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THE TRAVELTIME MODELLING IN FACTORIZED ANISOTROPIC INHOMOGENEOUS MEDIA BY THE PERTURBATION METHODS

Ivan de Araújo Simões Filho

Traveltime computations in factorized anisotropic inhomogeneous (FAI) media are performed by ray tracing and perturbation methods. In the FAI media the spatial variation of a 21 density normalized elastic parameters is assumed to be the same, $a_{ijkl}(x_s) = f^2(x_s)A_{ijkl}$, where $f(x_s)$ is a continuous smooth function of Cartesian coordinates x_s and A_{ijkl} are constants, independent of x_s. The types of anisotropy (A_{ijkl}) and inhomogeneties $[f(x_s)]$ are not restricted, but their effects are separated. The results of computations obtained by both methods are used to check the accuracy and limitations of the perturbation approach. The advantage of this method is that rays computed by the 2-D isotropic ray tracing can be used to estimate travel times in a more general 3-D anisotropic media. The least-squares criterium is used to minimize the differences between perturbed and background media. Examples of orthorhombic and transversely isotropic media are presented, with VSP and surface acquisition geometries, including layered media.

Key Words: Traveltimes; Ray tracing; Seismic anisotropy; Perturbation methods; Shear-waves.

MODELAGEM DE TEMPOS DE TRÂNSITO EM MEIOS HETEROGÊNEOS ANISOTRÓPICOS FATORADOS PELO MÉTODO DA PERTURBAÇÃO *Tempos de trânsito em meios heterogêneos anisotrópicos fatorados (HAF) foram calculados por traçado de raios e pelo método da perturbação. Nos meios HAF, considera-se que a variação espacial é a mesma para todos os 21 parâmetros elásticos divididos pela densidade, que são descritos por $a_{ijkl}(x_s) = f^2(x_s)A_{ijkl}$, onde $f(x_s)$ é uma função suave contínua das coordenadas cartesianas x_s, e A_{ijkl} são 21 constantes. Os tipos de heterogeneidade $[f(x_s)]$ e de anisotropia (A_{ijkl}) não sofrem restrições e seus efeitos são separados. Os tempos de trânsito obtidos pelos dois métodos foram comparados para a investigação da precisão e das limitações do método da perturbação. A vantagem deste é que os raios calculados por traçado de raios isotrópico 2-D podem ser usados para recalcular tempos de trânsito em meios anisotrópicos 3-D mais gerais. Em contrapartida, o método da perturbação se aplica apenas para meios fracamente anisotrópicos. Para diminuir as perturbações, a diferença entre os parâmetros elásticos divididos pela densidade do meio anisotrópico a ser modelado e do meio de referência isotrópico são minimizados pelo critério dos mínimos quadrados. Os exemplos apresentados são de meios anisotrópicos com simetria ortorrômbica e transversalmente isotrópicos; com geometria de aquisição de poço e superfície, incluindo meios estratificados.*

Palavras-Chaves: Tempos de trânsito; Traçado de raios; Anisotropia sísmica; Método de perturbação; Ondas cisalhantes.

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INTRODUCTION

The interest in seismic wave propagation in anisotropic media has considerably increased over the last few years. This is due to the development of shear-wave prospecting (shear-waves carry 3 or 4 times more information on anisotropy than compressional-waves, see Crampin, 1985), which show an anisotropic behaviour of most rocks within the crust and upper mantle (Leary, Crampin & McEvilly, 1990). Another source of interest in anisotropic propagation is the anisotropic response of media with aligned inclusions and fine layering, when investigated with wavelengths much larger than the dimensions of the inclusions or layers (Hudson, 1981; Crampin, 1984; Schoenberg & Muir, 1989). This opens up the possibility, with the use of multi-component seismic data, to study the behaviour of fluid-filled cracks to monitor fluid recovery and estimate reservoir parameters.

As the general anisotropic media are described by 21 independent elastic parameters $\mathbf{c}_{ijkl}$, which may all depend on coordinates in a different way, the parametrization of such media is complex, so that some simplifying assumptions related to the type fo heterogeneity (assuming that the elastic parameters follow some simple spatial variation) or to the type of anisotropy (considering higher order symmetry of elastic parameters) are usually done.

The concept of factorized anisotropic inhomogeneous (FAI) media considerably simplifies the parametrization of anisotropic inhomogeneous structures. In the FAI media, the spatial variation of all 21 density normalized elastic parameters is assumed to be the same,

$$a_{ijkl}(x_s) = c_{ijkl}(x_s) \; / \; \rho = f^2(x_s) A_{ijkl} \tag{1}$$

The function of Cartesian coordinates $f(x_s)$ represents the common spatial variation and A_{ijkl} are 21 constants representing the anisotropic properties (the reduced anisotropic constants). The types of anisotropic $[A_{ijkl}]$ and inhomogeneity ($f(x_s)$) are not restricted (within the validity of the ray method), but their effects are separated. The concept of FAI medium was introduced by Červený (1989), where the ray theory equations in FAI medium are treated in detail.

Even though the solution of the direct kinematic problem in anisotropic inhomogeneous media by the ray method is well known (Červený, 1972; Červený, Molotkov & Pšenčík, 1977), it is relatively time consuming, specially for two-point ray tracing. Therefore simpler approximate procedures based on perturbation theory may be very useful. These procedures considerably increase the time efficiency of traveltime computations in inhomogeneous slightly anisotropic media. They have been broadly applied in seismology (Backus, 1965; Červený, 1982; Červený & Firbas, 1984; Jech & Pšenčík, 1989).

The concept of FAI medium also simplifies the traveltime perturbation equations for anisotropic inhomogeneous media. In this paper, this concept is used to model traveltimes in FAI medium with an isotropic background. This case has very important practical applications, since rays and traveltimes can be very simply evaluated with the use of some very general and time efficient isotropic ray codes. In fact, no two-point ray tracing in three-dimensional anisotropic structures is performed. It is replaced by the very efficient two-dimensional isotropic two-points ray tracing. The obtained results are then used to recalculate the traveltimes for an anisotropic inhomogeneous medium.

Special attention was given to the qS-waves traveltime modelling, not only because they are more suitable for seismic anisotropy analysis than qP-waves, but also because the traveltime delay between the two split shear-waves in FAI media with isotropic background is independent of structure perturbations, depending only on anisotropy pertubations (Červený & Simões Filho, 1991).

