 0.15 | 1.25 | 1.25 | 1.60 | 0.25 |

The constants $\beta_o$, $d_o$, $c_N$, $c_{zl}$, $L_e$, $L_S$, consigned in Tab. 2 are also taken from Rishbeth (1967). The process of integration begins at a time $t_o$ where the contribution to the change of the concentration associated to collisional drag transport is assumed to be zero. For each season $t_o$ is determined according to the dynamics described by Kohl & King (1967). The initial value $N_o$ is determined from the observed critical frequency corresponding to the time $t_o$. A value for $q_o$ is chosen by iteration process, so that the $N_m$ value for the time $t = t_o + 24$ h differs at the most by 2% of the initial value $N_o$. The adopted values for $t_o$, $q_o$, and $N_o$ are listed in Tab. 3. Daily variations of the critical frequencies, as determined from model peak electron concentrations calculated with photochemical and diffusive contributions only, are shown in Figs. 1 and 2.

Values of the second term model, $\Delta N_m$ are computed as

$$\Delta N_m = 1.24 \times 10^4 [C_o + C_1 \cos ((2\pi/24)\, t - \phi_1) + C_2 \cos ((2\pi/12)\, t - \phi_2)]^2$$

**Table 3.** Proposed model initial values of $q_o$, $t_o$ and $N_o$ relating photochemical and diffusive contributions.

| | Low solar activity level | | | High solar activity level | | |
|---|---|---|---|---|---|---|
| | Winter | Equinox | Summer | Winter | Equinox | Summer |
| $q_o$ (cm$^{-3}$ s$^{-1}$) | 389 | 528 | 625 | 1528 | 1833 | 1764 |
| $t_o$ (LT hour, 75°W) | 10 | 9 | 7 | 10 | 9 | 7 |
| $N_o$ ($10^5$ cm$^{-3}$) | 3.60 | 5.22 | 4.74 | 14.40 | 18.08 | 13.35 |

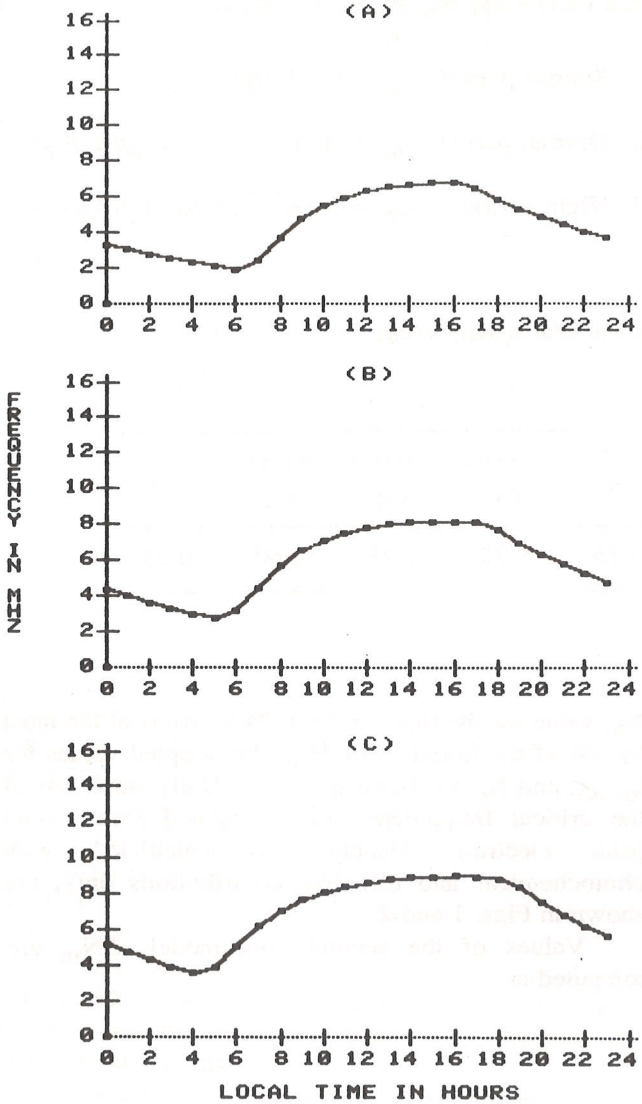


**Figure 1.** Daily variation of critical frequency determined from model peak electron concentrations calculated with photochemical and diffusive contributions only. Low solar activity level. (A) Winter; (B) Equinox; (C) Summer.

