doi: 10.22564/rbgf.v12i2.1169

INVERSION OF THE EARTH'S RADIAL DENSITY DISTRIBUTION BY AN INFORMATION THEORY APPROACH

Amin Bassrei

A new formulation called Minimum Relative Entropy (MRE) is applied to the determination of the density profile within the Earth. The question is defined as an inverse problem where the data are only the mass of the Earth and its moment of inertia. The solution of this inverse problem is based on a probabilistic philosophy and on the concept of entropy. It is defined an objective function that contains the relative entropy of the probability density function of the model parameters (density distribution). The objective function is then minimized under adequate constraints in order to give the output estimate of the model parameters. The results are compared to a standard inversion technique, showing that MRE gives a much better agreement with the reference models considered (Bullen A, HB1 and PREM).

Key words: Inversion; Earth's density; Entropy; Minimum relative entropy; Information theory.

INVERSÃO DA DISTRIBUIÇÃO RADIAL DE DENSIDADE DA TERRA POR UMA ABORDAGEM DA TEORIA DA INFORMAÇÃO *Um novo formalismo denominado princípio da entropia relativa mínima é aplicado na determinação da distribuição de densidade no interior da Terra. A questão é posta como um problema inverso, onde apenas a massa e o momento de inércia da Terra são utilizados como dados. A solução do problema inverso é baseada numa filosofia probabilística e no conceito de entropia. Define-se uma função objetivo que contém a entropia relativa da função de densidade de probabilidade dos parâmetros de modelo (distribuição de densidade). A função objetivo é então minimizada sob adequadas condições de restrição, de tal forma a fornecer a estimativa final nos parâmetros de modelo. Os resultados são comparados com um método clássico, onde se verifica uma concordância bem melhor com os modelos de referência considerados (Bullen A, HB1 e PREM).*

Palavras-chave: Inversão; Densidade da Terra; Entropia; Entropia relativa mínima; Teoria da informação.

Programa de Pesquisa e Pós-Graduação em Geofísica; Instituto de Geociências, Universidade Federal da Bahia, Rua Caetano Moura, 123, Federação. 40210-350 Salvador, BA.
On Leave at Earth Resources Laboratory, Department of Earth, Atmospheric and Planetary Sciences, Massachusetts Institute of Technology, 42 Carleton St., Bldg. E34, Cambridge MA 02142, USA

INTRODUCTION

The properties of the interiors of the planets are, of course, inaccessible for direct investigation. Even for the Earth we have to make use of indirect methods, or, in others words, we have to invert the data collected at the surface in order to estimate the internal constitution of our planet. The launching of artificial satellites and the development of planetary radars have provided more information about the internal structure of the planets, including the Earth. This information is obtained from the dynamic properties of the planets, associated with the knowledge we have about solids that are submitted to very high pressures. However, in the case of the Earth, we can determine the density profile among other physical quantities, from seismological data. Many authors have studied this problem, see for instance Bullen (1965), Stacey (1969), and the references quoted there. The work of Dziewonski & Anderson (1981) introduced the PREM model, which is the current reference model in the geophysical community.

Here we apply the Minimum Relative Entropy (MRE) approach to the inversion of the Earth density profile. As previous works that employed the concept of entropy with this problem, we quote Rietsch (1977,1986), who used the principle of maximum entropy (PME) to infer the radial density distribution of the Earth, using lower and upper limits. Graber (1977), based on the work of Rubincam (1978), used the maximum entropy approach to the radial density variation, considering not only mass and moment of inertia, but also torsional normal modes of the Earth. He performed several inversions. In some cases the inclusions of normal modes improved substantially in relation to the basic situation (only mass and moment of inertia). In other simulations the outcome is rather discrepant. Rubincam (1982) also used the PME to estimate the lateral distribution of density within the Earth, from the values of gravity field anomalies.

Howerer, here we use another entropy approach, and actually the main idea of this work lies on the concept of relative entropy, first defined by Kullback (Kullback & Leibler, 1951; Kullback, 1959). By using relative entropy we introduce a prior information in the same physical dimension of the model parameters. Among several applications of this principle – MRE – we quote its interesting application in the field of spectral analysis, as an inverse procedure (Shore, 1981). An axiomatic study of MRE can be found in Shore & Johnson (1980). This approach has been already applied in several geophysical inverse problems, mainly in exploration geophysics: inversion of interval velocities, seismic transmission tomography, data inversion in the τ-p domain, inversion of potencial data (gravity and magnetics), correction of the effect of aliasing, band limited extrapolation (Bassrei, 1990a,b; Bassrei, 1991a,b; Bassrei & Ulrych, 1989; Bassrei & Pestana, 1991; Ulrych et al., 1990a; Ulrych et al., 1990b). The MRE soluction is compared to reference models like the Bullen A (Bullen, 1965), HB1 (Stacey, 1969) and PREM (Dziewonski & Anderson, 1981), and it resulted in reasonable outputs.

CLASSICAL INVERSION OF DENSITY

The inversion of density is an old geophysical problem and there are several approaches to estimate the Earth's radial density distribution. Since the famous experiment of Cavendish it has been known that the average density of the Earth is about 5.5 *g/cm³*. It is also known that the density is not constant within the Earth, but it increases from the surface to the centre of the planet. The first models representing the variation of density were formal mathematical relations, sometimes arbitrary. Taking data after Bullen (1965), we produce the Tab.1, where the first entry is the density at the surface and the second at the centre of the Earth (in *g/cm³*).

Laplace	2.60	11.20
Roche	2.40	10.30
Wiechert	3.40	8.40
Jeffreys	4.27	12.04

Table 1. Classical models for the Earth's density.
Tabela 1. *Modelos clássicos de densidade da Terra.*

The density variation due to an increasing compression with depth in a chemically homogeneous layer (without phase change) can be calculated by the method of Williamson & Adams (1923), based on the integration of a differential equation, which nevertheless disagrees with the modern models. The differential equation is

$$\frac{d\rho}{dr} = \frac{d\rho}{dp}\frac{dp}{dr} = \frac{\rho}{k}(-g\rho) = -\frac{Gm(r)\rho^2}{kr^2}, \qquad (1)$$

where

$$m(r) = \int_0^r 4\pi r^2 \rho(r)\, dr \tag{2}$$

The quantity $m(r)$ is the total mass contained from the center of the Earth to the radius r, which determines the gravity g at this radius. Also, p is the pressure, k is the bulk modulus, and G is the universal gravitational constant. In each of the recognized zones of the Earth, for which the velocity profile is smooth, it is reasonable to assume cheminal and phase homogeneity. This allows the variation of density ρ, due to an increasing pressure p with depth z, to be calculated by the method of Williamson & Adams (1923). Many researchers improved the above formulation, in particular Bullen, who proposed several density distributions.

