doi: 10.22564/rbgf.v10i2.1126

GPS PHASE AND CODE SOLUTIONS

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The fully developed carrier phase observation equation is presented. The assumptions and conditions are given, which lead to the simplified form often found in the literature. The most popular linear combinations of the carrier and code observations are discussed. The wide laning technique is presented in detail, including its use in rapid static GPS surveying, and the benefits gained from dual frequency P code observations. The solutions are strengthened by imposing integer constraints on the ambiguities. Ellipses of standard deviation demonstrate the geometry of wide laning and its statistical strength. Several relationships between carrier phases and codes, which were already developed during the last decade, are now becoming increasingly important in view of modern multichannel P code receivers.

SOLUCÕES DE FASE E CÓDIGO PARA O SISTEMA DE POSICIONAMENTO GLOBAL (SPG) *Apresenta-se a dedução completa da equação de observação de fase da portadora do SPG. São apresentadas as condições e hipóteses que levam à forma simplificada, que é freqüentemente encontrada na literatura. As combinações lineares mais populares das observações de código e portadora são discutidas. Apresenta-se detalhadamente a técnica, incluindo uso em agrimensura de SPG estática e os benefícios obtidos com observações de código P em freqüência dupla. As soluções são otimizadas impondo-se limitações quando ocorrem ambigüidades. Elipses de desvio padrão demonstram a geometria da técnica e a força estatística do método. Várias relações entre fases e códigos de portadoras, já desenvolvidas durante a última década, tornam-se mais importantes agora com o uso de receptores modernos que usam os códigos P de vários canais.*

INTRODUCTION

The Global Positioning System (GPS) is entering the final stage of deployment. During 1993 the launch of all 24 satellites is expected to be completed. At that time, GPS will provide a world-wide 24 hour per day navigation and surveying capability available under all weather conditions. Although the GPS system is operated by the U.S. Department of Defense to provide support for military navigation, there is a tremendous utility of GPS for practical civilian and scientific applications. The wide spectrum of applications reaches from real-time low-accuracy positioning for Geographic Information Systems (GIS) and navigation of private cars to ultra-high accuracy applications in geodesy and geophysics.

Over the last 10 years GPS surveying techniques have continuously been refined. This process is still not completed. The pioneers of GPS surveying observed 4 to 5 hours to achieve one part per million (ppm) over 10 km base lines. This can now be accomplished in just minutes. Whereas the pioneers believed that 1 ppm was the limit of GPS surveying, recent experiments have demonstrated that 0.001 to 0.002 ppm, corresponding to about 1 cm to 2 cm in 10.000 km, is possible.

The GPS satellites transmit two carrier frequencies, f1=154*10.23 = 1575.42 MHz and f2=120*10.23=1227.60 MHz. Both carriers, usually referred to as L1 and L2 carriers, are modulated with P-codes having a chipping rate of 10.23 MHz. In addition, the L1 carrier is modulated with the C/A-code at a 1.023 MHz chipping rate and the navigation message. Details on the satellite transmissions and satellite constellation are found in the Interface Control Document ICD-GPS-200, issued by Rockwell International Corporation of Downey, California.

Recent advances in GPS surveying, as well as current research efforts, relate to the optimal combination of carrier phase and code phase (pseudo range) observations, improved techniques to estimate integer ambiguities, understanding the geometry of the solutions, and minimization of multipath. Especially attractive in this regard are receivers which observe the P-code on both frequencies. The prevailing strategies and their limitations are best explained in terms of mathematical expressions.

THE UNDIFFERENCED EQUATIONS

The detailed description of the various techniques for receivers to measure the carrier phase observable is not a subject of this paper. Let it suffice to say that each receiver contains a clock which drives oscillators which, in turn, run at the carrier frequencies. At a given instant, i.e. the epoch of measurement of true time t_r, the difference of the receiver phase, $\phi_k(t_r)$, and the received satellite carrier phase, $\phi^p(t_r)$, is measured by means of a phase delay-lock loop. Equation (1) contains all relevant terms:

$$\phi_k^p(t_r) = \phi_k(t_r) - \phi^p(t_r) + N_k^p(1) - \frac{a_k^p}{f} + \frac{f}{c}T_k^p \quad (1)$$

The phase carrier observable $\phi_k^p(t_r)$ refers to receiver k and satellite p. Once initial lock is achieved, i.e. the first difference is measured, the receiver provides the accumulated carrier phase observable by means of continuously measuring and integrating the differences between the receiver phase and the received phase. The accumulated carrier phase observations are stored in a data file at intervals set manually by the operator, i.e. 10 seconds.