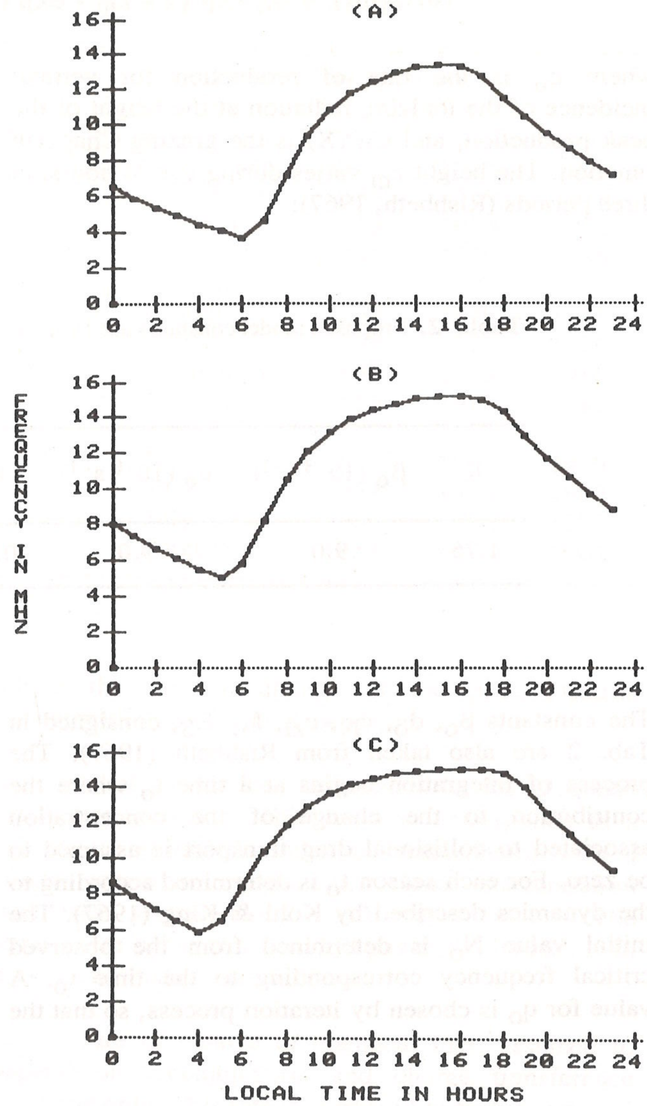


**Figure 2.** Daily variation of critical frequency determined from model peak electron concentrations calculated with photochemical and diffusive contributions only. High solar activity level. (A) Winter; (B) Equinox; (C) Summer.

where the amplitude coefficients $C_o$, $C_1$ and $C_2$, and the phases angles $\phi_1$ and $\phi_2$ are given in Tab. 4. Daily variations of the differences between critical frequencies, determined from model peak electron concentrations calculated with photochemical and diffusive contributions only, and those corresponding to the observational data base are compared with daily variations of $(\Delta N_m/1.24 \times 10^4)^{1/2}$ in Figs. 3 and 4. Although $\Delta N_m$ has been modelled using only two harmonics functions of 24 and 12 hours periods, respectively, the afforded fit is regarded as adequate.

Critical frequencies calculated using both terms of the proposed model ($N_m + \Delta N_m$) are compared with those corresponding to the observational base in Figs. 5 and 6.

## 4. CONCLUSION

In general, calculated values with the model and observed values are very similar. Largest differences occur in Winter and Equinox, for both solar activity levels. They range from 10 to 20%. For 62% of the cases, the positive and negative differences, symmetrically distributed, are smaller than 0.25 MHz (see Fig. 7a).

The critical frequency values calculated with CCIR model (1967) are systematically greater than the observed values. The differences from observed values show a greater dispersion than those corresponding to the model here proposed. Only 33% of the cases show differences smaller than 0.25 MHz (see Fig. 7b).