Free oscillations data have also been used to infer the radial distribution of density. Bullen & Haddon (1967), used for instance spherical oscillations up to the 24th order and toroidal oscillations up to the 18th order. A more recent work was performed by Dziewonski & Anderson (1981), where they have derived the so-called PREM – Preliminary Reference Earth Model. Their data were free-oscillation and long-period surface data, as well as body waves. The mass and the moment of inertia were used as constraints.

REDEFINING THE INVERSE PROBLEM

For the present approach to the problem, we will consider a model with spherical symmetry, where the values of the Earth's radius, mass, and moment of inertia are assumed to be known. The input data are only the mass of the Earth and its moment of inertia, and the kernel array will be built by the values of shell thickness. Dziewonski & Anderson (1981) have used those quantities as constraints. Constraints can, however, be introduced as data, or in other words, they can contribute directly to the determination or estimation of the model parameters, and not only to limit the range of variation in the model space.

A given density distribution with depth establishes the numerical values of the mass (M) and moment inertia (J). Nevertheless, the parameters M and J cannot be inverted to recover the true density profile. The problem is nonunique: there is an infinite number of density distributions which generate the observed values M and J. We will solve the nonuniqueness problem by using prior information in the MRE method.

For the forward modelling we consider the relationships,

$$M = 4\pi \int_0^R \rho(r)\, r^2\, dr, \tag{3}$$

and

$$J = \frac{8\pi}{3} \int_0^R \rho(r) r^4\, dr \tag{4}$$

Parker (1972) discusses the question of density inversion of planets with the knowledge of the mean density $\bar{\rho}$ and J/MR^2:

$$\bar{\rho} = \frac{3}{R^3} \int_0^R r^2 \rho(r) dr, \tag{5}$$

and

$$\frac{J}{MR^2} = \frac{2}{\bar{\rho} R^5} \int_0^R r^4 \rho(r)\, dr \tag{6}$$

which are equivalent to the Eqs. (3) and (4) respectively; and R is the Earth's radius.

Discretizing the above equations, by considering a geometry of spherical and concentric shells, we have (Rietsch, 1977)

$$M = \frac{4\pi}{3} \sum_{i=1}^{N} \rho_i \left(r_i^3 - r_{i-1}^3\right), \tag{7}$$

and

$$J = \frac{8\pi}{15} \sum_{i=1}^{N} \rho_i \left(r_i^5 - r_{i-1}^5\right). \tag{8}$$

The modelling (forward or inverse) is then expressed by

$$\mathbf{d} = \mathbf{Gm}, \tag{9}$$

or

$$d_j = \sum_{i=1}^{N} g_{ji} m_i, \qquad j = 1,2. \tag{10}$$

where

d_j = data vector;

g_{ji} = kernel array; and

m_i = model parameters (unknown).

Let us write the data elements explicitly:

$$d_1 = M = \sum_{i=1}^{N} g_{1i} \rho_i, \tag{11}$$

and

$$d_2 = J = \sum_{i=1}^{N} g_{2i}\rho_i, \tag{12}$$

where

$$g_{1i} = \frac{4\pi}{3}\left(r_i^3 - r_{i-1}^3\right) \tag{13}$$

and

$$g_{2i} = \frac{8\pi}{15}\left(r_i^5 - r_{i-1}^5\right) \tag{14}$$

In order to make comparisons we apply the generalized inversion (G1) by singular value decomposition (SVD), the solution of which has the minimum norm. The objective function to be minimized is

$$\mathbf{\Phi(m) = m^{T} \cdot m + t^{T} \cdot (d - Gm)}, \tag{15}$$

where **t** is the vector of Lagrange multipliers. The minimization yields

$$\mathbf{m = G^{T}(GG^{T})^{-1}\, d}. \tag{16}$$

The above equation is equivalent to the so called "pseudo-inverse" solution for underdetermined systems developed by Moore and later by Penrose (1955). One versatile way to calculate the pseudo-inverse is through Lanczos decomposition (Lanczos, 1961), where the kernel matrix is expressed as

$$\mathbf{G = U\Sigma V^{T}}, \tag{17}$$

where **U** is the matrix which contains the orthonormalized eigenvectors of $\mathbf{GG}^T$, **V** contains the orthonormalized eigenvectors of $\mathbf{G}^T\mathbf{G}$ and the diagonal matrix Σ is formed by the singular values of **G** . Thus, the pseudo-inverse $\mathbf{G}^+$ will be given by

$$\mathbf{G^{+} = V\Sigma^{-1}U^{T}}. \tag{18}$$

We can include prior information in a direct form, when for instance one wishes a solution close to the average one, and not necessarily a solution for which the norm is close to zero – the minimum norm. In this case the objective function will be

$$\mathbf{\Phi(m) = (m - m_0)^{T} (m - m_0) + 2t^{T} (d - Gm)}, \tag{19}$$

where $\mathbf{m}_0$ is the prior information of the model, **t** is again the vector of Lagrange multipliers, and the factor **2** is only to facilitate the algebraic calculations. The solution in this case is

$$\mathbf{m = G^{T}(GG^{T})^{-1} (d - Gm_0) + m_0}. \tag{20}$$

ENTROPY, MAXIMUM ENTROPY, AND MINIMUM RELATIVE ENTROPY

The Newtonian physics, which dominated from the 17th to the end of the 19th century, describe generally an universe in which everything should happen precisely in accordance with a law, where this universe is compact, organized, and all future should strictly depend on the past. However, we can never check, by means of our imperfect experiments, if this or that set of physical laws is verifiable up to the last decimal figure. The Newtonian conception, nevertheless, had to present and express the physical process, as if it had, in fact, been subjected to those laws (Wiener, 1973).

From the middle of the last century, a revolution without precedent began in the history of physics. This revolution, based on the idea of a contingent universe, changed the concept of physics. Now, instead of stating that some physical event will happen in any case, whatever the conditions, one states that there is an overwhelming probability that the event will happen.

The main principle of this process is the concept of entropy, which can be defined in several ways. According to the increase of entropy, the universe and all closed systems, tend naturally to deteriorate and lose their clarity, to change from a state for minimum probability to another of maximum probability, from a state of organization and differentiation, in which there exist forms and distinctions, to a state of chaos. In the contingent universe, the order is less probable. The role of entropy is so important that Jaynes (1957) states that entropy is a primitive physical concept, even more fundamental than the concept of energy. The concept of entropy was developed by the German Rudolf Clausius, in the context to classical thermodynamics. Later the Austrian physicist Ludwig Boltzmann gave the statistical interpretation of entropy.

