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PRINCIPLES OF ORBIT IMPROVEMENT AND GENERATION OF EPHEMERIDES FOR THE GLOBAL POSITIONING SYSTEM SATELLITES

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The ephemerides that describe the orbits of the Global Positioning System (GPS) satellites represent the solution of the equations of motion of the satellites. They require initial conditions (position and velocity) and a model which describes the forces that govern the motion of the satellites. A small offset in the initial conditions may cause hundreds, or even thousands, of metres of error in satellite position after a few days of integration. To minimize this problem, the initial conditions, plus some parameters of the force field, must be adjusted through a process known as orbit improvement. The improved initial conditions can then be used for the generation of the post-fitted ephemerides. This paper focuses on the orbit improvement and generation of ephemerides for GPS satellites. In our analysis, we processed GPS data from a network of North American stations contributing to the global network of the International GPS Service for Geodynamics (IGS). To obtain a measure of achieved accuracy of the geodetic network solution, we have compared the resulting baselines with published International Earth Rotation Service Terrestrial Reference Frame 1992 (ITRF92) values, and our improved orbits with the IGS orbits.

Key words: Global positioning system; Orbit improvement; Satellite positioning.

PRINCÍPIOS DE REFINAMENTO DE ÓRBITAS E GERAÇÃO DE EFEMÉRIDES PARA OS SATÉLITES DO SISTEMA DE POSICIONAMENTO GLOBAL - *As efemérides que descrevem a órbita dos satélites GPS representam uma solução particular das equações de movimento. A solução destas equações requer condições iniciais (posição e velocidade, agrupados no chamado vetor de estado inicial) e um modelo que descreva as forças que governam o movimento dos satélites. As condições iniciais têm que ser consistentes com a órbita a ser gerada. Uma pequena diferença nas condições iniciais pode acarretar erros na posição dos satélites da ordem de quilômetros após alguns dias de integração. Para minimizar este problema, as condições iniciais, bem como alguns parâmetros do modelo de força (por exemplo, os parâmetros da pressão da radiação solar) devem ser ajustados dentro de um processo aqui chamado de refinamento de órbitas. Neste contexto, as condições iniciais e os parâmetros do modelo de força constituem os parâmetros orbitais. No procedimento de refinamento de órbitas, os parâmetros orbitais são estimados usando observações coletadas por estações cujas coordenadas sejam conhecidas, ou que sejam estimadas junto com os parâmetros orbitais. As condições iniciais ajustadas podem então ser utilizadas para a geração das órbitas dos satelites GPS. Este artigo se ocupa do refinamento e geração de órbitas para os satelites GPS. As órbitas são determinadas usando-se dados de uma sub-rede do Serviço Internacional GPS para a Geodinâmica (IGS), composta por estações no Canadá e EUA. A partir desta solução, órbitas regionais são geradas, e comparadas com as órbitas do IGS. Uma avaliação da precisão externa do ajustamento desta rede é possivel através da comparação das bases ajustadas com os valores publicados do referencial ITRF92 (International Earth Rotation Service Terrestrial Reference Frame 1992).*

Palavras-chave: *Sistema de posicionamento global; Refinamento de órbitas; Posicionamento por satélites.*

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INTRODUCTION

The orbit of a satellite is the solution of a second order differential equation system, known as the equations of motion. The equations of motion represented in an inertial geocentric coordinate system have the form:

$$\ddot{\underline{r}} = -\frac{GM}{\|\underline{r}\|^3}\underline{r} + \ddot{\underline{p}}, \qquad (1)$$

where $\ddot{\underline{r}}$ the total acceleration vector of the satellite, GM is the earth's gravitational constant, $\underline{r}$ is the satellite geocentric postion vector, and $\ddot{\underline{p}}$ represents the sum of the perturbing accelerations that act on the satellite.

The equations of motion can be integrated, numerically or otherwise, provided the conditions of the motion at an initial time t_0 are given. The initial conditions are a vector composed of initial position $[x,y,z]^T$ and velocity $[\dot{x},\dot{y},\dot{z}]^T$ of the satellite or their equivalent osculating Keplerian elements (a_0, e_0, i_0, ϖ_0, Ω_0 and one of the anomalies), at the initial epoch. In this paper, this vector of initial conditions is referred to as the "state vector". The solution of the equations of motion yields satellite positions and velocities, at any other time, as a numerical or analytic function of these initial conditions.