The measurement process cannot account for the number of whole carrier wavelengths between the receiver and the satellite at the initial epoch. In fact, at the initial epoch only the phase fractions are measured. The initial arbitrary setting of the "counting register", which keeps track of the full wavelengths of the change in the measured differences, is denoted by $N_k^p(1)$. This value is also called the initial integer ambiguity; it is consequently only a function of the first epoch. If the continuous function $\phi^p(t_r)$ is temporarily interrupted because of blockage of the satellite signal, a cycle slip will occur. As lock of the signal is regained, the fractional part of the phase is measured correctly, but the number of full wavelengths by which the receiver phase and the received phase have changed, is not known. Various strategies exist to deal with cycle slips.

The last two terms in equation (1) represent the ionospheric and tropospheric effects on the phase observable. The carrier frequency and the velocities of light are denoted by f and c, respectively. The term a_k^p

varies with the Total Electron Content (TEC) along the path of the satellite transmission. Because of the strong variation of the TEC with time and direction, the ionosphere requires special attention. The tropospheric effects are computed from appropriate models using measured temperature, pressure, and relative humidity. Occasionally, experiments with water vapor radiometers are reported in the literature.

An equation can be written for the carrier phase observable of L1 and L2. One only needs to add a subscript 1 or 2 to all terms except a_k^p and T_k^p. Note that the integer ambiguities for $N_k^p.1$ and N_{k1}^p are independent. The fully developed form of equation (1) is (Leick, 1990, p.215):

$$
\begin{aligned}
\phi_k^p(t_r) &= -[\phi_T^p(t_o) + a^p(t-t_o) + \frac{b^p}{2}(t-t_o)^2] + \\
&+ \frac{f}{c}\rho_k^p(t) + \frac{f}{c}\dot{\rho}_k^p(t)dt_k + \\
&+ [a^p + b^p(t-t_o)\frac{k^{op(t)}}{c}] + \\
&+ \phi_k(t_o) + f dt_k + N_k^p(1) - \\
&- \frac{a_k^p}{f} + \frac{f}{c}T_k^p
\end{aligned}
\qquad (2)
$$

The symbol t denotes the nominal receiver time, which differs from the true time, t_r, by the receiver clock error, dt_k. The symbols a and b denote residual satellite frequency offset and drift at some arbitrary reference time t_o. The transmitted satellite phase and the receiver phase at t_o are $\phi_T^p(t_o)$ and $\phi_k(t_o)$. The topocentric distance to the satellite is $\rho_k^p(t)$, and $\dot{\rho}_k^p(t)$ is the respective distance rate.

Equation (2) can be simplified and written in a more compact form. Since t_o is arbitrary, it can be chosen such that $\phi_k(t_o)$ is zero for both frequencies. The term $[a^p + b^p(t-t_o)]\rho_k^p/c$ can be neglected because each satellite contains several rubidium and cesium clocks to provide stable time, and because the GPS control center monitors the satellite clocks and provides corrections for drift and offset as part of the navigation message. The first term in (2) can be written as:

$$f dt^p = [\phi_T^p(t_o) + a^p(t-t_o) + \frac{b^p}{2}(t-t_o)^2] \qquad (3)$$

thus defining the residual satellite clock error dt^p.

The derivation of the pseudo-range equations follows the same steps as outlined above. The codes generated within the receiver are compared with the incoming codes from the satellites. The code phases are measured by means of a code delay-lock loop. Thus, in equation (2) the carrier phase ϕ is to be replaced with the code phase Φ and the carrier frequency f with f_{code}. The resulting equation is usually multiplied with the code wavelength to convert the phases to distances, thus:

$$P_k^p(t_r) = \frac{c}{f_{code}}\Phi_k^p(t_r, f_{code}) \qquad (4)$$

There are two important differences. Equation (4) does not contain an integer ambiguity term because the wave ambiguity can be resolved from the unique code sequence. The actual number of code waves between the receiver and the satellite can be determined. Also, the code modulation experiences an ionospheric delay equal to that of the carrier phase but with opposite sign. Incorporating these changes and the simplifications mentioned above, the equation for the pseudo-range becomes:

$$
\begin{aligned}
P_k^p(t_r) &= \rho_k^p(t) + \dot{\rho}_k^p(t)dt_k - \\
&-cdt^p + cdt_k + \frac{ca_k^p}{f^2} + t_k^p
\end{aligned}
\qquad (5)
$$

This is the basic equation used in navigation. In these applications the residual satellite clock error is neglected, i.e. the error dt^p which remains once the clock corrections, provided in the navigation message, have been applied. The satellite positions follow from the broadcast ephemeris given by the navigation message. The ionospheric and tropospheric corrections are taken from appropriate models (recall that millimeter positioning is not expected for this type of solution). Thus, there are four unknowns in equation (5): three station coordinates implied in the topocentric distance ρ_k^p, and one receiver clock error. If the receiver measures pseudo-ranges to four different satellites at the same time, then the resulting four equations determine all four unknowns.