However, we make use of entropy within the framework of Information Theory. In 1948, Shannon (Shannon & Weaver, 1949) employed the concept of entropy in Information Theory: consider a source S emitting messages $m_1, m_2, \ldots, m_N$ with probabilities $p_1, p_2, \ldots, p_N$ respectively (where $p_1 + p_2 + \ldots + p_N = 1$). The information carried by each message is given by I_i, where

$$I_i = \log\left(\frac{1}{p_i}\right) \tag{21}$$

The entropy (H) is defined to be the average information of the source, i.e.,

$$H(S) = \sum_{i=1}^{N} p_i I_i = \sum_{i=1}^{N} p_i \log(p_i) \tag{22}$$

It would be interesting to determine the distribution that maximizes the entropy. Since entropy is a measure of uncertainty, the probability distribuiton which generates maximum uncertainty will have maximum entropy. In the absence of prior information, Jaynes (1957) stated that the maximum entropy is the least biased estimate from a given information. In the context of Prediction Theory, the maximization of entropy is not the application of a physical law, but merely a reasoning method which guarantees that no inconsistent assumptions were used.

The principle of maximum entropy (PME) is applicable to any inference problem with incomplete data, be it, or not, a repetitive situation like a random experiment (Jaynes, 1982). A proof of the consistency of PEM is given by Tikochinsky et al. (1984) where it is demostrated that the maximum entropy distribution, constrained to average values, is the only consistent induction from the data for any reproducible experiment.

The principle of minimum relative entropy (MRE) was developed in the field of statistical inference by Kullback & Leibler (1951); here we present it within the framework of the Information Theory. In the transmission of a message there is an undesirable signal called noise. Due to the noise, there is a probability called *a priori* (p), that a specific message will be emitted, and a probability called *a posteriori* (q), that this message will reach the addressee. Thus, the amount of information I obtained in a message transmission increases with the *a posteriori* probability, and is a ratio between the *a posteriori* and the *a priori*,

$$I = \log \frac{q}{p}, \tag{23}$$

in such a way that the entropy in the continuous case will be

$$H(q,p) = \int_D q(x) \log \frac{q(x)}{p(x)} dx \tag{24}$$

where x is the integration variable, in the domain D.

DENSITY INVERSION BY MRE

Here we present, in a condensed way, the MRE approach for the soluction of inverse problems, in particular the determination of the Earth's density distribution, given its mass and moment of inertia. For details, see Bassrei (1990a), or Ulrych et al. (1990a). We consider the kind of problem which can be described by a discretized Fredholm integral equation of the first kind

$$d_j = \sum_{n=0}^{N} f_j \, \overline{\rho}(n) \qquad j = 1,\dots, M \tag{25}$$

where d_j are the data, $f_j(n)$ are the kernel functions, and $\overline{\rho}(n)$, $n = 0,\dots, N$ are the estimates of the model parameters (density). Note that M in this section is not the Earth's mass, but the number of data points; in this particular problem $M = 2$. We consider that these estimates are the expected values of a random vector, $\boldsymbol{\rho}^T = \rho(n) = [\rho(0), \rho(1),\dots, \rho(N)]$, where we have that

$$\overline{\rho}(n) = \int_0^\infty \rho(n) q(\rho) d\rho, \qquad n = 0,\dots, N \tag{26}$$

The integral of relative entropy is given by

$$H(q,p) = \int_0^\infty q(\rho) \log \frac{q(\rho)}{p(\rho)} d\rho, \tag{27}$$

where $q(\rho)$ and $p(\rho)$ are respectively the posterior and prior pdf's (probability density function) of the model. With the last three equations plus an adequate prior pdf given by

$$p(\rho) = \prod_{n=0}^{N} \frac{1}{\rho_0(n)} e^{-\frac{\rho(n)}{\rho_0(n)}} \tag{28}$$

and a normalization condition given by

$$\int_0^\infty q(\rho) d\rho = 1, \tag{29}$$

we can build an objective function with $M+1$ Lagrange multipliers

$$\Phi(\rho) = \int_0^\infty q(\rho)\log\frac{q(\rho)}{p(\rho)}d\rho + \mu\left[\int_0^\infty q(\rho)d\rho - 1\right]$$

$$+\sum_{j=1}^{M}\lambda_j \int_0^\infty \left[q(\rho)\sum_{n=0}^{N} f_j(n)\rho(n)d\rho - d_j\right]. \qquad (30)$$

The minimization of the above equation with the use of the constraints yields

$$\bar{\rho}(n) = \frac{1}{\frac{1}{\rho_0(n)} + \sum_{j=1}^{M}\lambda_j f_j(n)}, \qquad n = 0,\dots, N \qquad (31)$$

where λ_j are the Lagrange multipliers used in the objective function, and $\rho_0(n)$ is the prior information in the same physical dimension of the model. The prior pdf $p(\rho)$ used here comes from the entropy maximization constrained by expected values (Eq.26) and normalization (Eq.29). Since we have a nonlinear system of equations, we can, for instance, use the Newton-Raphson technique for the determination of λ's, and then obtain the posterior estimate. For the data vector, we have used the following values from Stacey (1969): $M = 5.976 \times 10^{27} g$ and $J = J_{POLE} = 8.068 \times 10^{44} g.cm^2$. In order to make comparisons, we have considered the Bullen A, HB1 and PREM models as the "true" or reference model. These reference models generate by forward modeling M and J that are very close to the ones as input data. The HB1 model (Stacey, 1969) for instance, yielded the values of mass of the Earth and its moment of inertia with an error about 1 and 1.6%, respectively.

In any reference model we note that there is an abrupt change in the value of density from the mantle to the outer core. The neglect of this change in the selection of the prior estimate (for instance as in a linear increase in the estimate from the surface to the Earth's centre) leads to unsatisfactory results. Thus, we will take the discontinuity into account in the choise of the prior information. We stress, nevertheless, that such discontinuity has been known already for a long time and the fact that we incorporate it in to the prior information does not mean that we are making use of the entire reference model. Thus, as a general criterion, in all inversions, we will consider a linear increase prior before, and a flat prior after the discontinuity point. The model A (Bullen, 1965) has 22 points in the model space. The HB1 model (Stacey, 1969) has 51 points, and the PREM (Dziewonski and Anderson, 1981), 94 points.

The results are shown in Figs. 1 to 9. Since the knowledge of density variation is more uncertain in the layers nearest to Earth centre, we have used here weighted initial estimates (Johnson et al., 1984). The second value of weight (W2) is for the distance between the surface and the discontinuiy, and the first value (W1) for the part limited by the discontinuity and the Earth centre. In this manner, the usage of weights 0.5 and 1.0 (Figs. 2,5 and 8) resulted in a better output when compared to the result associated with weights 1.0 and 0.5 (not shown). For shallow depths, we have large radius values associated to the kernel array, producing thus large kernel array values. Mathematically, this means that each datum has a major physical contribution from points closest to the surface. Consequently, there is a higher sensibility in this region as compared to large depths. It is interesting to observe that the application of weight led to previously expected results.