THE TRAVELTIME MODELLING IN GENERAL INHOMOGENEOUS ANISOTROPIC MEDIA

Rays and traveltime in inhomogeneous anisotropic media are obtained by the solution of a system of ordinary differential equations of the first order, which can be simply written using time T as integration parameter (Červený, Molotkov & Pšenčík, 1977):

$$\frac{dx_i}{dT} = \frac{1}{2}\frac{\partial G^{(m)}}{\partial p_i} = a_{ijkl}\, p_l\, g_k^{(m)}\, g_j^{(m)},$$

$$\frac{dp_i}{dT} = \frac{1}{2}\frac{\partial G^{(m)}}{\partial x_i} = -\frac{1}{2}\frac{\partial a_{sjkl}}{\partial x_i}\, p_s\, p_l\, g_k^{(m)}\, g_j^{(m)}. \tag{2}$$

Here p_i are the Cartesian components of the slowness vector and $g_j^{(m)}$ are the Cartesian components of the m-th eigenvector of the 3X3 symmetric Christoffel matrix, defined by its components $\Gamma_{ik} = a_{ijkl} p_j p_l$. Its eigenvalues $G^{(m)}$(m = 1, 2 or 3) correspond to the three types of body-waves which can propagate in the general inhomogeneous anisotropic media (qS1, qS2 or qP). For isotropic media, the two eigenvalues corresponding to the qS-waves coincide, $G^{(1)} =$

$G^{(2)} = \beta^2 p_i p_i$ and $G^{(3)} = \alpha^2 p_i p_i$. This case corresponds to a degeneracy of the Christoffel matrix.

Although straighforward, the solution of (2) requires the evaluation of 63 partial derivatives of elastic parameters, which is rather cumbersome. The use of the perturbation approach will considerably simplify the traveltime modelling in inhomogeneous slightly anisotropic media, as ray computations may be performed in a simpler medium and the obtained results used to recalculate traveltimes in a more complex medium.

The density normalized elastic parameters a_{ijkl} will be called from now *elastic parameters* for simplicity. I hope there will be no confusion with stiffness c_{ijkl}. Consider an inhomogeneous anisotropic medium $M^{(0)}$, described by elastic parameters $a^{(0)}_{ijkl}$. Construct the ray $\mathcal{L}^{(0)}$ of one of the three types of body waves which can propagate in this medium between points S and R, and compute the corresponding traveltime $T^{(0)}(S,R)$ or the ray $\mathcal{L}^{(0)}$. The medium $M^{(0)}$ is then slightly perturbed and the resulting medium is denoted by M. The new parameters are a_{ijkl} and the new ray is $\mathcal{L}$ (Fig.1).

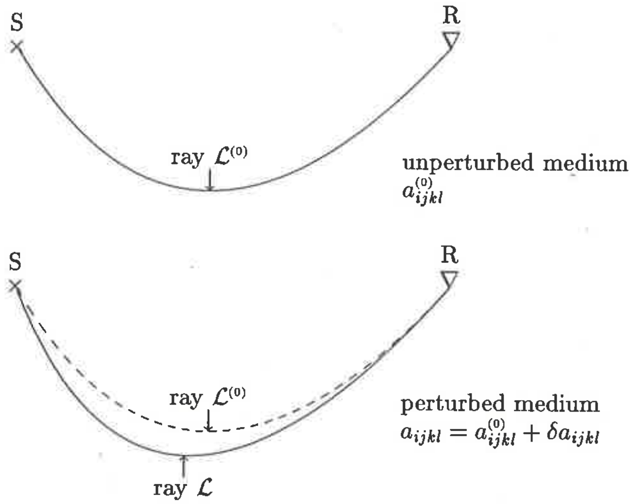


Figure 1 - Rays $\mathcal{L}^{(0)}$ and $\mathcal{L}$ in the background isotropic medium $M^{(0)}$ and in the perturbed medium M [modified from Cervený & Firbas (1984)].

Figura 1 - *Raios $\mathcal{L}^{(0)}$ e $\mathcal{L}$ no meio de referência $M^{(0)}$ e no meio perturbado M [modificado de Cervený & Firbas (1984)].*

The perturbed elastic parameters and traveltime are given by

$$a_{ijkl} = a^{(0)}_{ijkl} + \delta a_{ijkl}$$
$$T(S,R) = T^{(0)}(S,R) + \delta T(S,R) \qquad (3)$$

The general relation between the perturbation of the traveltime δT and the perturbation of the elastic parameters δa_{ijkl} is linear for the nondegenerate case (Červený, 1982).

$$\delta T(S,R) = -\frac{1}{2} \int_{\mathcal{L}^{(0)}\,S}^{R} \delta a_{ijkl}\, p_i\, p_l\, g_k^{(m)} g_j^{(m)}\, dT \qquad (4)$$

The integration is performed along the ray $\mathcal{L}^{(0)}$ in the unperturbed medium, where the quantitites p_i, p_l, $g_k^{(m)}$ and $g_j^{(m)}$ are evaluated. Ray $\mathcal{L}^{(0)}$ is called the frozen ray.

If the eigenvalues of the Christoffel matrix corresponding to the two qS-waves coincide ($G_1 = G_2$), the medium is degenerate. Such situation may occur along shear-wave singularity directions in anisotropic media, or globally in isotropic media. In this case (4) is not valid for qS-waves and non-linear equations must be used (Jech & Pšenčík, 1989).

$$\delta T_{qS1,qS2}(S,R) = -\frac{1}{4} \int_{\mathcal{L}^{(0)}\,S}^{R} \Big\{ D_{11} + D_{22} \pm$$
$$\pm \left[(D_{11} - D_{22})^2 + 4 D_{12}^2 \right]^{1/2} \Big\}\, dT, \qquad (5)$$

with

$$D_{mn} = \delta a_{ijkl}\, p_i\, p_l\, e_j^{(m)} e_k^{(n)}$$

The equations for qS1 and qS2 differ only by the sign "+" or " – " in the integral. Vectors $e^{(1)}$ and $e^{(2)}$ are any two mutually perpendicular unit vectors which are also perpendicular to $g^{(3)}$, the polarization vector of the P-wave. They may be chosen as the polarization vectors of S-waves as such vectors are usually computed in ray tracing codes for isotropic structures.

THE TRAVELTIME PERTURBATIONS IN FACTORIZED ANISOTROPIC INHOMOGENEOUS MEDIA WITH AN ISOTROPIC BACKGROUND

In the equations presented in the previous section, the traveltime perturbations are influenced by both anisotropy and structure perturbations, without a possibility to separate the contribution of each of these perturbations to the final traveltime perturbation. The concept of FAI medium offers the possibility to study separately the contributions of both perturbations. The use of an isotropic background has also very important practical applications, as ray and traveltime computations may be performed in an isotropic inhomogeneous medium and the obtained results used to simply recalculate the traveltimes for an anisotropic inhomogeneous medium.