The correction to the receiver clock can be computed with about 0.1 μ sec accuracy from this type of a solution. Thus, the $\dot{\rho}_k^p(t)$ term in equation (2) can be moved to the left side, i.e. the observed phases (and ranges) are corrected for this component of the receiver clock error. Since $\dot{\rho} < 800$ m/sec for GPS satellites, residual receiver clock errors smaller than 0.1 μ sec are negligible with regard to this term; the phase error would be smaller than the measurement accuracy of 1/100 of a cycle. With the understanding that this procedure is always followed, the $\dot{\rho}$ terms are dropped and the equations for the pseudo-range and the carrier phase observable become:

$$P_k^p(t_r) = \rho_k^p(t) - cdt^p + cdt_k + \frac{ca_k^p}{f^2} + t_k^p \qquad (6)$$

$$\phi_k^p(t_r) = \frac{f}{c}\rho_k^p(t) - fdt^p + fdt_k + \\ + N_k^p(1) - \frac{a_k^p}{f} + \frac{f}{c}T_k^p \qquad (7)$$

Because the receiver clock correction can be computed from the pseudo-ranges, it is possible to use relatively inexpensive crystal clocks in the receivers. Even a drift of several milliseconds in the receiver clock during an observation session will present no problem.

DOUBLE DIFFERENCES

Assume that two receivers located at different stations observe two satellites at the same nominal time (as indicated by the respective receiver clocks). Denoting the receivers by k and m, and the satellites by p and q, the following difference of the observables, called the double difference,

$$\phi_{km}^{pq}(t_r) = [(\phi_k^p(t_r) - \phi_m^p(t_r)] - [(\phi_k^q(t_r) - \phi_m^q(t_r)] \quad (8)$$

can be formed. Applying this differencing scheme to equations (6) and (7) yields:

$$\phi_{km}^{pq}(t_r) = \frac{f}{c}\rho_{km}^{pq}(t) + N_{km}^{pq}(1) - \frac{ca_{km}^{pq}}{f} + \frac{f}{c}T_{km}^{pq} \quad (9)$$

$$P_{km}^{pq}(t_r) = \rho_{km}^{pq}(t) + \frac{ca_{km}^{pq}}{f^2} + T_{km}^{pq} \qquad (10)$$

The clock terms dt^p, dt^q, dt_k, and dt_m, which are still present in the undifferenced equations (6) and (7), cancel during the differencing operation. These clock errors can be in the range of thousands of cycles. Thus, simultaneity of observations is defined in terms of the nominal receiver clock time. The error in the receiver clocks must be small enough so that $\dot{\rho}$ can be assumed constant for the purpose of correcting the phase via the terms (f/c) $\dot{\rho}$. A more general scheme could be presented requiring only near-simultaneity in nominal times, but no additional insight to GPS techniques would be gained for the purpose of this paper.

Because the satellites are at least 20.000 km away, the signals received at nearby stations k and m have traveled essentially through the same portion of the ionosphere and the troposphere, thus experiencing the same propagation effects. Because of the differencing operation (8) the ionospheric term a_{km}^{pq} can be neglected for short base lines of about 15 km. This is important for the development of modern rapid static techniques which have the goal of shortening the observation time to just minutes per base line.

The integer term $N_{km}^{pq}(1)$ is a linear combination of the undifferenced integer ambiguities. This term is, indeed, estimable. The absolute value of

$N_{km}^{pq}(1)$ is not relevant because of the arbitrary initial counter register setting. It is, however essential to recognize that $N_{km}^{pq}(1)$ is an integer. Modern techniques take full advantage of this fact. For example, the estimation process yields a rational value which might or might not be close to an integer depending on the magnitude of unmodelled errors such as residual ionosphere and orbital errors. A particularly accurate solution is found if it is possible to constrain the ambiguities to the correct integer values in subsequent solution. The goal of many modern approaches is not only being able to "fix the integer ambiguities" but to do this as soon as possible, thus shortening the observation time.

If R receivers observe S satellites simultaneously, (R-1)(S-1) independent double differences of the type (9) or (10) are possible. Of course, double differences can be formed separately for the carrier phases on L1 and L2, the P-codes on L1 and L2, and the C/A code on L1. Double differences of the same epoch are correlated.