**Table 4.** Proposed model amplitude coefficients, $C_0$, $C_1$, $C_2$ and phase angles $\phi_1$ and $\phi_2$.

| | Low solar activity level | | | High solar activity level | | |
|---|---|---|---|---|---|---|
| | Winter | Equinox | Summer | Winter | Equinox | Summer |
| $C_0$ (MHz) | -0.5 | -0.3 | -0.1 | -1.0 | 0.0 | -1.0 |
| $C_1$ (MHz) | 1.2 | 1.1 | 0.8 | 2.0 | 1.6 | 3.1 |
| $C_2$ (MHz) | 0.1 | 0.6 | 0.8 | 0.5 | 0.3 | 1.0 |
| $\phi_1$ (degrees) | 102.74 | 118.58 | 47.32 | 95.28 | 62.36 | 51.64 |
| $\phi_2$ (degrees) | 127.49 | 57.89 | 74.16 | 258.45 | 39.36 | 52.53 |

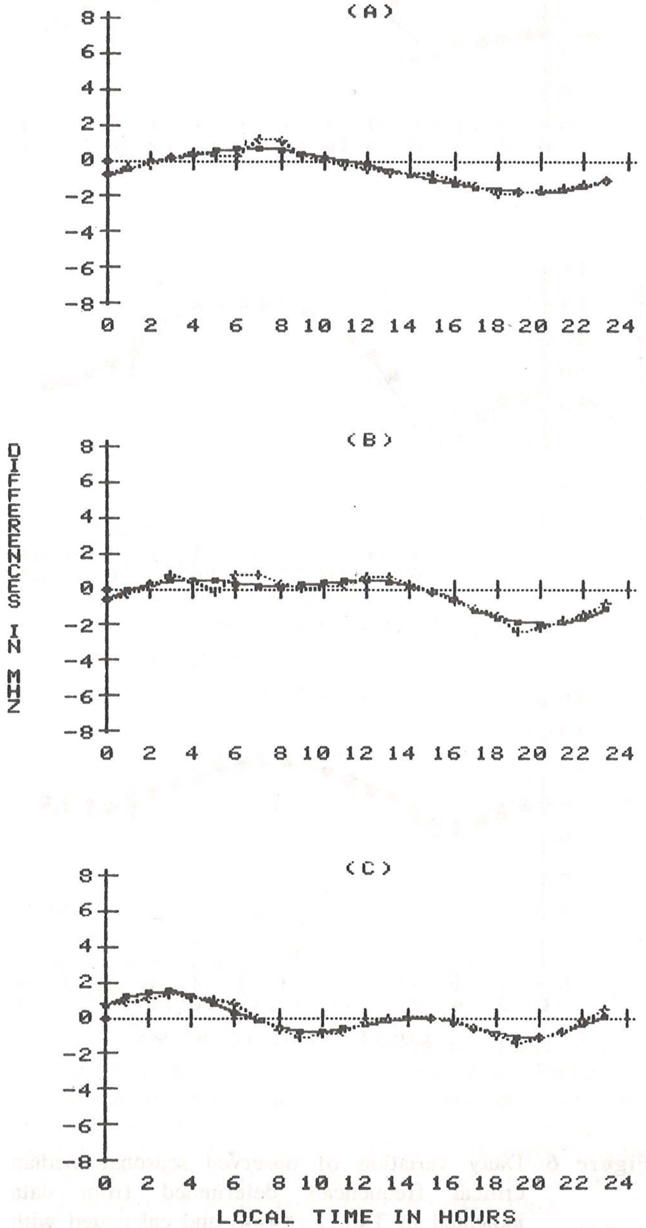


**Figure 3.** Daily variations of the differences between observed seasonal median critical frequencies and those determined from model peak electron concentrations calculated with photochemical and diffusive contributions only (- - - -). Modelled contribution associated to collisional drag transport (see text) (——). Low solar activity level. (A) Winter; (B) Equinox; (C) Summer.