Both the isotropic inhomogeneous and the perturbed anisotropic media must satisfy the conditions of the FAI media,

$$a_{ijkl}^{(0)} = f^{(0)^2}(x_s)A_{ijkl}^{(0)}, \quad a_{ijkl} = f^2(x_s)A_{ijkl}. \tag{6}$$

To satisfy the conditions of the FAI medium, the S- to P-velocity ratio must be constant through the whole background isotropic medium,

$$v = \beta/\alpha = \text{constant}. \tag{7}$$

The structure perturbations $\delta f^2(x_s)$ and the anisotropy perturbations A_{ijkl} are introduced in the following way,

$$f^2(x_s) = f^{(0)^2}(x_s) + \delta f^2(x_s),$$

$$A_{ijkl} = A_{ijkl}^{(0)} + \delta A_{ijkl}. \tag{8}$$

Considering only first-order perturbations, the perturbation of elastic parameters is

$$\delta a_{ijkl} = A_{ijkl}^{(0)}\,\delta f^2(x_s) + f^2(x_s)\,\delta A_{ijkl} \tag{9}$$

The function $f^{(0)}(x_s)$ is chosen as the P-velocity distribution in the background medium,

$$f^{(0)}(x_s) = \alpha(x_s) \tag{10}$$

The isotropic elastic parameters are given by the well known relation (Aki & Richards, 1981).

$$a_{ijkl}^{(0)} = \frac{\lambda}{\rho}\delta_{ij}\delta_{kl} + \frac{\mu}{\rho}\left(\delta_{ik}\delta_{jl} + \delta_{il}\delta_{jk}\right)$$
$$= \left(\alpha^2 - 2\beta^2\right)\delta_{ij}\delta_{kl} + \beta^2\left(\delta_{ik}\delta_{jl} + \delta_{il}\delta_{jk}\right) \tag{11}$$

where λ and μ are Lamé elastic parameters, ρ the density and δ_{ij} the Kronecker delta symbol.

The perturbation equations in FAI medium with an isotropic background then read (Červený & Simões Filho, 1991).

$$\delta T_q p(S,R) = -\int_{\mathcal{L}^{(0)}\,S}^{R} \frac{\delta f(x_s)}{\alpha(x_s)}\,dT -$$
$$-\frac{1}{2}\,\delta A_{ijkl}\int_{\mathcal{L}^{(0)}\,S}^{R} N_i N_l N_j N_k\,dT. \tag{12}$$

$$\delta T_{qS1,qS2}(S,R) = -\int_{\mathcal{L}^{(0)}\,S}^{R} \frac{\delta f(x_s)}{\alpha(x_s)}\,dT -$$
$$-\frac{1}{4v^2}\int_{\mathcal{L}^{(0)}\,S}^{R}\left(C_{11} + C_{22}\right)dT$$
$$\pm\frac{1}{4v^2}\int_{\mathcal{L}^{(0)}\,S}^{R}\left[\left(C_{11} - C_{22}\right)^2 + 4C_{12}^2\right]^{1/2} dT, \tag{13}$$

with

$$C_{mn} = \delta A_{ijkl}\,N_i\,N_l\,e_j^{(m)}\,e_k^{n},$$

where N equals $g^{(3)}$, the eigenvector of the Christoffel matrix corresponding to the polarization vector of the compressional-wave ($N_i = \alpha p_i$).

Eqs.(12) and (13) show that the effects of structure perturbations and of anisotropy perturbations are fully separated with a very simple expression for the integral corresponding to the contribution of structure perturbations to the total traveltime perturbation.This distinction between structure and anisotropy perturbations has meaning only in FAI media.

From Eq.(13), the traveltime delay between the two split shear-waves can be easily written,

$$\left|\delta T_{qS1}(S,R) - \delta T_{qS2}(S,R)\right| =$$
$$= \frac{1}{2v^2}\int_{\mathcal{L}^{(0)}\,S}^{R}\left[\left(C_{11} - C_{22}\right)^2 + 4C_{12}^2\right]^{1/2} dT. \tag{14}$$

The above equation shows a very interesting result: within the framework of first-order perturbation theory, *the traveltime delay between the two split shear-waves in a FAI medium with an isotropic background is independent of the structure perturbations*, it depends only on the anisotropy perturbations. This result will considerably simplify inversion procedures based on the delay time between split shear-waves (Simões Filho, 1992).

THE CHOICE OF THE ELASTIC PARAMETERS IN THE BACKGROUND MEDIUM

The background medium must be chosen in such a way that its elastic parameters are as close as possible to those of the anisotropic medium. In this section, a simple way to choose the background medium is proposed, which minimizes the relative perturbations of elastic parameters in the least-square sense. The maximum relative perturbation of the elastic parameters is proposed as a measure of the degree of anisotropy, useful only when using perturbation theory.

As the elastic parameters a_{ijkl} have the well-known symmetries $a_{ijkl} = a_{jikl} = a_{ijlk} = a_{klij}$, it is possible to use a more compact 6X6 notation a_{pq} for the elastic parameters, where the suffixes p and q correspond to the pairs i, j and k, l, respectively. The correspondence is as follow: 1, 1 → 1; 2,2 → 2; 3,3 → 3; 2,3 → 4; 1,3 → 5 and 1,2 → 6. In this reduced notation, the symmetric matrix of elastic parameters in the background isotropic medium (11) reads,

$$a_{pq}^{(0)} = \begin{pmatrix} \alpha^2 & \alpha^2-2\beta^2 & \alpha^2-2\beta^2 & 0 & 0 & 0 \\ & \alpha^2 & \alpha^2-2\beta^2 & 0 & 0 & 0 \\ & & \alpha^2 & 0 & 0 & 0 \\ & & & \beta^2 & 0 & 0 \\ & & & & \beta^2 & 0 \\ & & & & & \beta^2 \end{pmatrix} \quad (15)$$

The perturbed medium must be only slightly anisotropic, i.e. the density normalized elastic parameters in the perturbed medium (a_{pq}) must not differ too much from those in the background medium ($a_{pq}^{(0)}$). Given the a_{pq} of the medium to be modelled, α and β are chosen in the background medium in a way to minimize E, the sum of the squares of the relative perturbations of elastic parameters,

$$E = \sum\left(\frac{\delta a_{pq}}{a_{pq}^{(0)}}\right)^2. \quad (16)$$

Deriving (16) with respect to $a_{pq}^{(0)}$ and equating to zero, the P-velocity in the background medium is given by (for a given υ):

$$\alpha^2 = \frac{Q_1 + Q_2/\left(1-2\nu^2\right)^2 + Q_3/\nu^4}{R_1 + R_2/\left(1-2\nu^2\right)^2 + R_3/\nu^2} \quad (17)$$

with

$$Q_1 = a_{11}^2 + a_{22}^2 + a_{33}^2, \qquad R_1 = a_{11} + a_{22} + a_{33},$$
$$Q_2 = a_{23}^2 + a_{13}^2 + a_{12}^2, \qquad R_2 = a_{23} + a_{13} + a_{12},$$
$$Q_3 = a_{44}^2 + a_{55}^2 + a_{66}^2, \qquad R_3 = a_{44} + a_{55} + a_{66}.$$

The value of υ may be chosen arbitrarily. The background medium may be simply considered as a Poisson solid ($\upsilon^2 = 1/3$), or E, α and β may be computed for several values of υ within its range ($0 < \upsilon < \sqrt{2}/2$). The value of υ which minimizes E is then chosen for the background medium.

The perturbation approach naturally offers a simple way of estimating the anisotropic strength of an elastic medium. If fact, the maximum absolute value of the relative perturbation of the elastic parameters with respect to the background isotropic medium is a quantity which measures the distance, in the elastic parameters space, from the perturbed to the background medium. If the isotropic background is chosen as to minimize this distance, as in Eq. (17), this maximum absolute value may indicate the deviation of the medium from isotropy.

$$P_{\max} = \max\left|\frac{\delta a_{pq}}{a_{pq}^{(0)}}\right|. \quad (18)$$

However, (18) is defined only with respect to the background medium defined by (17), and may have different values for the same anisotropic medium, if related to different background media. It may even be non-zero for isotropic media, if the isotropic background is different from the isotropic perturbed medium.