As a rule of thumb, the measurement accuracy of phases is about 1/100 of a cycle. This translates to about 2 mm, 30 cm, and 3 m of equivalent distance accuracy for the carrier phases, the P codes, and the C/A code respectively. In addition, the carrier is least affected by multipath, followed by the P code and then the C/A code. It follows that receivers observing carrier phases and P codes at both frequencies are most desirable.

CODE AND CARRIER PHASE COMBINATIONS

Depending on the type of receiver, various combinations of the observables are possible. Because P-code receivers are becoming more widely available, this section concerned primarily with the combination of carrier phases and P codes.

Two P codes

From equation (6) it follows immediately that:

$$P_{k.1}^{p}(t_r) - P_{k.2}^{p}(t_r) = ca_k^p\left(\frac{f_2^2-f_1^2}{f_1^2 f_2^2}\right) \qquad (11)$$

$$\frac{f_2^2}{f_2^2-f_1^2}P_{k.1}^{p}(t_r) - \frac{f_2^2}{f_2^2-f_1^2}P_{k.2}^{p}(t_r) = $$
$$= \rho_k^p(t) - cdt^p + cdt_k + t_k^p \qquad (12)$$

Equation (11) provides a direct measure of the ionosphere. The receiver and satellite clock errors cancel and there is no dependency on the receiver - satellite geometry. In equation (12), on the other hand, the ionosphere has been eliminated. Applying the double difference operation will also cancel the clock errors. Thus, equation (12) is ideal for continuous relative positioning of modest accuracy over long lines.

Two Carrier Phases

The equivalent of equations (11) and (12) for the case of carrier phases is:

$$\phi_{k.IF}^{p}(t_r) = \frac{f_1^2}{f_1^2-f_2^2}\phi_{k.1}^{p}(t_r) - \frac{f_1 f_2}{f_1^2-f_2^2}\phi_{k.2}^{p}(t_r) =$$
$$= \frac{f_1}{c}\rho_k^p(t) - f_1 dt^p + f_1 dt_k + \frac{f_1^2}{f_1^2-f_2^2}N_{k.1}^{p}(1)$$
$$- \frac{f_1 f_2}{f_1^2-f_2^2}N_{k.2}^{p}(1) + \frac{f_1}{c}T_k^p \qquad (13)$$

$$\phi_{k.I}^{p}(t_r) = \phi_{k.1}^{p}(t_r) - \frac{f_1}{f_2}\phi_{k.2}^{p}(t_r)$$
$$= N_{k.1}^{p}(1) - \frac{f_1}{f_2}N_{k.2}^{p}(1) - \frac{a_k^p}{f_1}\left(1 - \frac{f_1^2}{f_2^2}\right) \qquad (14)$$

Unfortunately, a variation in one of the integer ambiguities causes a non-integer step in the ionospheric-free combination (13). The respective factors are $f_1^2/(f_1^2-f_2^2) = 2.54$ and $f_1 f_2/(f_1^2-f_2^2) = 1.98$. Similarly, the ionospheric combination (14) depends on the ambiguities; the fraction $f_1/f_2 = 77/60$.

Applying the double difference operation eliminates the clock errors in (13) and most of the ionosphere in (14) for short station separation. Equation

(14) is frequently used to check for cycle slips assuming that the ionosphere is eliminated in the double differences or varies slowly. This kind of analysis is sometimes ambiguous because certain combinations of integers in L1 and L2 generate almost the same change in either the ionospheric or the ionospheric-free combination. See Leick (1990, p. 238) for a listing of integer combinations.

The following combination:

$$\begin{aligned}\phi^p_{k.w}(t_r) &= \phi^p_{k.1}(t_r) - \phi^p_{k.2}(t_r) = \\ &= \tfrac{f_1-f_2}{c}\rho^p_k(t) - (f_1-f_2)dt^p + (f_1-f_2)dt_k + \\ &+ N^p_{k.w}(1) - a^p_k\left(\tfrac{1}{f_1}-\tfrac{1}{f_2}\right) + \tfrac{f_1-f_2}{c}T^p_k \end{aligned} \quad (15)$$

with:

$$N^p_{k.w}(1) = N^p_{k.1}(1) - N^p_{k.2}(1) \quad (16)$$

is very important in modern GPS surveying. It is called the wide lane observable because the effective wavelength of this combination is:

$$\lambda_w = \frac{c}{f_1 - f_2} = 0.86m \quad (17)$$

A popular strategy is to attempt to estimate the wide lane ambiguities (16), round the estimates to integers, and constrain the rounded values in subsequent solutions. The wide lane observable in units of length is:

$$\begin{aligned}L^p_{k.w}(t_r) &= \lambda_w \phi^p_{k.w}(t_r) = \\ &= \rho^p_k(t) - cdt^p + cdt_k + \lambda_w N^p_{k.w} + \\ &+ \tfrac{c}{f_1 f_2}a^p_k + T^p_k \end{aligned} \quad (18)$$