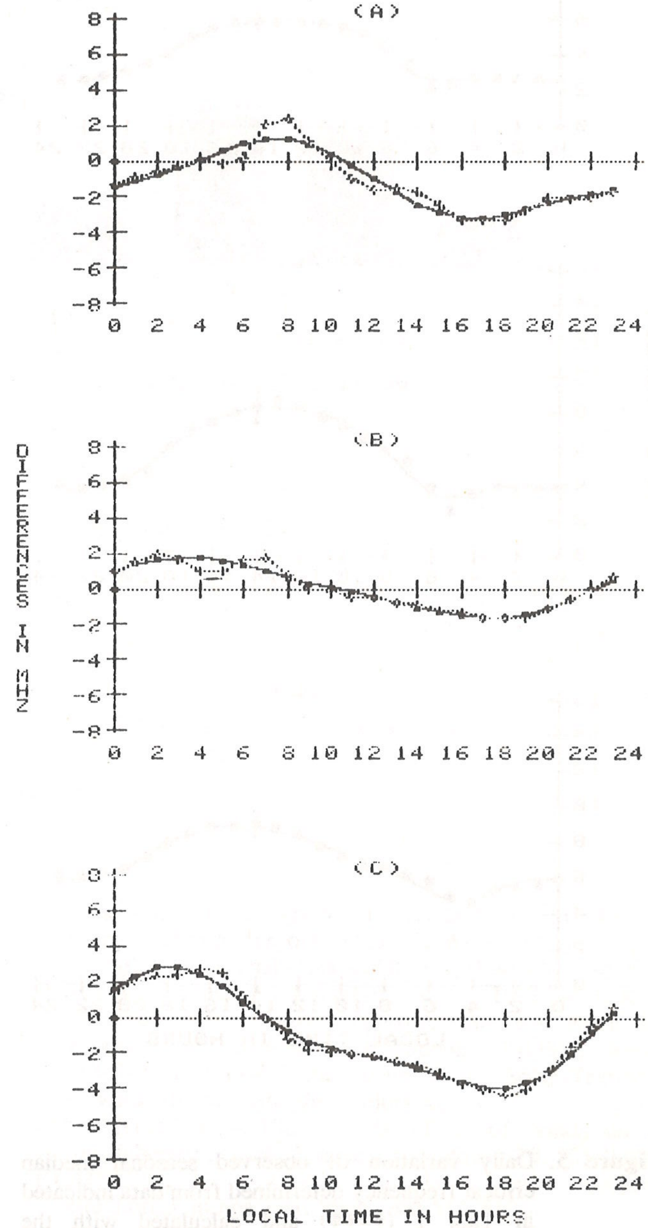


**Figure 4.** Daily variations of the differences between observed seasonal median critical frequencies and those determined from model peak electron concentrations calculated with photochemical and diffusive contributions only (- - - -). Modelled contribution associated to collisional drag transport (see text) (——). High solar activity level. (A) Winter; (B) Equinox; (C) Summer.