The first term on the right-hand side of Eq. (1) describes the Keplerian motion of a satellite in a central field (i.e., under the influence of the central, or radial, part of the earth's gravitational field only), the orbit being a conic section, typically an ellipse. The second term ($\ddot{\underline{p}}$) represents the sum of the effects caused by the non-central part of the earth's gravitational field, the attraction of the moon, the sun, and other celestial bodies, the direct and indirect effects of the solar radiation pressure, the atmospheric drag effect, the ocean and earth tides, relativistic effects, electromagnetic effects, thruster firings and out-gassing, etc. These perturbing accelerations cause a departure from the (elliptical) Keplerian orbit. If these perturbations were perfectly modelled, the integrated orbit would pinpoint the satellite position at any given time without an error.

Unfortunately, this utopian situation does not exist and so we must contend with potential orbit errors. The three sources of error in the orbit determination are:

1. the numerical integration technique, reflecting the stability of the integrator itself or coming from the numerical integration step size;
2. the force model used; and
3. the initial conditions (a small offset in the initial conditions may cause hundreds or even thousands of metres of error after a few days of integration).

These errors can be overcome by:

1. choosing a stable integrator that makes use of a step size large enough to save computing time yet avoiding larger integration errors;
2. adopting a complete force model that accounts for all significant perturbations; and
3. improving the initial conditions with respect to observations to the satellite, a process known as orbit improvement.

In this paper, we outline a technique for carrying out improvement of the orbits of a GPS satellite. By orbit improvement, we understand the procedure by which orbital parameters of a satellite (the initial state vector and the parameters that describe solar radiation pressure) are estimated using observations to the satellite collected by stations whose coordinates are known, or estimated together with the satellite's orbital parameters. To test the capability of our technique, we use a regional network of GPS receivers. The procedure of orbit improvement helps us to obtain better results in the network adjustment by allowing the orbital parameters to "learn" from the past satellites' trajectories. These trajectories are defined by the observations and regarded as extra parameters in the adjustment; they help to absorb possible mis-modellings of the observations. The numerical integration technique and the force model used in this orbit improvement analysis are capable of overcoming the error sources 1 and 2 enumerated above. Section 2 of this paper briefly describes the models we have used. Section 3 explains the principles of orbit improvement. The processing of data from a North American network and discussion of the results are presented in Sections 4, 5 and 6. Concluding remarks are made in Section 7.

ADOPTED MODEL

The model we have adopted for our orbit improvement has the following components: the geopotential represented by the GEM-T3 model (Lerch et al., 1992) up to 8th degree and order; the sun and the moon, regarded as point masses and their effects modelled according to Rizos & Stolz (1985); the direct and y-bias effects of the solar radiation pressure (Beutler et al., 1986); the solid earth tides (Rizos & Stolz, 1985); and the relativistic effect (Zhu & Groten, 1988). The effects caused by the earth's reflectivity, ocean tides, atmospheric drag, satellite maneuvering and gravitational fields of the other planets were disregarded.

The solar radiation pressure represents the most problematic aspect of orbit improvement of GPS satellites. Program PREDICT (Santos, 1995) handles it by using selectively three different models. The first model takes into account only the direct solar radiation pressure $\ddot{\underline{p}}_{dir}$ (Beutler et al., 1986):

$$\ddot{\underline{p}}_{dir} = \nu \; p_0 \, \underline{n} \,, \tag{2}$$

where ν is the eclipse factor, p_0 is the direct solar radiation pressure parameter and $\underline{n}$ is a unit vector pointing from the sun through the spacecraft. The eclipse factor ν is equal to zero when the satellite is in the earth's shadow, equal to one when it is in sunlight, and somewhere in between 0 and 1 during its passage through the penumbral zone. The parameter p_0 is given an a priori value, and the correction to it becomes one of the estimated quantities in the orbit improvement process.