The complement of the wide lane is, of course, the narrow lane, given by:

$$\begin{aligned}\phi^p_{k.n}(t_r) &= \phi^p_{k.1}(t_r) + \phi^p_{k.2}(t_r) = \\ &= \tfrac{f_1+f_2}{c}\rho^p_k(t) - (f_1+f_2)dt^p + (f_1+f_2)dt_k + \\ &+ N^p_{k.n}(1) - a^p_k[\tfrac{1}{f_1}+\tfrac{1}{f_2}] + \tfrac{f_1+f_2}{c}T^p_k \end{aligned} \quad (19)$$

$$N^p_{k.n}(1) = N^P_{k.1}(1) + N^p_{w.2}(1) \quad (20)$$

$$\lambda_n = \frac{c}{f_1 + f_2} = 0.11m \quad (21)$$

$$\begin{aligned}L^p_{k.n}(t_r) &= \lambda_n \phi^p_{k.n}(1) = \\ &= \rho^p_k(t) - cdt^p + cdt_k + \lambda_n N^p_{k.n}(1) - \\ &- \tfrac{c}{f_1 f_2}a^p_k + T^p_k \end{aligned} \quad (22)$$

The wide lane and narrow lane observables relate to the ionospheric-free and ionospheric combinations as follows:

$$\frac{f_1}{2c}[L^p_{k.w}(t_r) + L^p_{k.n}(t_r)] = \phi^p_{k.IF}(t_r) \quad (23)$$

$$\begin{aligned}\tfrac{c}{2f_2}[\tfrac{1}{\lambda_w}\phi^p_{k.n}(t_r) - \tfrac{1}{\lambda_n}\phi^p_{k.w}(t_r)] &= \\ = \phi^p_{k.I}(t_r) \end{aligned} \quad (24)$$

Wide Lane Ambiguity from Code and Carrier

Given P code receivers the wide lane ambiguity can be computed directly from:

$$\begin{aligned}\lambda_w \phi^p_{k.w}(t_r) - \tfrac{f_1 P^p_{k.1}(t_r) + f_2 P^p_{k.2}(t_r)}{f_1+f_2} &= \\ = \lambda_w N^p_{k.w}(1) \end{aligned} \quad (25)$$

This particular combination is independent of clock errors, ionosphere, and the station - satellite geometry. For very accurate P code observations one might be tempted to solve the wide lane ambiguity directly each epoch. In practice, several epochs will be combined to estimate the wide lane ambiguity, while the receiver remains at the same location. Equation (25) was already considered by Melbourne (1985) when the first TI-4100 dual frequency data became available. With the availability of new multichannel P code receivers, this remarkably simple relationship between codes, carrier phases, and the wide lane ambiguity will become even more useful.

The Range Difference Equation

Evaluate the undifferenced equations (6) and (7) at a general epoch t_r and at some initial epoch 1. The respective differences have the range difference

$\rho_k^p(t_r) - \rho_k^P(1)$ in common. Differencing the differences gives:

$$P_k^p(t_r) - P_k^p(1) = \frac{c}{f}[\phi_k^p(t_r) - \phi_k^p(1)] + \\ + \frac{2c}{f^2}[a_k^p(t) - a_k^p(1)] \quad (26)$$

which nicely shows the dependency of the code and carrier phases on the changes of the ionosphere. The receiver location at epochs t_r and 1 can be different. In fact, the receiver can be moving at both epochs and during the interval. A more practical form of equation (26) is:

$$P_k^p(1) = P_k^p(t_r) - \frac{c}{f}[\phi_k^p(t_r) - \phi_k^p(1)] \\ - \frac{2c}{f^2}[a_k^p(t) - a_k^p(1)] \quad (27)$$

This equation shows how the pseudo-range can be mapped back in time. For example, the observations at various epochs can be used to average or smooth the pseudo-range at the initial epoch 1, thus producing a more accurate value. If equation (27) is subjected to the double difference operation, it will be possible to compute an accurate epoch 1 position using a navigation-type of solution. Double differencing, of course, eliminates most of the ionosphere when applied over short distances. Over longer distance it is necessary to combine carrier and code observations of both frequencies.