Since we have an under-determined problem, the discontinuity in the prior information may result in unrealistic discontinuity in the obtained solutions – and it actually does. This problem can be partially solved by the use of filters, and we have used a convolutional filter ($\rho(n) = 0.25\rho(n-2) + 0.50\,\rho(n-1) + 0.25\rho(n)$). The code SOFT5 appearing in Tab.2 and figure captions indicates that this filter was applied five times to the solution (both MRE and SVD). The use of the filter yielded good results, for all cases: Bullen A (Fig. 3), HB1 (Fig. 6) and PREM (Fig. 9).

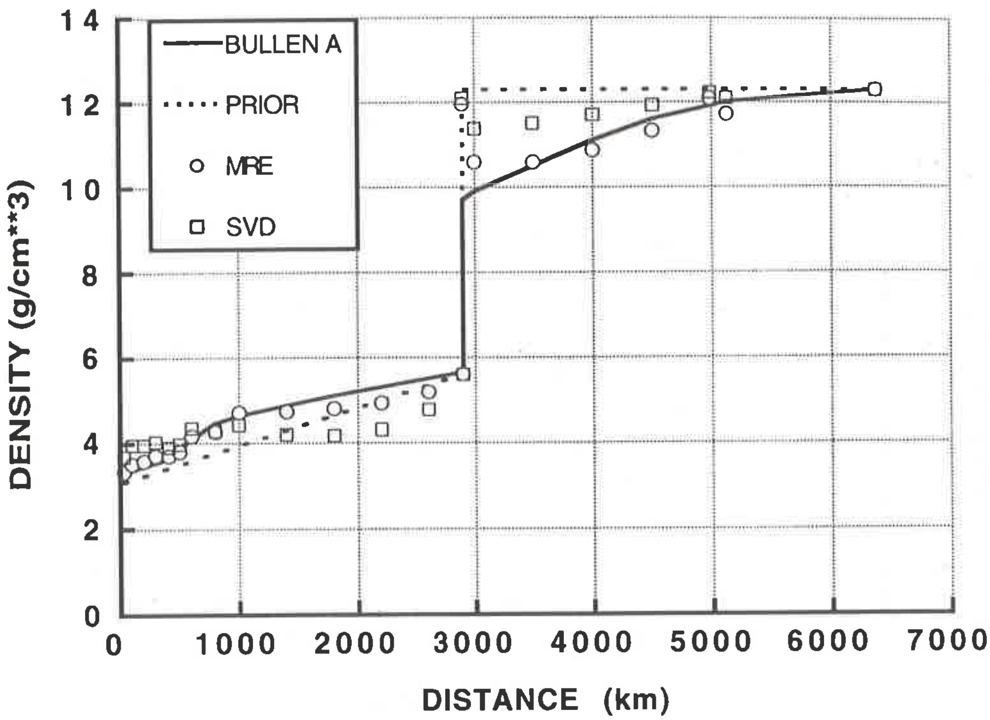

Figure 1 - Comparison of the curves: reference BULLEN A, the input PRIOR, and the results by MRE and SVD. Weights: 1.0 and 1.0.

Figura 1 *- Comparação entre as curvas: referência BULLEN A, informação prévia PRIOR, e os resultados por SVD e MRE. Pesos: 1,0 e 1,0.*

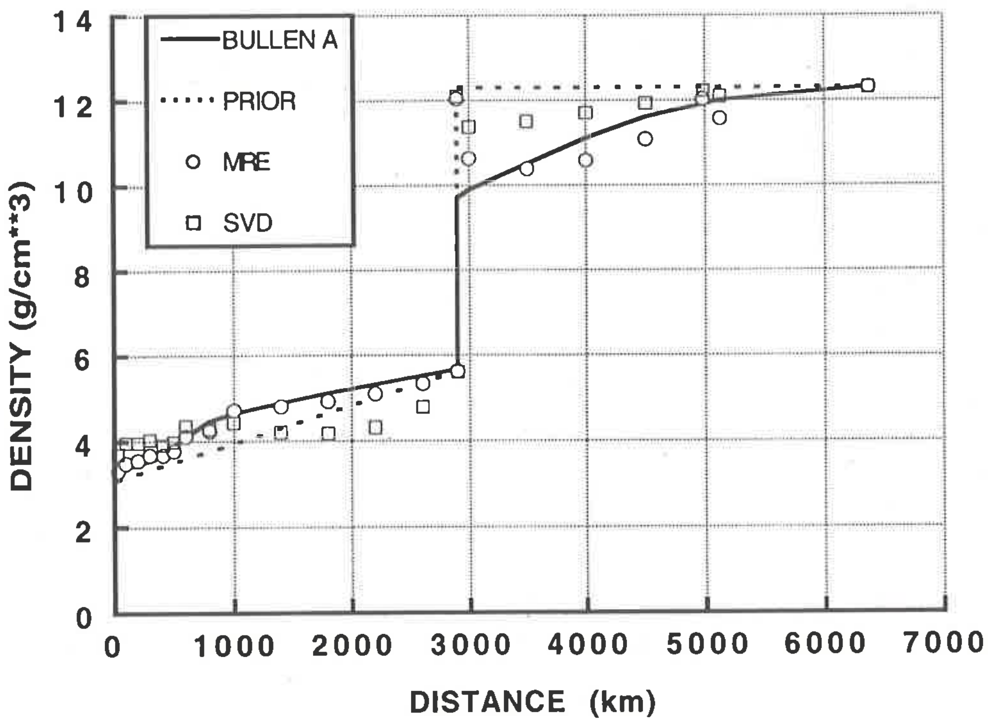

Figure 2 - Comparison of the curves: reference BULLEN A, the input PRIOR, and the results by MRE and SVD. Weights: 0.5 and 1.0.

Figura 2 *- Comparação entre as curvas: referência BULLEN A, informação prévia PRIOR, e os resultados por SVD e MRE. Pesos: 0,5 e 1,0.*