The second solar radiation pressure model is given by:

$$\ddot{\underline{p}}_{srp} = \ddot{\underline{p}}_{dir} + \ddot{\underline{p}}_{y} \,, \tag{3}$$

where $\ddot{\underline{p}}_{dir}$ is as above and $\ddot{\underline{p}}_{y}$ takes into account the acceleration along the solar panel rotation axis of the GPS satellite (the y-axis direction of the satellite-fixed coordinate system), and can be expressed by (Beutler et al., 1986):

$$\ddot{\underline{p}}_{y} = \nu \; p_y \, \underline{e}_y \,, \tag{4}$$

where $\ddot{\underline{p}}_{y}$ is the y-bias parameter and $\underline{e}_y$ is the unit vector in the direction of the solar panel rotation axis of the GPS satellite. The parameter p_y, along with p_0, is given an a priori value, and the correction to it becomes one of the estimated quantities in the orbit improvement.

The third solar radiation model is given by (Lichten & Border, 1987):

$$\ddot{\underline{p}}_{srp} = \nu \left[\frac{AU^2}{\left\| \underline{r} - \underline{r}_s \right\|^2} \left(G_x a_x \underline{e}_x + G_z a_z \underline{e}_z \right) + G_y \underline{e}_y \right] , \tag{5}$$

where AU is the astronomical unit, $\underline{r}_s$ is the position vector of the sun, G_x and G_z are solar pressure scaling factors, G_y is the p_y y-bias parameter, and a_x and a_z are the satellite-centered accelerations represented by short Fourier series known as equations T10 and T20 (Fliegel et al., 1992). The vectors $\underline{e}_x$, $\underline{e}_y$, $\underline{e}_z$ are unit vectors in a satellite-centered inertial coordinate system (Fliegel et al., 1992). The parameters G_x ,G_y and G_z are estimated quantities in an orbit improvement process.

In the research described in this paper, we used the model represented by Eq. (3). We have also made use of Ash's model (Ash, 1972) for the computation of the eclipse factor ν. This model takes into account the satellite's passage through the penumbra zone of the earth.

The solution of the equations of motion (Eq. 1) requires that the numerical integration be carried out in an inertial coordinate system (ICS). The adopted ICS for the numerical integration of the equations of motion is the true right ascension system (Vanícek & Krakiwski, 1986) at a reference epoch t_0, which is the initial epoch of the equations of motion. The ICS keeps a constant orientation with respect to the Conventional Inertial System at *J2000.0*. The orbit improvement is carried out in the Conventional Terrestrial System (CTS). The relation between the CTS and the ICS reads:

$$\underline{r}^{CTS} = \underline{W}\,\underline{G}\,\underline{N}\,\underline{P}\,(\underline{N}^*\,\underline{P}^*)^T\,\underline{r}^{ICS(t_0)} \,, \tag{6}$$

where $\underline{P}$, $\underline{N}$, $\underline{G}$ and $\underline{W}$ represent rotation matrices for precession, nutation, Greenwhich Apparent Sideral Time (GAST) and polar motion, respectively, and $\underline{P}^*$ and $\underline{N}^*$ are the precession and nutation matrices used in the transformation between *J2000.0* and the initial epoch t_0 of the equations of motion (International Earth Rotation Service, 1992).

The integration techniques, which program PREDICT allows us to choose from, are either the Adams-Moulton or the Störmer-Cowell methods of 11th order (Velez & Maury, 1970). Both techniques are multi-step methods (Kreyszig, 1988). The Adams-Moulton method is a first-order method used for velocity computation (if position is also desired the equations of motion have to be integrated twice). The Störmer-Cowell method is a second-order method used for position computation. The starting values required by these methods are computed following Velez & Maury (1970). In the research described in this paper, we used the Störmer-Cowell method.

PRINCIPLES OF ORBIT IMPROVEMENT

As mentioned in Section 1, by the term "orbit improvement" we understand the procedure by which the initial state vector and initial dynamical parameters of a satellite are estimated using observations on this satellite collected by stations whose coordinates are known, or which are to be estimated together with the satellite's initial state vector and initial dynamical parameters.