Equation (27) is fundamental to a technique called "integer fixing on the fly". Assume that station k is located on the ground and that receiver m is located on a plane. Let epoch 1 refer to an instant after take off or before landing. Repeated back-mapping of the pseudo-ranges gives an estimate of the epoch 1 position which, in turn, is used to define a search volume which contains the correct epoch 1 position. Various techniques have been developed to determine the exact location within the search volume and, by doing so, to obtain the correct integer ambiguities. See Mader (1992) and Remondi (1991) for implementation details.

Of course, receiver m can be on the ground and be stationary. The same technique can be used. Thus, it is readily seen how modern GPS unifies navigation and surveying. The ambiguity function technique, developed by Counselman and Gourevitch (1981), is likely to be widely used in these developments.

WIDE LANING TECHNIQUES

Given dual frequency carrier phase observations, the simplest approach is to use equation (8) for each frequency and to estimate station positions and integer ambiguities together. Frei and Beutier (1990) describe an optimized approach for using statistical tests to determine the integers from the real-valued estimates. The integer identification is complicated, because the estimated ambiguities are correlated, and several sets of integers might not be distinguishable statistically.

It is always best to estimate the wide lane ambiguity first, then determine the wide lane integer values and constrain these values in subsequent solutions. The strength added to the adjustment with such a procedure is very significant. It is, in a sense, a "back door" to neutralizing unavoidable model errors caused, e.g. by the residual ionosphere. This technique makes it possible to fix integers over longer base lines or shorten the observation span for short base lines. The major advantage of dual frequency P code receivers is the ability to shorten the time necessary to resolve the wide lane ambiguity, e.g. by means of equation (25).

Another level of sophistication is possible. Consider equation (28) which shows a particular combination of L1 and the wide lane phases, and consider the rearrangements (29) and (30). Further assume that the double difference

$$\phi^p_{k.1}(t_r) - \frac{f_1}{f_1-f_2}\phi^p_{k.w}(t_r) = N^p_{k.1}(1) - \\ - \frac{f_1}{f_1-f_2}N^p_{k.w}(1) - \frac{f_1+f_2}{f_1 f_2}a^p_k \qquad (28)$$

$$N^p_{k.1}(1) = [-\phi^p_{k.w}(t_r) + N^p_{k.w}(1)] \times \\ \times \frac{f_1}{f_1 f_2} + \frac{f_1+f_2}{f_1 f_2}a^p_k \qquad (29)$$

$$N^p_{k.2}(1) = [-\phi^p_{k.w}(t_r) + N^p_{k.w}(1)] \times \\ \times \frac{f_2}{f_1 f_2} + \frac{f_1+f_2}{f_1 f_2}a^p_k \qquad (30)$$

operations will be applied, resulting in a cancellation of the ionospheric term in (29) and (30) over short base lines. If the wide lane ambiguity $N^{pq}_{km.w}(1)$ is available from a prior solution, i.e. code phase solution, the integers $N^{pq}_{km.1}(1)$ and $N^{pq}_{km.2}(1)$ follow from (29) and (30). Assume that the estimate of the wide lane ambiguity is wrong by one (wide lane) cycle. The resulting errors on the L1 and L2 ambiguities are $f_1/(f_1 - f_2) = 4.52$ and $f_2/(f_1 - f_2) = 3.53$ respectively. We know that over short base lines the ambiguities estimate must be very close to an integer value. Thus, an erroneous wide lane by 1 cycle would result in ambiguity estimates with fractions close to one half (.5). In practice, this fact would raise a flag, warning of a possible error in the wide lane ambiguity. Another wide lane ambiguity value could be used. This procedure implies that the permissible uncertainty in the wide lane ambiguity is 2, i.e. a minor uncertainty in the wide lane ambiguity is allowed. This technique is often referred to as extra wide laning (Wuebbena, 1985). Because fewer observations are required to identify two wide lane candidates than are needed to determine the wide lane uniquely, extra wide laning is particularly attractive for making rapid static GPS surveying even more rapid.