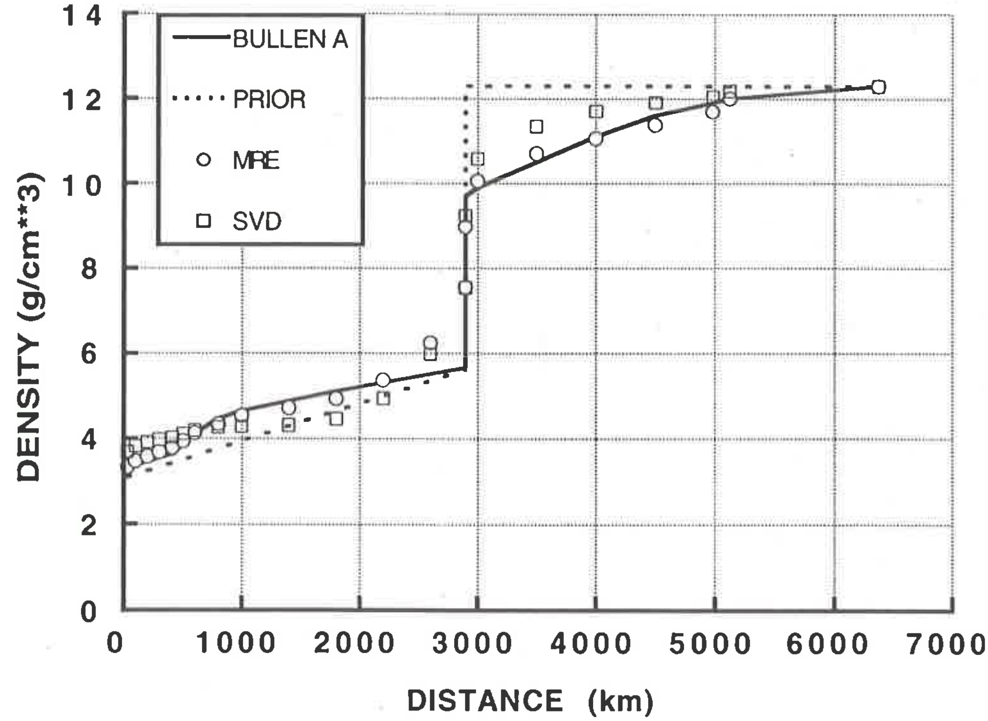

Figure 3 - Comparison of the curves: reference BULLEN A, the input PRIOR, and the results by MRE and SVD. Weights: 1.0 and 1.0. Results filtered by SOFT5.

***Figura 3** - Comparação entre as curvas: referência BULLEN A, informação prévia PRIOR, e os resultados por SVD e MRE. Pesos: 1,0 e 1,0. Resultados filtrados por SOFT5.*

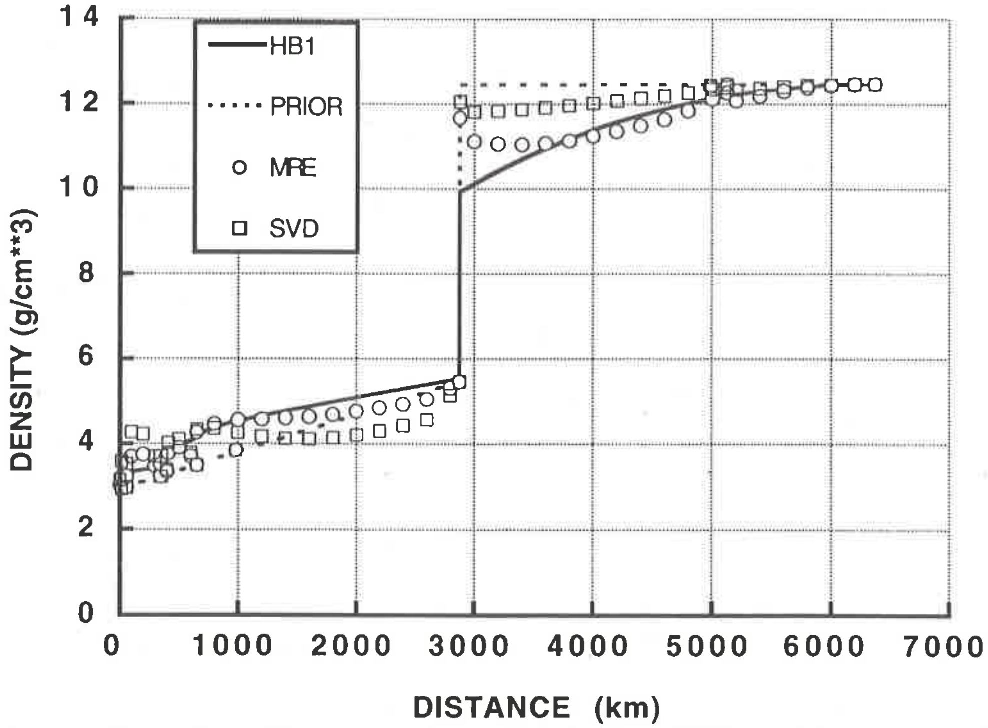

Figure 4 - Comparison of the curves: reference HB1, the input PRIOR, and the results by MRE and SVD. Weights: 1.0 and 1.0.

***Figura 4** - Comparação entre as curvas: referência HB1, informação prévia PRIOR, e os resultados por SVD e MRE. Pesos: 1,0 e 1,0.*

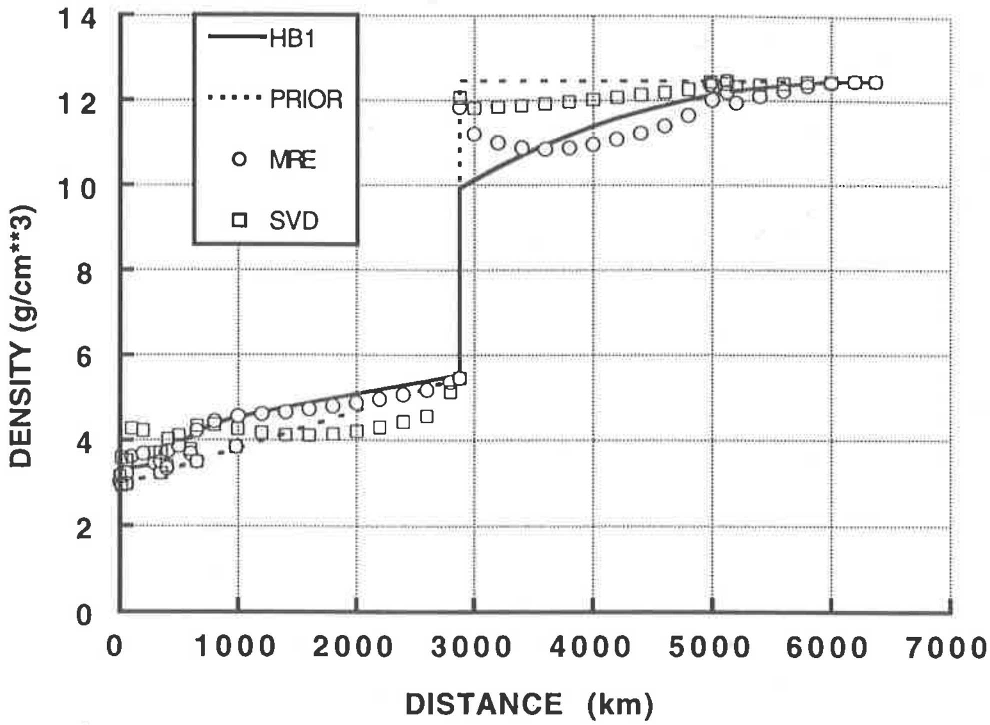

Figure 5 - Comparison of the curves: reference HB1, the input PRIOR, and the results by MRE and SVD. Weights: 5.0 and 1.0.