NUMERICAL EXAMPLES

In the following examples, the Cartesian coordinate system used to describe the model is defined with the z-axis chosen positive downwards and the x-and y-axes situated in the horizontal plane, oriented such that the coordinate system is right-handed. The elastic parameters are also represented in this same system. The ray computations are always performed in the xz-plane (the saggital plane), where the elastic parameters perturbations are also computed. For traveltime computations along other planes, these perturbations are obtained by a series of suitable rotations.

As an indication of the anisotropy strength, two approaches will be used. First, and most familiar, Thomsen (1986) dimensionless parameters δ, ε,and γ,defined as combinations of elastic parameters, are used.

$$\delta = \frac{\left(a_{13} + a_{44}\right)^2 + \left(a_{33} + a_{44}\right)^2}{2a_{33}\left(a_{33} + a_{44}\right)},$$

$$\varepsilon = \frac{a_{11} - a_{33}}{2a_{33}}, \qquad \gamma = \frac{a_{66} - a_{44}}{2a_{44}}. \quad (19)$$

These parameters are defined with respect to the plane in which they are represented and may therefore be different depending on the orientation of the source-receiver plane. They are all zero for isotropic media and their deviation from zero gives an estimation of the anisotropy strength. They are <0.2 for weak to moderate anisotropy.

Second, the maximum relative perturbations of elastic parameters with respect to the isotropic background, given by (18), is used. This approach is appropriate for anisotropy estimation only in the framework of perturbation theory, as it depends on the background isotropic medium. On the contrary, Thomsen parameters are intrinsic to the anisotropic medium under consideration and may be used to different types of anisotropy analysis, not only with perturbations.

Layered model with orthorhombic symmetry

The model consists of two horizontal layers overlying an isotropic half-space. The top layer is homogeneous isotropic, 500 m thick, with velocities $\alpha = 1500$ m s^{-1}, $\beta = 860$ m s^{-1} and density $\rho = 1500$ kg m^{-3}. The bottom layer is anisotropic, 500 m thick, with $\rho = 1750$ kg m^{-3} and the following elastic parameters at depth $z = 500$ m (in 10^6 m^2 s^{-2}),

$$a_{pq} = \begin{pmatrix} 4.35 & 1.37 & 1.22 & 0 & 0 & 0 \\ & 4.88 & 1.29 & 0 & 0 & 0 \\ & & 3.97 & 0 & 0 & 0 \\ & & & 1.29 & 0 & 0 \\ & & & & 1.23 & 0 \\ & & & & & 1.62 \end{pmatrix} \tag{20}$$

These parameters are similar to those presented in Mallick & Frazer (1990), to which a linear vertical gradient is superposed. It is not intended here to reproduce or make any conclusions on Mallick & Frazer's experiment with multi-component ocean sub-bottom data, the elastic parameters are used here only to model traveltimes in a realistic anisotropic model.

The orthorhombic medium is the equivalent medium of a periodic sequence of thin horizontal layers with a vertical set of fractures, for long wavelengths with respect to the layer thickness and fracturing. Thomsen parameters in the xz-plane are $\delta = -0.041$, $\varepsilon = 0.071$ and $\gamma = 0.128$, while in the yz-plane they are $\delta = -0.015$, $\varepsilon = -0.062$ and $\gamma = -0.102$, thus satisfying the conditions of weak anisotropy.

In the background model, the anisotropic layer is replaced by an isotropic layer with the following values of individual parameters at depth 500 m: $\alpha = 2060$ m s^{-1}, $\beta = 1208$ m s^{-1} and $\rho = 1750$ kg m^{-3}. The ratio $\upsilon = 0.59$ remains constant through the layer. This layer has a linear vertical gradient of P-wave velocity, $c = 1$ s^{-1}, corresponding to a gradient of S-wave velocity $vc = 0.59$ s^{-1}. The P-wave velocity is then 2560 m s^{-1} at depth $z = 1000$ m, which corresponds to the following density normalized elastic parameters:

$$a_{pq} = \begin{pmatrix} 7.00 & 2.12 & 1.88 & 0 & 0 & 0 \\ & 7.54 & 1.99 & 0 & 0 & 0 \\ & & 6.13 & 0 & 0 & 0 \\ & & & 1.99 & 0 & 0 \\ & & & & 1.90 & 0 \\ & & & & & 2.50 \end{pmatrix}. \tag{21}$$

Using (6) and (10), the reduced anisotropic parameters are obtained from (20) and (21),

$$A_{pq} = \frac{a_{pq}}{\alpha^2} = \begin{pmatrix} 1.07 & 0.32 & 0.29 & 0 & 0 & 0 \\ & 1.15 & 0.30 & 0 & 0 & 0 \\ & & 0.94 & 0 & 0 & 0 \\ & & & 0.30 & 0 & 0 \\ & & & & 0.29 & 0 \\ & & & & & 0.38 \end{pmatrix} \tag{22}$$

The reduced parameters of the isotropic background are given from (6) and (15),

$$A^{(0)}_{pq} = \frac{a^{(0)}_{pq}}{\alpha^2} = \begin{pmatrix} 1 & 0.31 & 0.31 & 0 & 0 & 0 \\ & 1 & 0.31 & 0 & 0 & 0 \\ & & 1 & 0 & 0 & 0 \\ & & & 0.34 & 0 & 0 \\ & & & & 0.34 & 0 \\ & & & & & 0.34 \end{pmatrix} \tag{23}$$

The anisotropy perturbations $\delta A_{pq} = A_{pq} - A^{(0)}_{pq}$ are:

$$\delta A_{pq} = \begin{pmatrix} 0.07 & 0.01 & -0.02 & 0 & 0 & 0 \\ & 0.15 & -0.01 & 0 & 0 & 0 \\ & & -0.06 & 0 & 0 & 0 \\ & & & -0.04 & 0 & 0 \\ & & & & -0.05 & 0 \\ & & & & & -0.04 \end{pmatrix} \tag{24}$$

It is easy to see that the maximum individual perturbation of elastic parameters does not exceed 15% of the corresponding parameter in the background isotropic medium.

The 2-D program package BEAM87-A, based on (12) and (13) and briefly described in Červený & Simões Filho (1991), was used to compute traveltimes of both quasi-shear waves in the model described above, at four different surface profiles: one situated at the saggital plane and the others obtained by rotation around the z-axis by 30°, 60° and 90°. Note that ray tracing is performed only once in the background medium (Fig.2) and the obtained results are used to recalculate the traveltimes of both qS-waves along several profiles in the anisotropic medium, by suitable rotation of the elastic parameters perturbations.

The results obtained by perturbation methods were compared with exact ray computations based on (2), obtained with the program package ANRAY89 (Gajevski & Pšenčík, 1987), designed for ray tracing in 3-D anisotropic inhomogeneous structures. Both results are displayed in Figs.3-6. The loss in accuracy is compensated by a great save of computer work: eight more time consuming 3-D anisotropic ray computations are replaced by a single 2-D isotropic ray computation, for which very general and efficient ray codes are available.