An alternative implementation of the extra wide laning concept is seen from equations (31) and (32). Equation (31):

$$\phi^p_{k.IF}(t_r) = \frac{f_1}{c}\rho^p_k(t) - f_1 dt^p + f_1 dt_k \\ - \frac{f_1}{2(f_1-f_2)}[N^p_{k.1}(1) - N^p_{k.2}(1)] \\ + \frac{f_1}{2(f_1+f_2)}[N^p_{k.1}(1) + N^p_{k.2}(1)] \\ + \frac{f}{c}T^p_k \qquad (31)$$

$$\phi^{pq}_{km.IF}(t_r) = \frac{f_1}{c}\rho^{pq}_{km} \\ - \frac{f_1}{2(f_1-f_2)}[N^{pq}_{km.1}(1) - N^{pq}_{km.2}(1)] \\ + \frac{f_1}{2(f_1+f_2)}[N^{pq}_{km.1}(1) + N^{pq}_{km.2}(1)] \qquad (32)$$

follows directly from the ionospheric-free combination (13) with only a rearrangement of the ambiguities into a difference (wide lane) and a sum (narrow lane). Extra wide laning is implemented by utilizing the fact that there exists an odd/odd and even/even relationship between the wide lane and the narrow lane ambiguities. If the candidate wide lane is odd, then the narrow lane solved from (32) must be odd as well, otherwise the candidate wide lane is wrong.

There are other variations for implementing extra wide laning. But no additional insight would be gained by dealing with them in this paper.

CODE-PHASE GEOMETRY

The variance covariance matrix is readily available for studying the code-phase geometry. From equations (6) and (7) it follows that:

$$[P^p_{k.1}(t_r)P^p_{k.2}(t_r)\phi^p_{k.1}(t_r)\phi^p_{k.2}(t_r)]^T \\ = A[\rho^p_k(t)I^p_{k.1}N^p_{k.1}N^p_{k.2}]^T \qquad (33)$$

with:

$$I^p_{k.1} = \frac{c}{f_1^2}a^p_k \qquad (34)$$

and:

$$A = \begin{bmatrix} 1 & 1 & 0 & 0 \\ 1 & f_1^2/f_2^2 & 0 & 0 \\ 1 & -1 & \lambda_1 & 0 \\ 1 & -f_1^2/f_2^2 & 0 & \lambda_2 \end{bmatrix} \tag{35}$$

The clock error terms are ignored in these equations because they will cancel after the double difference operation is applied. The ionosphere is expressed in terms of the L1 correction (34). Equation (33) is written as:

$$Y_k^p(t_r) = AX_k^p(t_r) \tag{36}$$

The solution for the unknowns is:

$$X_k^p(t_r) = A^{-1}Y_k^p(t_r) \tag{37}$$

Applying the low of variance covariance propagation (Leick, 1990, p. 79) gives:

$$\sum_x = A^{-1}\sum_y (A^{-1})^T \tag{38}$$

The variance covariance matrix of the observations and the unknowns are Σ_y and Σ_x respectively. The variance of ρ_k^p or ρ_{km}^{pq}, in case of the double difference, expresses the position accuracy. Assuming that the observations are uncorrelated, Σ_y is given by:

$$\sum_y = \sigma_\phi^2 \begin{bmatrix} k^2 & 0 & 0 & 0 \\ 0 & k^2 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{39}$$

This stochastic model implies that the P codes and the carrier phases are of the same quality respectively, with the ratio:

$$\frac{\sigma^P}{\sigma_\phi} = k \tag{40}$$

The analytical expressions for Σ_x are listed in the appendix. Using $\sigma_\phi = 0.002$ m and k = 154, which corresponds to the ratio of the L1 frequency and the P code chipping rate, the following numerical values are obtained for the standard deviations and the correlation matrix:

$$[\sigma_\rho, \sigma_I, \sigma_{N.1}, \sigma_{N.2}] = $$
$$= [0.917m, 0.673m, 8.29cyc, 8.23cyc] \tag{41}$$

$$C_x = \begin{bmatrix} 1.000 & -0.9714 & -0.9949 & -0.9914 \\ & 1.0000 & 0.9904 & 0.9941 \\ & & 1.0000 & 0.9995 \\ & & & 1.0000 \end{bmatrix} \tag{42}$$

The equations listed in the appendix readily confirm that k acts basically as scaling factor for the variance covariance matrix. The only exception is a small term in the variances of the ambiguities. Of particular importance is the high correlation between the ambiguities. Thus, the ellipse of standard deviation degenerates almost into a straight line with semiaxis of 11.68 and 0.17 cycles respectively, and an orientation of 44.78 degrees. Transforming to wide lane and narrow lane ambiguities according to:

$$\left(N_1 - N_2, N_1 + N_2\right)^T =$$
$$\begin{pmatrix} 0 & 0 & 1 & -1 \\ 0 & 0 & 1 & 1 \end{pmatrix} \left(\rho_k^p I_{k.1}^p N_{k.1}^p N_{k.2}^p\right) \tag{43}$$

and applying variance covariance propagation gives:

$$[\sigma_\rho, \sigma_I, \sigma_{N.w}, \sigma_{N.n}] =$$
$$= [0.916m, 0.673m, 0.25cyc, 16.52cyc] \tag{44}$$

$$C_x = \begin{bmatrix} 1.000 & -0.9714 & -0.9949 & -0.9914 \\ & 1.0000 & 0.9904 & 0.9941 \\ & & 1.0000 & 0.2441 \\ & & & 1.0000 \end{bmatrix} \tag{45}$$