***Figura 5** - Comparação entre as curvas: referência HB1, informação prévia PRIOR, e os resultados por SVD e MRE. Pesos: 0,5 e 1,0.*

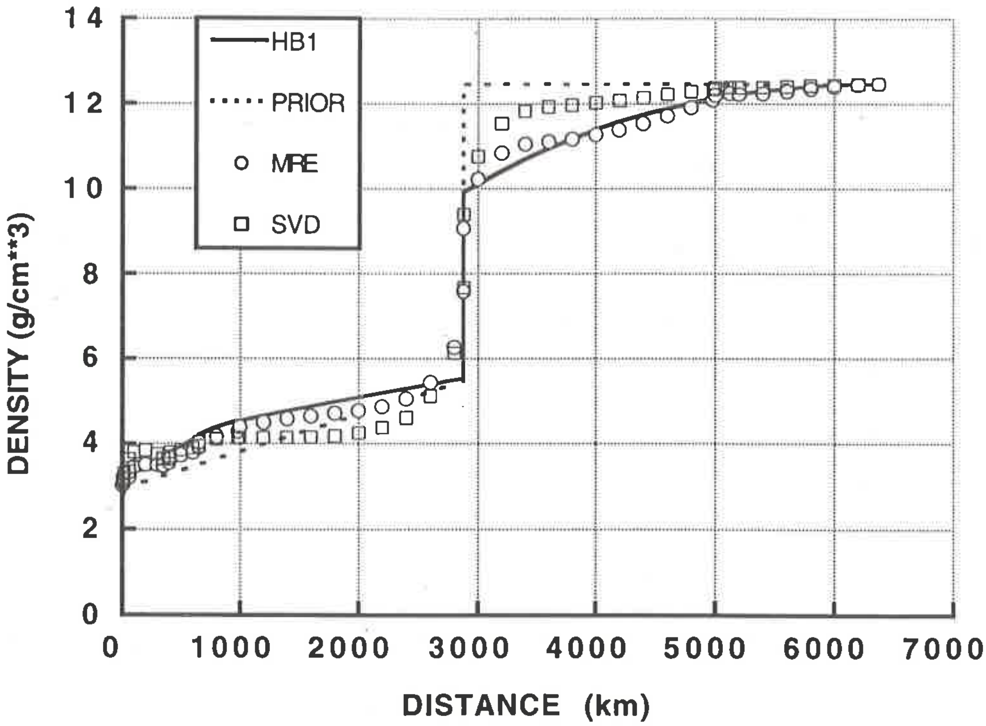

Figure 6 - Comparison of the curves: reference HB1, the input PRIOR, and the results by MRE and SVD. Weights: 1.0 and 1.0. Results filtered by SOFT5.

***Figura 6** - Comparação entre as curvas: referência HB1, informação prévia PRIOR, e os resultados por SVD e MRE. Pesos: 1,0 e 1,0. Resultados filtrados por SOFT5.*

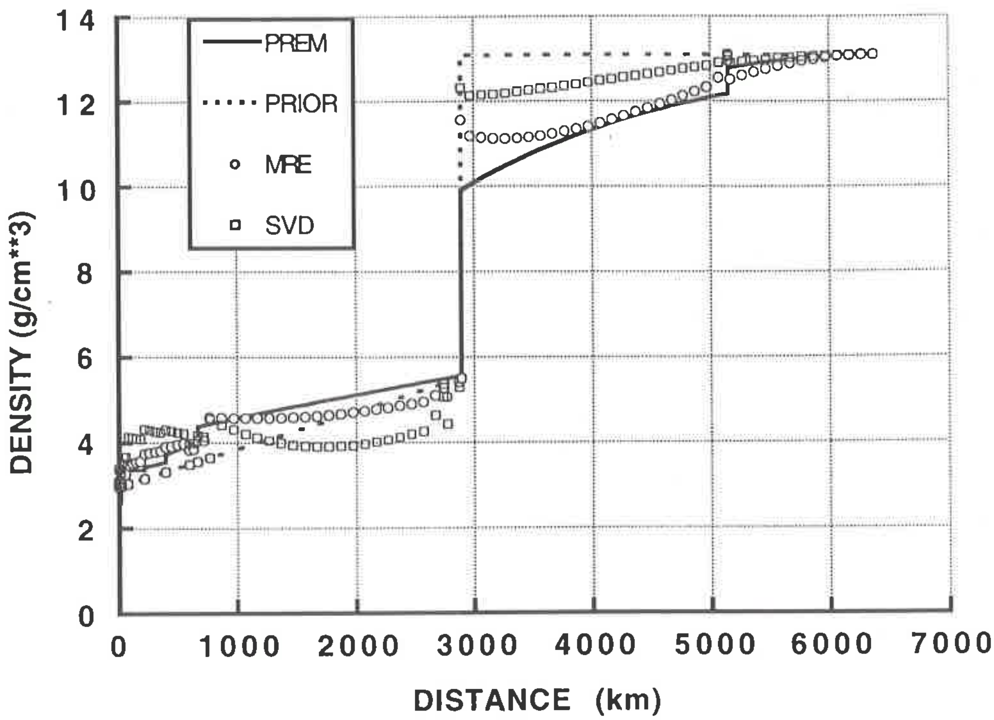

Figure 7 - Comparison of the curves: reference PREM, the input PRIOR, and the results by MRE and SVD. Weights: 1.0 and 1.0.

***Figura** 7 - Comparação entre as curvas: referência PREM, informação prévia PRIOR, e os resultados por SVD e MRE. Pesos: 1,0 e 1,0.*