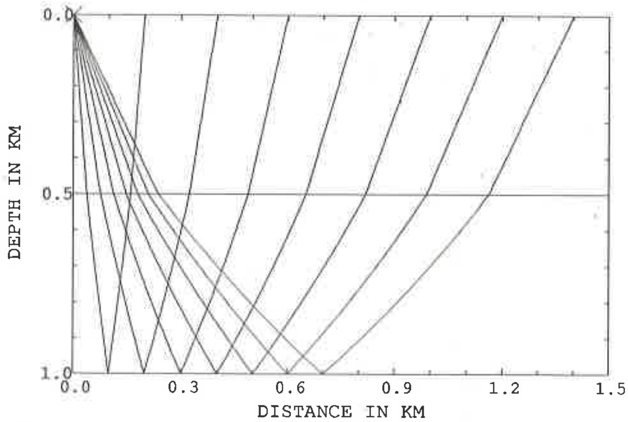


Figure 2 - Frozen rays in the isotropic background medium. The elastic parameters of the bottom layer are perturbed to acquire orthorhombic symmetry.

Figura 2 *- Diagrama de raios no meio de referência isotrópico. Os parâmetros elásticos da camada inferior são perturbados para adquirir simetria ortorrômbica.*

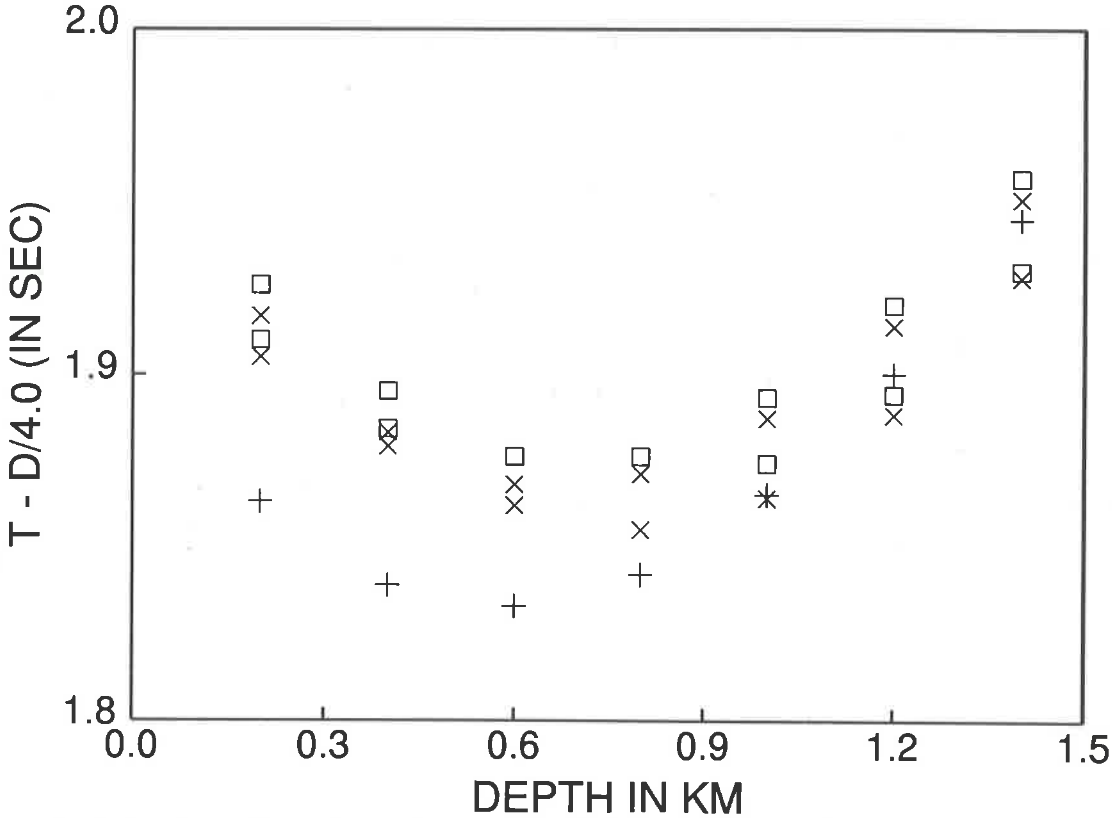


Figure 3 - Reduced traveltime of quasi shear-waves in the background isotropic medium (denoted by +) and in the perturbed anisotropic medium for the profile in the saggital plane. The reduced velocity is 4000 m s^{-1}. The traveltimes in the anisotropic medium were computed by perturbation (denoted by ×) and exact methods (denoted by).

Figura 3 *- Tempos de trânsito das ondas quase cisalhantes, reduzidos por uma velocidade de 4000 m s^{-1}, no meio de referência isotrópico (símbolo +) e no meio perturbado anisotrópico, para o perfil situado no plano sagital. Os tempos de trânsito no meio anisotrópico foram obtidos pelo método da perturbação (símbolo ×) e pelo traçado de raios exato (símbolo).*

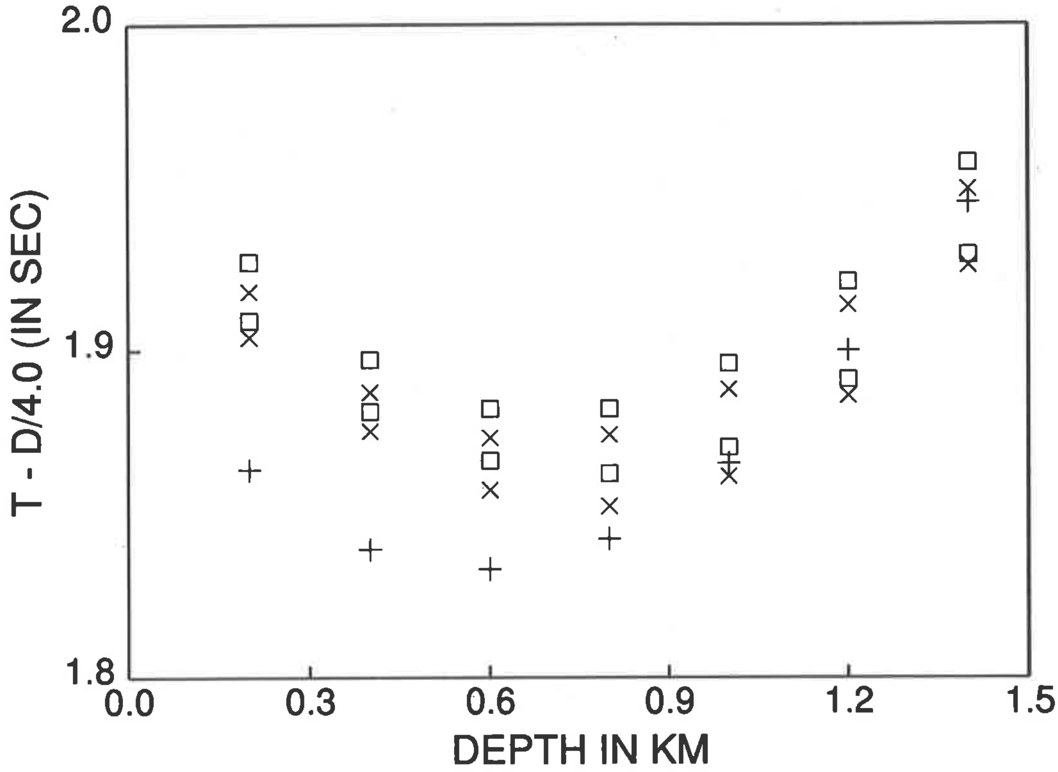


Figure 4 - As Fig. 3, for a profile rotated by 30° around z-axis.
Figura 4 - *Como na Fig.3, para um perfil obtido por rotação de 30° em torno do eixo z.*