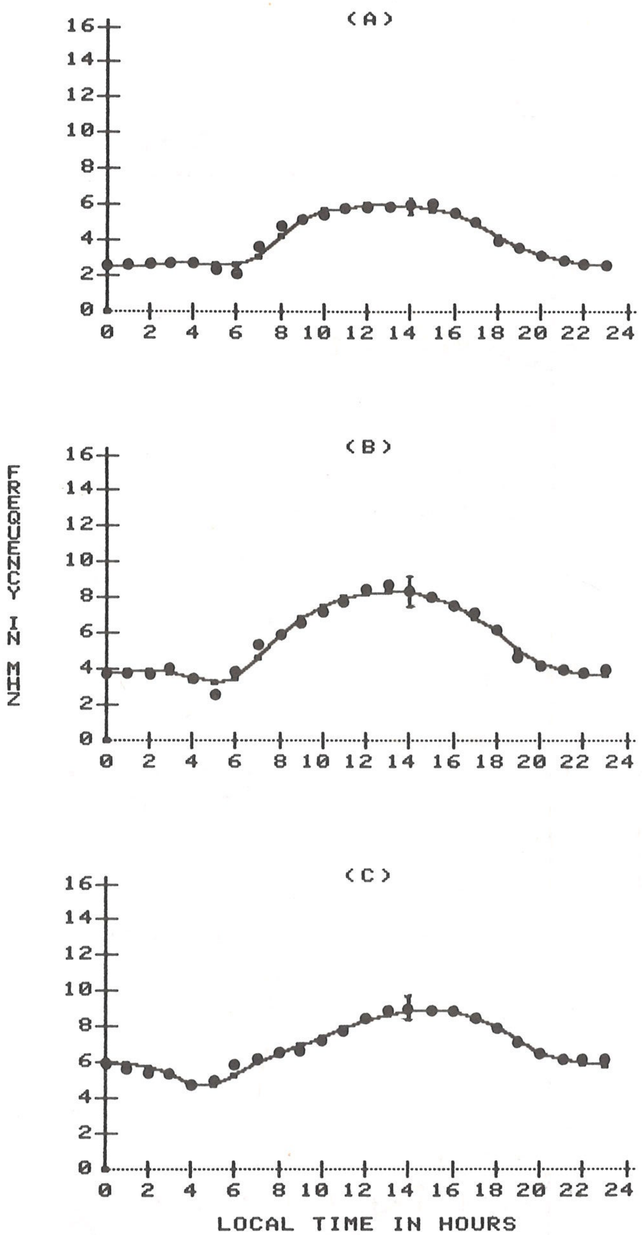


**Figure 5.** Daily variation of observed seasonal median critical frequency determined from data indicated in Tab. 1 (●●●●) and calculated with the proposed semiempirical model (———). Low solar activity level. (A) Winter; (B) Equinox; (C) Summer. Vertical bars give largest standard deviation of estimated median.

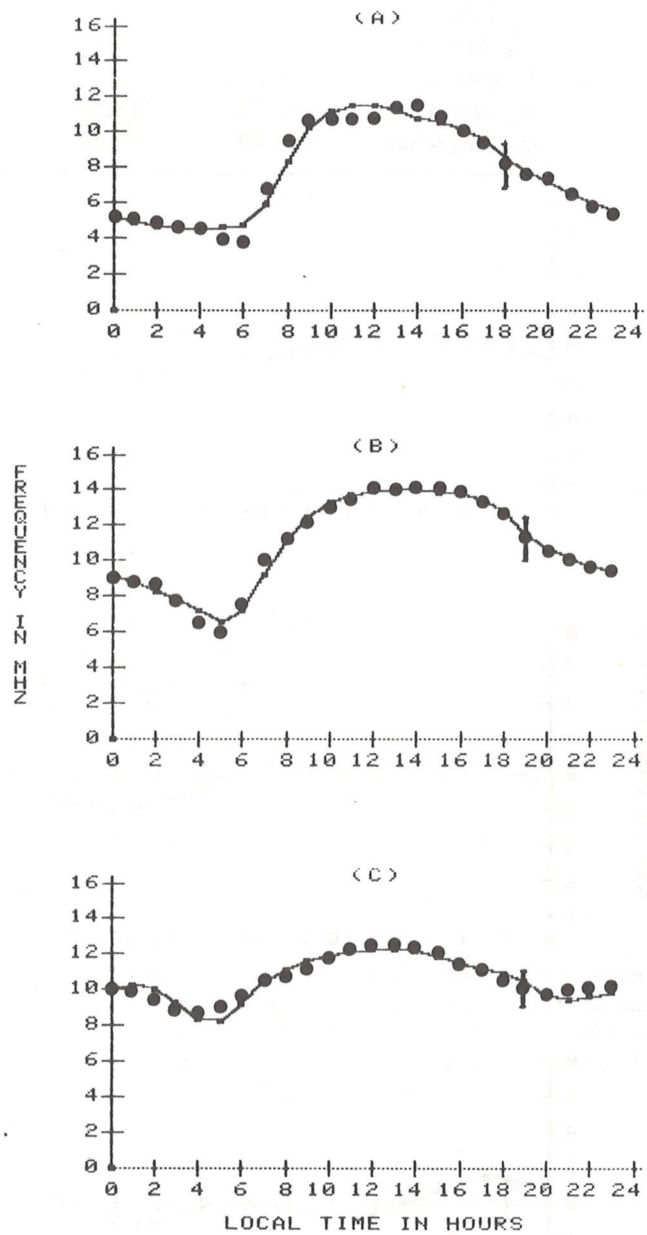


**Figure 6.** Daily variation of observed seasonal median critical frequencies determined from data indicated in Tab. 1 (●●●●) and calculated with the proposed semiempirical model (———). High solar activity level. (A) Winter; (B) Equinox; (C) Summer. Vertical bars give largest standard deviation of estimated median.