Let the linearized GPS observation equation be written as (Santos, 1995):

$$\underline{A}_R \underline{\delta}_R + \underline{A}_r \underline{B}_1^* \underline{\delta}_s + \underline{A}_r \underline{B}_2^* \underline{\delta}_p + \underline{A}_y \underline{\delta}_y + \underline{w} = \underline{v} \ , \tag{7}$$

where $\underline{A}$ represents the design matrices, $\underline{\delta}$ is the vector of corrections to the estimated parameters, $\underline{v}$ is the vector of observation residuals and $\underline{w}$ the vector of observation misclosure. The subscripts R, r, s, p and y represent the receiver position, the satellite positions, the initial state vector, the initial dynamical parameters (only solar radiation pressure parameters in the case of GPS satellites) and the nuisance parameters, respectively. The Jacobian matrices $\underline{B}_1$* and $\underline{B}_2$* contain the *variational partials*, i.e., the partial derivatives of a satellite position in the inertial system with respect to the initial state vector $\underline{s}$ and to the vector of initial solar radiation pressure parameters $\underline{p}$. They are grouped together into matrix $\underline{B}$*:

$$\underline{B}^* = \left[\underline{B}_1^* \ \underline{B}_2^*\right] = \left[\frac{\partial \underline{r}}{\partial \underline{s}} \ \frac{\partial \underline{r}}{\partial \underline{p}}\right] . \tag{8}$$

Matrix $\underline{B}$* is thus a transformation operator between the vector space populated by the initial state vectors $\underline{\delta}_s$ and the vectors of initial solar radiation pressure parameters $\underline{\delta}_p$ and the vector space containing the satellite positions at some observation epoch $\underline{\delta}_r$, where:

$$\underline{\delta}_r = \underline{B}^* \begin{bmatrix} \underline{\delta}_s \\ \underline{\delta}_p \end{bmatrix} . \tag{9}$$

The variational partials are the solution of a system of second-order differential equations known as *variational equations*. The variational equations can be written in a matrix form as (McCarthy et al., 1993):

$$\underline{F} = \underline{W}\,\underline{B}^* + \underline{K} \tag{10}$$

The matrix $\underline{F}$ is composed of partial derivatives of the satellite acceleration vector $\ddot{\underline{r}}$ with respect to the initial state vector $\underline{s}$ and to the initial solar radiation pressure parameters $\underline{p}$. The matrix $\underline{W}$ has entries which correspond to second order derivatives of the earth's gravitational potential with respect to the satellite position vector. The matrix $\underline{K}$ consists of partial derivatives of the solar radiation pressure at any subsequent epoch with respect to the initial solar radiation pressure parameters $\underline{p}$.

The variational partials can be computed by either numerically integrating the variational equations, in which case all elements of matrix $\underline{F}$ are integrated (McCarthy et al., 1993), or by a hybrid solution, in which case the Keplerian part of $\underline{B}$* is solved analytically and the solar radiation pressure part of $\underline{F}$ is numerically integrated. The integration is commonly carried out along with the solution of the equations of motion for computational efficiency.

The second approach for the solution of the variational equations, i.e., the hybrid solution, is the one we have chosen. In this case, matrix $\underline{B}$* is spelled out as:

$$\underline{B}^* = \left[\frac{\partial \underline{r}}{\partial \underline{\kappa}} \ \frac{\partial \underline{r}}{\partial \underline{p}}\right] , \tag{11}$$

where $\underline{\kappa}$ is a vector containing the initial satellite position in Keplerian elements and $\underline{p}$ is a vector of initial solar radiation pressure parameters (Cf. Eq. 11). The first submatrix (that depends on the Keplerian elements) is computed analytically following Langley et al. (1984) and Parrot (1989). The second submatrix (that contains the solar radiation pressure parameters) is computed by numerically integrating the variational equations. For this purpose, the variational equations are written as:

$$\frac{\partial \ddot{r}_i}{\partial p_\kappa} = A_{ij} \frac{\partial r_j}{\partial p_\kappa} + \frac{\partial \ddot{p}_i}{\partial p_\kappa} , \tag{12}$$

where $r_{i,j}$=1,2,3 are the Cartesian components of $\underline{r}$, p_κ is equal to (p_0, p_y), for κ =1,2, at t_0, $\ddot{p}$ represents the x, y, z components of the solar radiation pressure contribution, cf. Eqs. (3) and (5), and $\underline{A}$ is the part of matrix $\underline{W}$ containing only the radial gravitational field contribution:

$$\underline{A} = -\frac{GM}{r^3}\left(\underline{I} - 3\frac{\underline{r}\,\underline{r}^T}{r^2}\right), \tag{13}$$

with $\underline{I}$ being a unit matrix of dimension 3 and r the norm of $\underline{r}$. The initial condition for the solution of Eqs. (12) is:

$$\frac{\partial r_j}{\partial p_\kappa} = 0 \,. \tag{14}$$

The orbit improvement is carried out by first predicting an orbit using the a priori initial state vector. This predicted orbit is then improved (adjusted) using the GPS observations. This process yields a least-squares correction vector $\underline{\delta}_x$ to be applied to the initial state vector. The improved state vector is then used for the ephemeride's generation through a prediction of a new (improved) orbit.

An example of the effect of orbit improvement can be seen in Figs. 1 and 2. Fig. 1 shows the radial, along-track, and cross-track components of the difference between the predicted orbit of GPS satellite PRN 25, as computed by program PREDICT, and a reference orbit for the same satellite obtained from IGS. The initial conditions used for the prediction were taken from the reference orbit, but the prediction was carried out totally independent on the reference orbit. It can be seen that after one day, a difference of up to 30 m is encountered. Fig. 2 shows the radial, along-track and cross-track components of the difference between a new predicted orbit of satellite PRN 25, using initial conditions improved with respect to the reference orbit (used as "pseudo-observations"), and the reference orbit itself. The peak-to-peak difference is now below the 20 cm level. The two trajectories are very close because the new prediction was carried out using improved initial conditions, which are a function of the reference orbit. Orbits generated using improved initial conditions are usually referred to as "improved orbits".

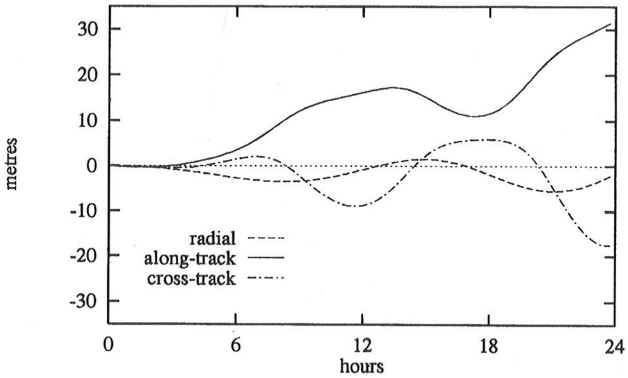

Figure 1 - Difference between predicted orbit and reference orbit.

Figura 1 - *Diferença entre a órbita predita e a órbita de referência.*

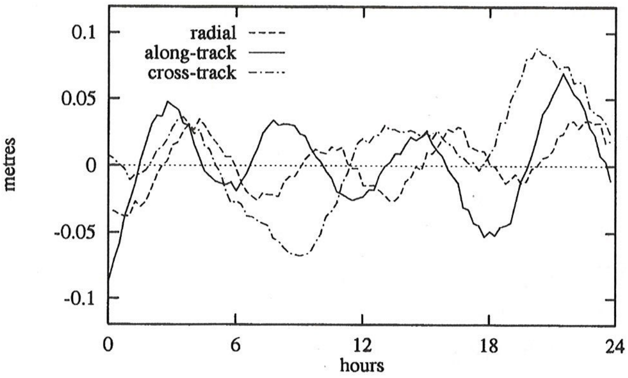

Figure 2 - Difference between improved orbit and reference orbit.

Figura 2 - *Diferença entre a órbita refinada e a órbita de referência.*

DATA SET DESCRIPTION

The GPS data used for this analysis, covering day 003 of GPS week 730, corresponding to 3 January, 1994, were collected by a subset of the global IGS network. Fig. 3 shows the geographical distribution of the eight IGS stations used

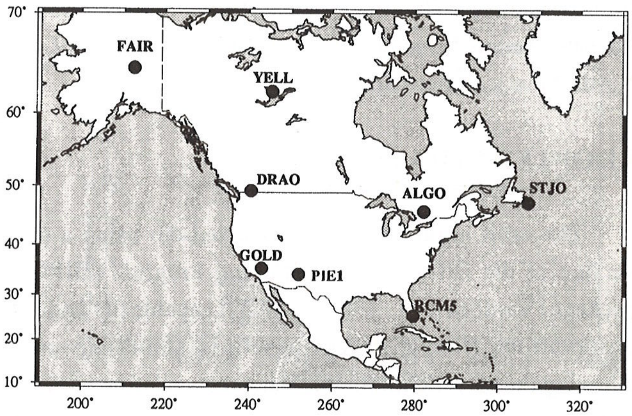

Figure 3 - North-American network configuration.