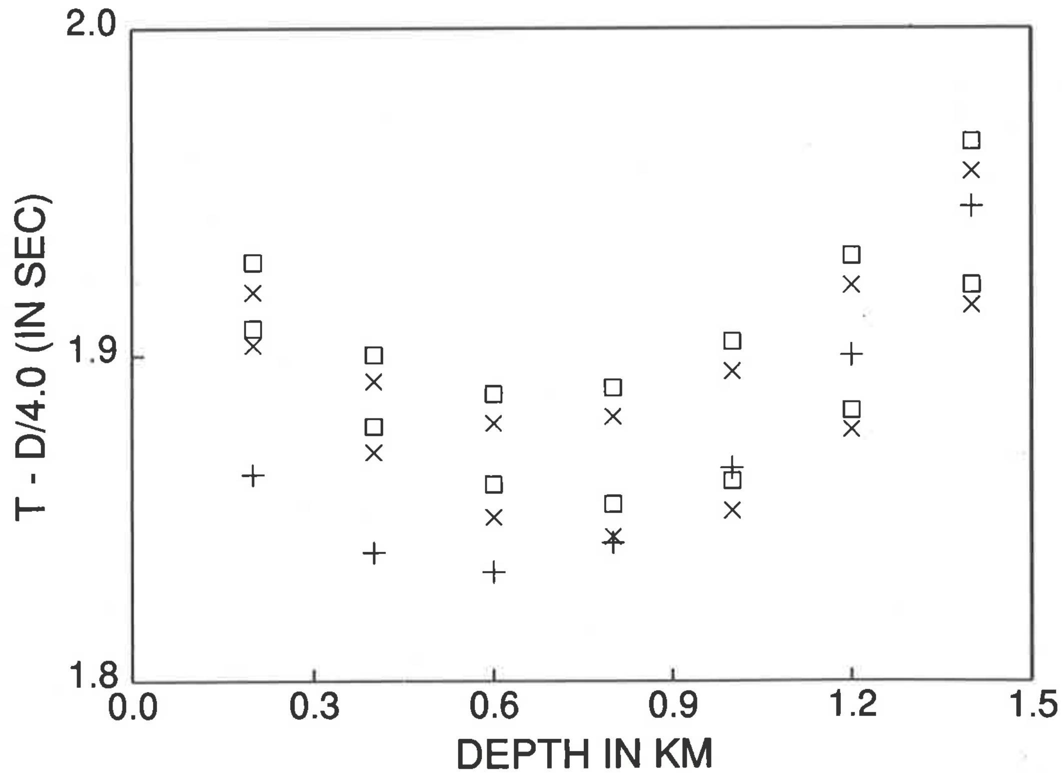


Figure 5 - As Fig.3, for a profile rotated by 60° around z-axis.
Figura 5 - *Como na Fig.3, para um perfil obtido por rotação de 60° em torno do eixo z.*

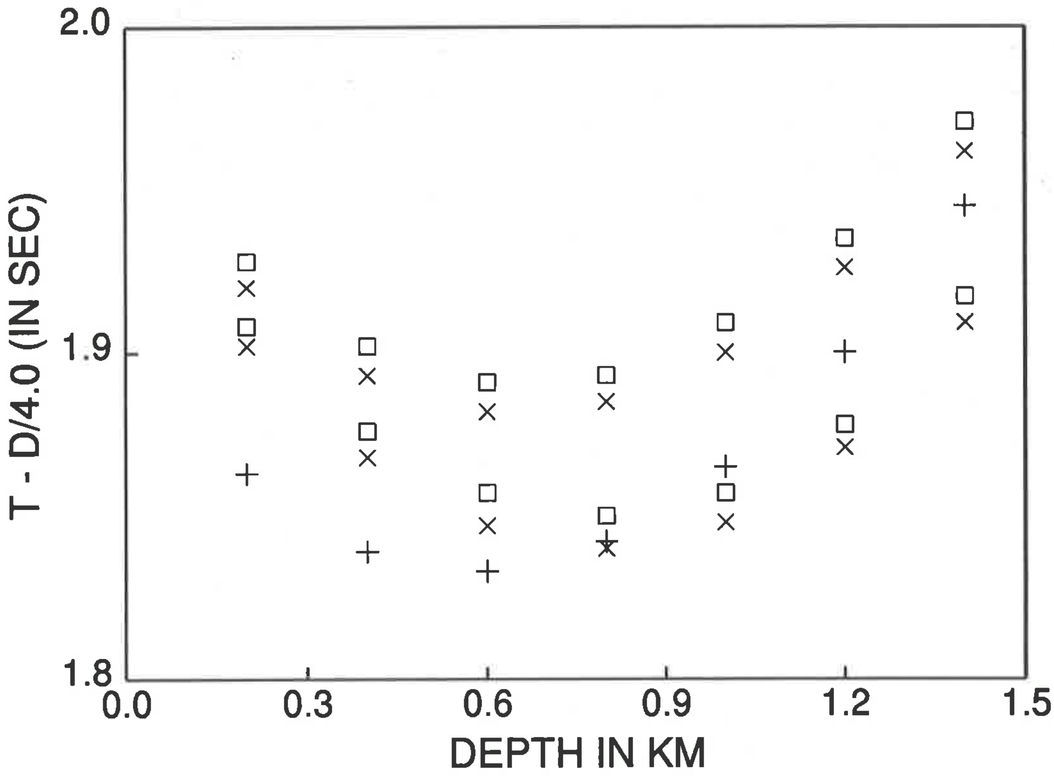


Figure 6 - As Fig.3, for the profile in the *yz*-plane.
Figura 6 - *Como na Fig.3, para o perfil do plano yz.*

The traveltimes obtained by exact ray tracing do not differ from those computed by perturbation methods by more than 25 ms (less than 2% of the exact traveltime), and in most cases by less than 10 ms, even for the profiles situated outside the planes of orthorhombic symmetry (Figs. 4 and 5), for which the ray are 3-D. The biggest misfit between the traveltimes computed by both methods appears close to the coinciding shear-wave traveltimes (Fig.3).