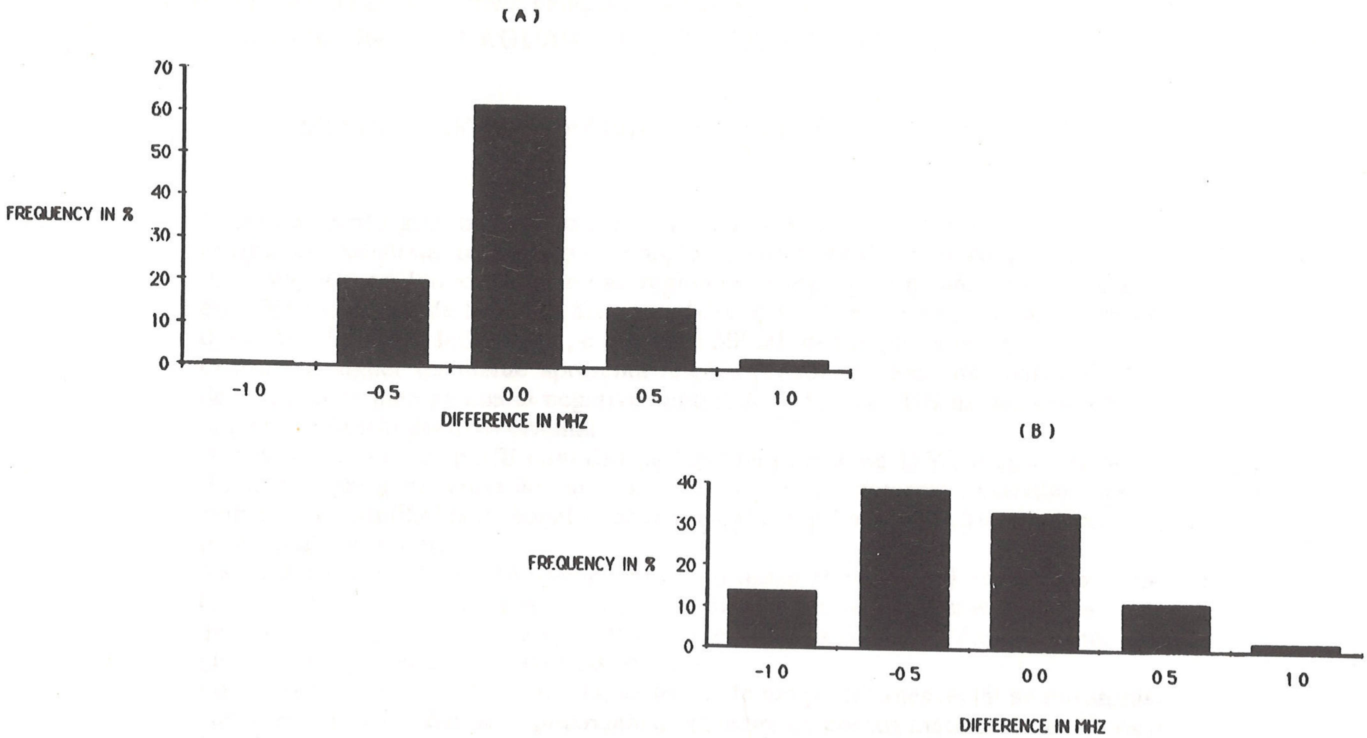


**Figure 7.** Frequency distribution of the differences between observed and calculated critical frequencies. (A) Proposed semiempirical model. (B) CCIR model (1967).

The simplicity of the model allows its use in a personal computer and even in a pocket calculator.

The results obtained seem to confirm that the use of simple conceptual models is still valid, at least for operatives purposes.

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