Figura 3 - *Configuração da rede Norte Americana.*

in our study. The station names are listed in Tab. 1. The test network is composed of the baselines (number between parenthesis is baseline length) ALGO-STJO (1931 km), ALGO-PIE1 (2822 km), GOLD-PIE1 (810 km), PIE1-RCM5 (2811 km), GOLD-DRAO (1556 km), FAIR-DRAO (2374 km) and YELL-DRAO (1495 km). The criteria for selecting these baselines were: the maximization of double difference observations, and the shortest baseline lengths. These baselines thus allow us to use as many observations as possible and simplify the ambiguity resolution.

IGS code	*Location*
ALGO (F)	Algonquin
DRAO	Penticton
FAIR	Fairbanks
GOLD (F)	Goldstone
PIE1	Pie Town
RCM5	Richmond
STJO (F)	Saint John's
YELL	Yellowknife

Table 1 - IGS stations used in our analysis (F = fiducial stations).

Tabela 1 - *Estações IGS utilizadas nesta análise (F = estações fiduciais).*

We have used the International Earth Rotation Service Terrestrial Reference Frame of 1992 (ITRF92) at epoch 1994.0 to refer the station coordinates to (Altamini & Boucher, 1993). We have also followed the IGS choice of fiducial stations (Kouba, 1993). Table 1 indicates the stations used as fiducial stations for the processing of our North American network.

ADOPTED STRATEGY

The orbit improvement was carried out using a new version of the Differential Positioning Program (DIPOP) package (Vanícek el al., 1985), which is capable of handling observations from different baselines simultaneously, allowing thus for the full mathematical correlation between baselines to be taken into account (Santos, 1995). This network-orientated DIPOP incorporates several other modifications that have been made recently in support of other on-going research at the University of New Brunswick, such as the option to choose from a variety of tropospheric propagation delay models, the estimation of residual tropospheric delay correction parameters and taking into account the different antenna heights for the L1 and L2 phase centres (Mendes & Langley, 1994; van der Wal, 1995; Komjathy, 1995).

The processing strategy applied for the orbit improvement is summarized below:

- Fiducial stations weighted according to the ITRF92 standard deviations (around 5mm); floating stations weighted at 10 m.
- Satellites used: all observed satellites are improved.
- Double difference carrier phase measurement noise: 12 millimetres.
- Troposphere dry zenith delay model: Saastamoinen (Saastamoinen, 1973).
- Troposphere wet zenith delay model: Saastamoinen (Saastamoinen, 1973).
- Troposphere dry mapping function: Ifadis (Ifadis, 1986).
- Troposphere wet mapping function: Ifadis (Ifadis, 1986).
- A priori standard deviation for tropospheric zenith delay correction: 20 cm.
- Number of tropospheric parameters estimated: 1 per station per 24 hours.
- Observation elevation cut-off angle: 15 degrees.
- Data sampling interval: 120 seconds.
- Solution type: ionosphere-free linear combination of phase double differences.
- Carrier phase cycle ambiguities: estimated as real-valued parameters;
- Adopted models: GEM-T3 geopotential model up to degree and order 8 with C21 and S21 consistent with the mean pole, as defined by the International Earth Rotation Service (1992); gravitational effect of the sun and moon regarded as point masses; solar radiation pressure as given by Eq. (3), with penumbral effect included; solid earth tides with Love number equal to 0.29; relativistic effect.

DISCUSSION OF THE RESULTS

The data set described in Section 4 was used for the orbit improvement following the processing strategy described in Section 5. As an outcome of the adjustment, adjusted station coordinates and the improved initial conditions of the orbits were obtained.