VSP Geometry in a transversely isotropic medium

Consider 9 receivers, separated by a constant 100 m offset, situated in a borehole. The shallower receiver is at depth $z = 200$ m. Two different shot positions are used, at the surface and 500 m depth. The medium presents an effective transverse isotropy with the ∞-fold simmetry axis oriented parallel to the x-axis. The elastic parameters are taken from Shearer & Chapman (1988), as the equivalent medium of aligned water-filled cracks in an isotropic host rock. They read (in $10^6 m^2 s^{-2}$),

$$a_{pq} = \begin{pmatrix} 20.04 & 7.41 & 7.41 & 0 & 0 & 0 \\ & 20.22 & 7.46 & 0 & 0 & 0 \\ & & 20.22 & 0 & 0 & 0 \\ & & & 6.38 & 0 & 0 \\ & & & & 6.38 & 0 \\ & & & & & 5.10 \end{pmatrix} \qquad (25)$$

The background medium is chosen with velocities $\alpha = 4428$ m s^{-1} and $\beta = 2396$ m s^{-1}. Similarly as in the previous examples, the anisotropic perturbations $\delta A_{pq} = A_{pq} - A^{(0)}_{pq}$ may be easily computed,

$$\delta A_{pq} = \begin{pmatrix} 0.02 & -0.04 & -0.04 & 0 & 0 & 0 \\ & 0.03 & -0.03 & 0 & 0 & 0 \\ & & 0.03 & 0 & 0 & 0 \\ & & & 0.03 & 0 & 0 \\ & & & & 0.03 & 0 \\ & & & & & -0.03 \end{pmatrix}. \qquad (26)$$

Here the maximum individual relative perturbation of elastic parameters do not exceed 11%, and Thomsen parameters are $\delta = -0.002$, $\varepsilon = -0.004$ and $\gamma = -0.100$. Both measures of anisotropy strength indicate weaker anisotropy than in the orthorhombic example. Fig.7 shows the ray diagrams in the background isotropic medium. The computed traveltimes by exact and perturbed methods are displayed in Fig.8. The weaker anisotropy of this example is responsible for the better fit between traveltimes computed by both methods than in the previous one. In fact, the misfit between exact and perturbed traveltimes does not exceed 5 ms here (less than 1% of the exact traveltime). Further analysis of the accuracy of traveltime perturbations with respect to the medium deviation from isotropy is presented in Simões Filho (1993).

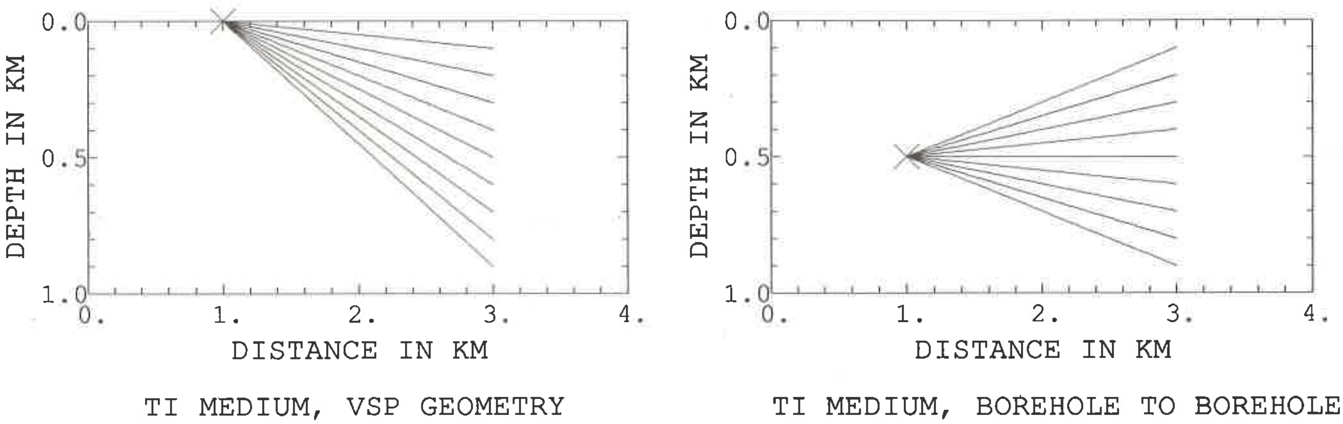
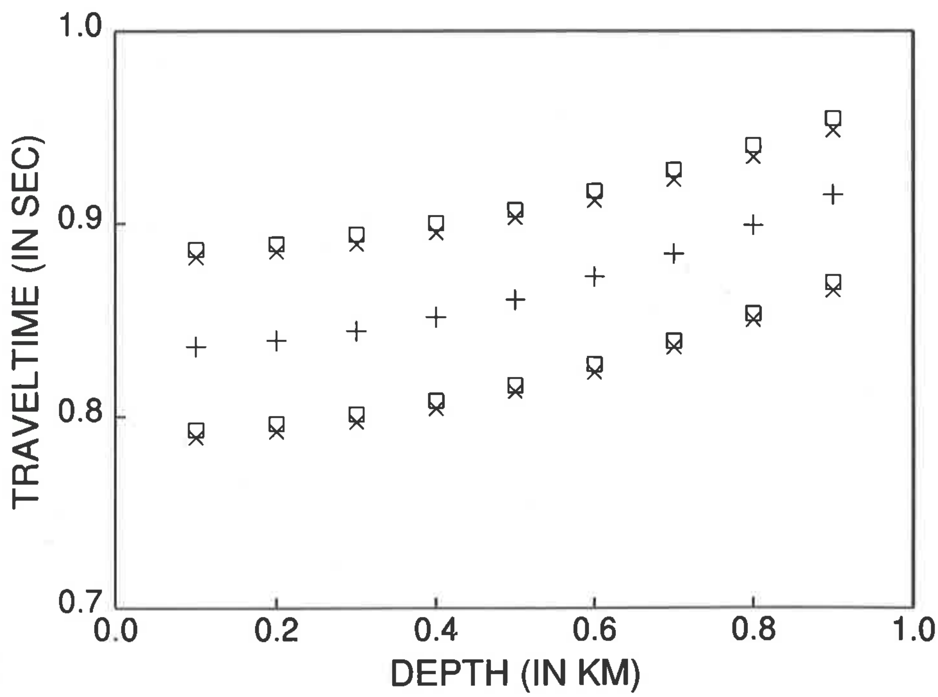


Figure 7 - Frozen rays in the isotropic background medium. The elastic parameters are perturbed to acquire transverse isotropy with a horizontal axis of symmetry.