The orientation of the ellipse of standard deviation for the ambiguities is 89.78 degrees with the semiaxis being 16.62 and 0.24 cycles respectively. There is little correlation between the wide lane and the narrow lane ambiguities. The semiminor axis essentially coincides with the direction of the wide ambiguity.

These are the statistical reasons why it is possible to estimate the wide lane ambiguity with only a short set of observations and then to take advantage of the additional strength provided by subsequent integer constraints.

The ratio of the semiaxis is constant in both cases, the value being very close to the squared ratio of the wide lane and narrow lane wavelength. The orientation and shape of the elipse of standard deviation are quite insensitive to variations of k and σ_ϕ.

CONCLUSION

Much of the improvement of GPS surveying in recent years is due to the availability of dual frequency receivers and more sophisticated processing. Newly available P code receivers, which measure P codes at both frequencies, will advance GPS surveying even more. This paper summarizes some of the techniques which are used most effectively with such modern receivers. As the observation time for short base lines decreases, it will be critical to control the multipath (which tends to average out over longer observation times). Mission planning will, again, become important, in order to avoid weak geometry during the short observation time.

Algorithmic developments clearly tend toward a unified approach for surveying and high precision navigation. The respective developments belong to the area of "integer fixing on the fly". This paper has not focused on these techniques, nor has it addressed issues which are particular to long base lines. Much has been learned in this area as evidenced by results of recent international GPS campaigns.

ACKNOWLEDGEMENTS

This work was prepared while Alfred Leick was visiting the University of São Paulo, Department of Transportation, on a Fulbright grant. The support given by Fulbright during the July/August 1991 and 1992 visits is gratefully acknowledged.

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APPPENDIX

$$F = \sigma_\phi^2 \frac{1}{(f_1^2 - f_2^2)^2} \tag{1}$$

$$\sigma_{11}^2 = \sigma_\rho^2 = Fk^2\left(f_1^4 + f_2^4\right) \tag{2}$$

$$\sigma_{12} = \sigma_{\rho.I.1} =$$
$$= -Fk^2 f_2^2\left(f_1^2 + f_2^2\right) \tag{3}$$

$$\sigma_{19} = \sigma_{\rho.N.1} =$$
$$= -F\tfrac{k^2}{\lambda_1}\left(f_1^4 + 2f_2^4 + f_1^2 f_2^2\right) \tag{4}$$

$$\sigma_{14} = \sigma_{\rho.N.2} =$$
$$-F\tfrac{k^2}{\lambda_2}\left(2f_1^4 + f_2^4 + f_1^2 f_2^2\right) \tag{5}$$

$$\sigma_{22}^2 = \sigma_{I.1}^2 = 2Fk^2 f_2^4 \tag{6}$$

$$\sigma_{23} = \sigma_{I.1.N.1} =$$
$$F\tfrac{k^2}{\lambda_1} f_2^2\left(f_1^2 + 3f_2^2\right) \tag{7}$$

$$\sigma_{24} = \sigma_{I.1.N.2} =$$
$$F\tfrac{k^2}{\lambda_2} f_2^2\left(3f_1^2 + f_2^2\right) \tag{8}$$

$$A = \left[k^2\left(5f_2^4 + f_1^4 + 2f_1^2 f_2^2\right) + \left(f_1^2 - f_2^2\right)^2\right]$$
$$\sigma_{33}^2 = \sigma_{N.1}^2 = \tfrac{F}{\lambda_1^2} A \tag{9}$$

$$\sigma_{34} = \sigma_{N.1.N.2} =$$
$$= 2F\tfrac{k^2}{\lambda_1\lambda_2}\left(f_1^2 + f_2^2\right)^2 \tag{10}$$

$$B = \left[k^2\left(5f_1^4 + f_2^4 + 2f_1^2 f_2^2\right) + \left(f_1^2 - f_2^2\right)^2\right]$$
$$\sigma_{44}^2 = \sigma_{N.2}^2 = \tfrac{F}{\lambda_2^2} B \tag{11}$$

Submetido em: 28.08.92
Revisado em: 27.09.92
Aceito em: 21.01.93
Editor Associado: Luiz Rijo