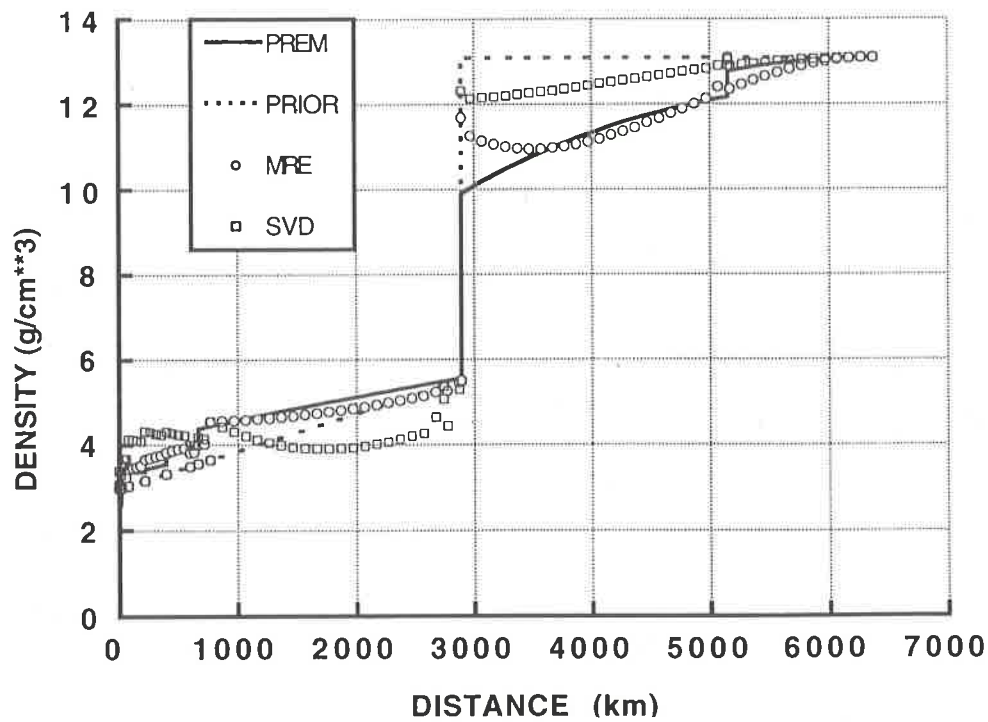

Figure 8 - Comparison of the curves: reference PREM, the input PRIOR, and the results by MRE and SVD. Weights: 0.5 and 1.0.

***Figura** 8 - Comparação entre as curvas: referência PREM, informação prévia PRIOR, e os resultados por SVD e MRE. Pesos: 0,5 e 1,0.*

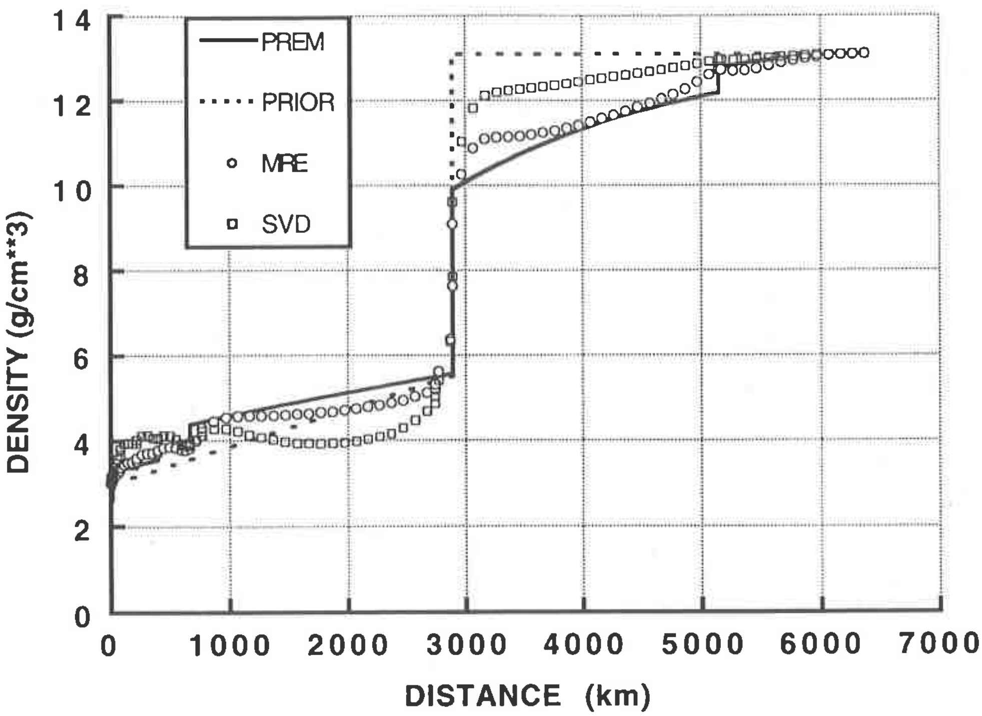

Figure 9 - Comparison of the curves: reference PREM, the input PRIOR, and the results by MRE and SVD. Weights: 1.0 and 1.0. Results filtered by SOFT5.

Figura 9 - *Comparação entre as curvas: referência PREM, informação prévia PRIOR, e os resultados por SVD e MRE. Pesos: 1,0 e 1,0. Resultados filtrados por SOFT5.*

The nine figures presented are in fact a selection from a large number of simulations that can be done by mixing several kinds of weight and filtering. However, we believe that the results are enough to give a feeling for the use of this approach. The results are condensed in Tab. 2, in particular so as to compare the model misfit, i.e. the percentual error between the reference models (Bullen A, HB1 and PREM) and the input prior, or the outputs by MRE and SVD, as given by the following equations:

$$\Delta_1 = \frac{1}{N} \sum_{n=0}^{N} \frac{\left|\rho_{MODEL}(n) - \rho_{PRIOR}(n)\right|}{\rho_{MODEL}(n)} \times \quad (32)$$

$$\Delta_2 = \frac{1}{N} \sum_{n=0}^{N} \frac{\left|\rho_{MODEL}(n) - \rho_{MRE}(n)\right|}{\rho_{MODEL}(n)} \times \quad (33)$$

and

$$\Delta_3 = \frac{1}{N} \sum_{n=0}^{N} \frac{\left|\rho_{MODEL}(n) - \rho_{SVD}(n)\right|}{\rho_{MODEL}(n)} \times 1 \quad (34)$$

Another feature that was tested in the algorithm was the hydrostatic stability (Stacey, 1969), that is, density cannot decrease significantly toward the center without demanding implausibly high strength. This was implemented approximately by not allowing decreasing values of density with depth. The result was not so different relative to those not using this constraint.

FIG.	MODEL	W1	W2	FILTER	Δ_1 (%)	Δ_2 (%)	Δ_3 (%)
1	BULLEN A	1.0	1.0	NO	9.38	3.90	9.42
2	BULLEN A	0.5	1.0	NO	9.38	3.63	9.42
3	BULLEN A	1.0	1.0	SOFT5	9.38	4.13	8.35
4	HB1	1.0	1.0	NO	8.76	4.25	8.35
5	HB1	0.5	1.0	NO	8.76	4.20	8.08
6	HB1	1.0	1.0	SOFT5	8.76	3.92	6.96
7	PREM	1.0	1.0	NO	13.96	8.81	15.27
8	PREM	0.5	1.0	NO	13.96	8.09	15.27
9	PREM	1.0	1.0	SOFT5	13.96	8.52	14.62

Table 2. Model misfit correspondent to Figs.1 to 9 (See text for W1, W2, Δ_1 , Δ_2 , Δ_3).
Tabela 2. *Erros no modelo referente às Figs. 1 a 9. (Vide texto para* W1, W2, Δ_1 , Δ_2 , Δ_3 *).*

CONCLUSIONS

A stochastic tecnique based on an entropy principle for the inversion of the Earth's radial density distribution has been demostrated. The general procedure to get the radial density profile of the Earth requires the use of several kinds of data. We simplified this problem considerably by using only the mass and the moment of inertia, as well as a good prior information. However, we are not proposing a new reference model for the density distribution. We explored the use of a probabilistic approach to a well known geophysical problem presented here as an ill-posed, underdetermined inverse problem. MRE is a method that allows the easy inclusion of the *a priori* information; its posterior estimate is the solution of the minimization problem, that is consistent with the input data. We consider the results satisfactory as long as the MRE output is close, both in a qualitative and quantitative sense, to the reference model, and the results were better when compared to the generalized inverse, using the same prior information. We stress that only two data were used to reconstruct the model (with 22, 51 and 94 points). This means that the problem is "critically" underdetermined, thence the disagreeing results for inconsistent priors. The consistent prior which we used, applied the knowledge of an abrupt difference of density between mantle and outer core, of a linear increasing curve between the surface and that discontinuity, and, of an uniform value for the outer and inner core. In particular, that a flat prior for the latter region modeled reasonably the values of density closer to Eath center.