Figura 7 - *Diagrama de raios no meio de referência isotrópico. Os parâmetros elásticos são perturbados para obter um meio transversalmente isotrópico com eixo de simetria horizontal.*



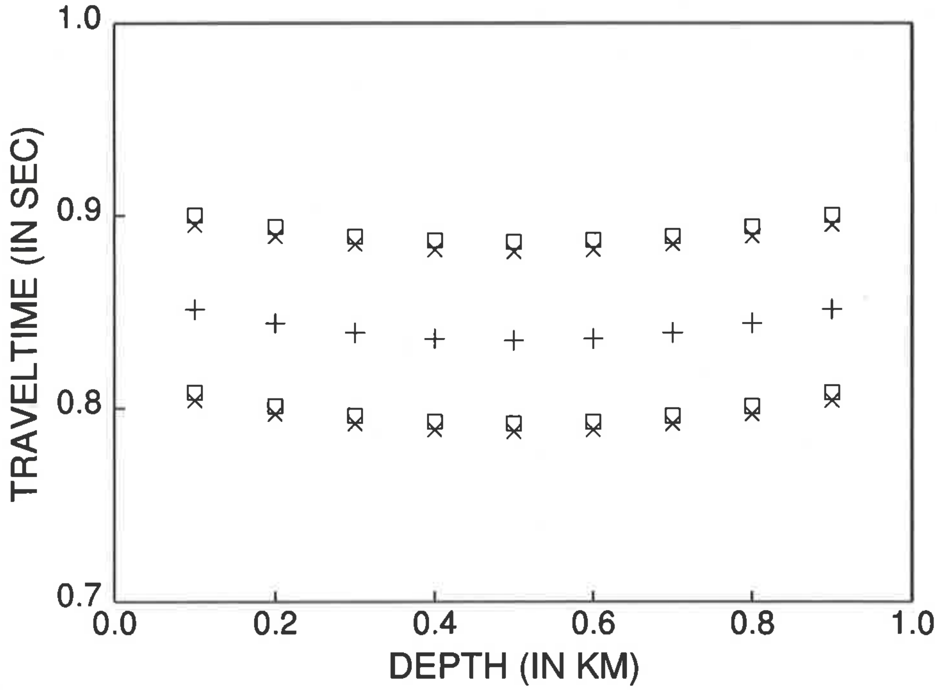


Figure 8 - Traveltimes of quasi shear-waves for a VSP in a transversely isotropic medium. The traveltimes were computed by perturbation (denoted by ×) and exact methods (denoted by).
Figura 8 - *Tempos de trânsito das ondas quase cisalhantes, em um VSP em meio transversalmente isotrópico. Os tempos de trânsito foram obtidos pelo método da perturbação (símbolo ×) e pelo traçado de raios exato (símbolo).*

CONCLUSION

The parameterization of inhomogeneous anisotropic media is highly simplified by the concept of FAI media. Further, perturbation theory in FAI media with isotropic background shows that the effects of anisotropy perturbations and structure perturbations on the total perturbed traveltimes are completely separated. In such case, the traveltime delay between the two split shear-waves depends only on the anisotropy perturbations. This result will be useful for inversion of the traveltime delay between the split shear-waves in FAI media, and will provide a way to investigate separetly the contribution of structure and anisotropy perturbations to the perturbed traveltimes.

The forward modelling in FAI media is radically simplified by the use of perturbation methods. In fact, ray tracing is performed only once in an isotropic background and the computed rays are used to recalculate the traveltimes in a slightly anisotropic medium along several profile directions. The use of exact ray tracing requires as many ray computations for each receiver as the number of profiles considered, for both quasi-shear waves.

The restriction to weak anisotropy must be kept in mind. Therefore the elastic parameters of the background isotropic medium should be chosen in a way to minimize the parameters perturbation. The example of the orthorhombic medium shows that 15% may be close to the perturbation limit, as the accuracy in such case has considerably decreased when compared to the transversely isotropic example, where the perturbations do not exceed 11%. The perturbation approach, when applied to model traveltimes in media with stronger anisotropy (in which the maximum perturbation of the elastic parameters can not be lower than 30% of the corresponding parameter in the background isotropic model) lead to uncorrect results. Even though the unperturbed rays are 2D, the accuracy is the same of 2D or 3D perturbed rays.

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REFERENCES

AKI, K. & RICHARDS, P.G. - 1981 - Quantitative Seismology. Freeman, New York.

BACKUS, G.E. - 1965 - Possible forms of seismic anisotropy in the uppermost mantle under the oceans. J. Geophys. Res., **70:** 3429-3439.

ČERVENÝ, V. - 1972 - Seismic rays and ray intensities in inhomogeneous anisotropic media. Geophys. J.R. astr. Soc., **29:** 1-13.

ČERVENÝ,V. - 1982 - Direct and inverse kinematic problems for inhomogeneous anisotropic media – linearization approach. Contr. Geophys. Inst. Slov. Acad. Sci., **13:** 127-133.

ČERVENÝ, V. - 1989 - Ray tracing in factorized anisotropic inhomogeneous media. Geophys. J. Int., **99:** 91-100.

ČERVENÝ, V. & FIRBAS, P. - 1984 - Numerical modelling and inversion of travel times of seismic body waves in inhomogeneous anisotropic media. Geophys. J.R. astr. Soc., **76:** 41-51.

ČERVENÝ, V. & SIMÕES FILHO, I.A. - 1991 - The travel time perturbations for seismic body waves in factorized anisotropic inhomogeneous media. Geophys. J. Int., **107:** 219-229.

ČERVENÝ, V., MOLOTKOV, I.A. & PŠENČÍK, I. - 1977 - Ray method in seismology. Charles University Press, Praha.

CRAMPIN, S. - 1984 - Effective anisotropic elastic constants for wave propagation through cracked solids. Geophys. J.R. astr. Soc., **76:** 135-145.

CRAMPIN, S. - 1985 - Evaluation of anisotropy by shear-wave splitting. Geophysics, **50:** 142-152.

GAJEWSKI, D. & PŠENČÍK, I. - 1987 - Computation of high frequency seismic wavefield in 3-D laterally inhomogeneous anisotropic media. Geophys. J.R. astr. Soc., **91:** 383-411.

HUDSON, J.A. - 1981 - Wave speeds and attenuation of elastic waves in material containing crack. Geophys. J.R. astr. Soc., **64:** 133-150.

JECH, J. & PŠENČÍK, I. - 1989 - First order perturbation method for anisotropic media. Geophys. J. Int., **99:** 369-376.

LEARY, P.C., CRAMPIN, S. & McEVILLY, T.V. - 1991 - Seismic fracture anisotropy in the Earth's crust: An overview. J. Geophys. Res., **95:** 11105-11 114.

MALLICK, S & FRAZER, L.N. - 1990 - Fracture orientation from marine air-gun OSS data. Expanded abstracts, 60th international SEG meeting, S. Francisco, 1448-1451.

SCHOENBERG, M. & MUIR, F. - 1989 - A calculus of finely layered anisotropic media. Geophysics, **54:** 581-589.

SHEARER, P.M. & CHAPMAN, C.H. - 1988 - Ray tracing in anisotropic media with a linear gradient. Geophys. J., **94:** 575-580.

SIMÕES FILHO, I.A. - 1992 - Modelagem e inversão de tempos de trânsito em meios heterogêneos anisotrópicos fatorados. Ph.D. thesis, PPPG/UFBA.

SIMÕES FILHO, I.A. - 1993 -The accuracy of traveltime perturbations from isotropic to anisotropic medium as a function of the medium deviation from isotropy. Geophys. Res. Letters, submitted.

THOMSEN, L. - 1986 - Weak elastic anisotropy. Geophysics, **51:** 1954-1966.

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