ACKNOWLEDGMENTS

A preliminary version of this work was presented at the 2nd International Congress of the Brazilian Geophysical Society held at Salvador, Brazil, in October 1991. This work was supported by the Brazilian agency CNPq – Conselho Nacional de Desenvolvimento Científico e Tecnológico through a research scholarship (300.209-90/8) and a post-doctoral scholarship (202.601-91/9), and also initially, by a doctorate scholarship from PETROBRÁS. The author wishes to thank Prof. V. Červený for interesting discussions, Prof. T. Ulrych for kindly introducing to him the MRE method, and two anonymous reviewers for several suggestions for the improvement of this manuscript. Finally, the author had the benefit of the kind hospitality and computational facilities at the Earth Resources Laboratory of MIT.

REFERENCES

BASSREI, A. - 1990a - Minimum relative entropy inversion of one-dimensional geophysical data. Doctoral thesis (in Portuguese, with abstract in English), Federal University of Bahia.

BASSREI, A. - 1990b - Inversion of seismic data by a relative entropy algorithm. Expanded abstract, 60th Annual International SEG Meeting, San Francisco, USA, volume **II:** 1111-1114.

BASSREI, A. - 1991a - Inversion of 2-D gravity data by a relative entropy algorithm. 53rd EAEG Meeting, Florence, Italy, 410-411.

BASSREI, A. - 1991b - Inversion of the Earth radial density distribution by an Information Theory Approach. Expanded abstract, 2nd International Congress of the Brazilian Geophysical Society, Brazil, volume **I:** 205-211.

BASSREI, A. & PESTANA, R.C., - 1991 - Informative inversion of seismic acoustic data in the tau-p domain. Expanded abstract, 61st Annual International SEG Meeting, Houston, USA, volume **II:** 951-954.

BASSREI, A. & ULRYCH, T.J. - 1989 - Minimum relative entropy inversion of geophysical data. Expanded abstract, 1st International Congress of the Brazilian Geophysical Society, Rio de Janeiro, Brazil, volume **I:** 159-166.

BULLEN, K.E. - 1965 - An Introduction to the Theory of Seismology. 3rd. ed., Cambridge University Press, London.

BULLEN, K.E. & HADDON, R.A.W. - 1967 - Derivation of an Earth model from free oscillation data. Proceedings of the National Academy of Sciences, **58:** 846-852.

DZIEWONSKI, A.M., & ANDERSON, D.L. - 1981 - Preliminary reference Earth model. Phys. Earth Planet-Inter, **25:** 297-356.

GRABER, M.A. - 1977 - An Information Theory approach to the density of the Earth. NASA Technical Memorandum 78034, Goddard Space Flight Center.

JAYNES, E.T. - 1957 - Information theory and statistical mechanics. Phys. Rev., **106:** 620-630.

JAYNES, E.T. - 1982 - On the rationale of maximum-entropy methods. Proceedings of the IEEE, **70:** 939-952.

JOHNSON, R.W., SHORE, J.E., & BURG, J.P. - 1984 - Multisignal minimum cross entropy spectrum analysis with weighted initial estimates. IEEE Transactions on ASSP, **32:** 531-539.

KULLBACK, S. - 1959 - Information Theory and Statistics. Wiley, New York, 1969, Dover.

KULLBACK, S. & LEIBLER, R.A. - 1951 - On information and sufficiency. Annals of Mathematical Statistics. **22:** 79-86.

LANCZOS, C. - 1961 - Linear Differential Operators. Van Nostrand, London.

PARKER, R.L. - 1972 - Inverse theory with grossly inadequate data. Geophys. J.R. astr. Soc., **29:** 123-138.

PENROSE, R. - 1955 - A generalized inverse for matrices. Proceedings of the Cambridge Philosophical Society, **51:** 406-413.

RIETSCH, E. - 1977 - The maximum entropy approach to inverse problems. J. Geophys., **42:** 489-506.

RIETSCH, E. - 1986 - Maximum entropy and the Earth's density, in Maximum Entropy and Bayesian Methods in Applied Statistics. Justice et al. (ed.). 254-268.

RUBINCAM, D.P. - 1978 - Information Theory and the Earth's density distribution. NASA Technical Memorandum 78088, Goddard Space Flight Center (also revised edition, 1979, number 80586).

RUBINCAM, D.P. - 1982 - Information Theory lateral density distribution for Earth inferred from global gravity field. J. Geophys. Res., **87:** 5541-5552.

SHANNON, C.E. & WEAVER, W. - 1949 - The Mathematical Theory of Communication. University of Illinois Press, Urbana.

SHORE, J.E. - 1981 - Minimum cross-entropy spectral analysis. IEEE Transactions on ASSP, **29:** 230-237.

SHORE, J.E. & JOHNSON, R.W. - 1980 - Axiomatic derivation of the principle of the minimum cross entropy. IEEE Transactions on IT, **26:** 26-37.

STACEY, F.D. - 1969 - Physics. of the Earth. Wiley, New York.

TIKOCHINSKY, Y., TISHBY, N.Z. & LEVINE, R.D. - 1984 - Consistent inference of probabilities for reproducible experiments. Phys. Rev. Lett., **52:** 1357-1360.

ULRYCH, T.J., BASSREI, A., & LANE, M. - 1990a - Minimum relative entropy inversion of one-dimensional data with applications. Geophysical Prospecting, **38**: 465-487.

ULRYCH, T.J., LEANEY, W.S., JENSEN, O.G., & BASSREI, A. - 1990b - Inference, inversion, Bayes and entropy. Expanded abstract, 60th Annual International SEG Meeting, San Francisco, USA, volume II: 1115-1118.

WIENER, N. - 1973 - Cibernética e Sociedade (The Human Use of Human Beings). Cultrix, São Paulo.

WILLIAMSON, E.D. & ADAMS, L.H. - 1923 - Density distribution in the Earth. Journal of the Washington Academy of Sciences, **13**: 413-428.

Submetido em: 16-10-92
Revisado pelo(s) autor(es) em: 21-02-94
Aceito em: 04-03-94