The accuracy of the adjusted station coordinates was measured by comparing the components of the baselines with their published ITRF92 counterparts. A summary of the accuracy, by means of the relative error in baseline length, is shown in Fig.4. The average relative error is 2.27x 10^{-8}, the smallest is equal to 2.57x10^{-10} and the largest is equal to 4.76x10^{-8}.

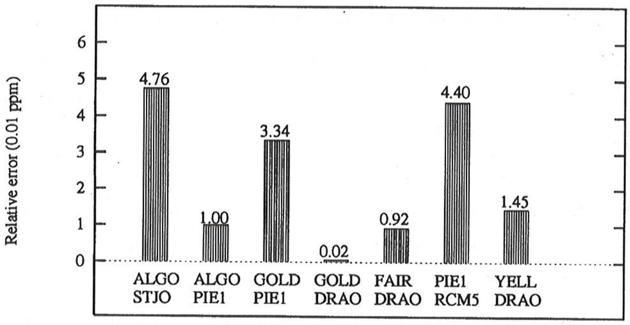

Figure 4 - Relative error in baseline length.

Figura 4 - *Erro relativo em termos de comprimento das bases.*

The set of improved initial conditions of the orbits was used to generate the post-fitted (improved) ephemerides. These ephemerides were then compared with the IGS orbits, regarded in this study as a benchmark. The differences, termed “orbital residuals”, were expressed in radial, along-track and cross-track components in a satellite-fixed coordinate system. Figs. 5, 6, 7, 8, 9 and 10 depict the orbital residuals for satellites PRN 1, 15 and 28. Figs. 5, 7 and 9 encompass the whole day (24 hours), indicating the period of time each particular satellite was observed by the network. Figs. 6, 8 and 10 concentrate on the period of data coverage only. Due to the regional (as opposed to global) extent of the North American network, the GPS satellites have not been observed continuously by all stations throughout the 24-hour observing session. This lack of simultaneous observations for a particular satellite for a certain period of time results in larger orbital residuals for the period during which the satellite was not observed, which is very evident particularly for satellites PRN 1 and 28. The orbital residuals illustrate the difference between the strategies used to generate the compared orbits: the IGS orbits are generated from observations in a global network,

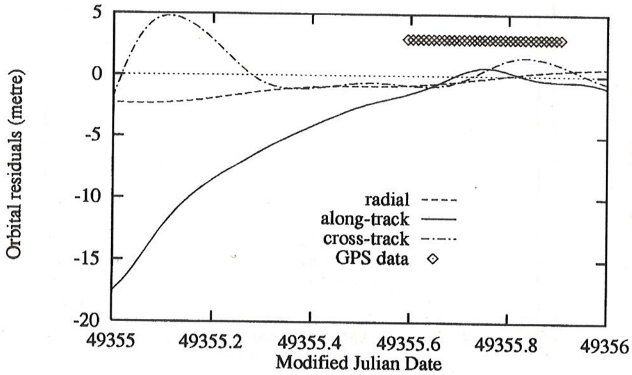

Figure 5 - Orbital residuals for PRN 1 - the whole day.

Figura 5 - *Resíduos orbitais para o satélite PRN 1, para o período de 1 dia.*

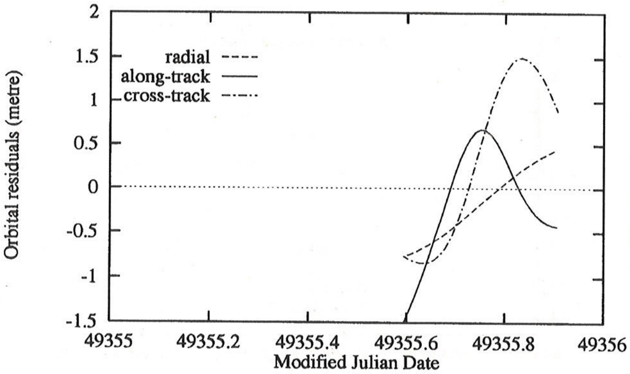

Figure 6 - Orbital residuals for PRN 1 - data coverage only.

Figura 6 - *Resíduos orbitais para o satélite PRN 1, para o período coberto por observações.*

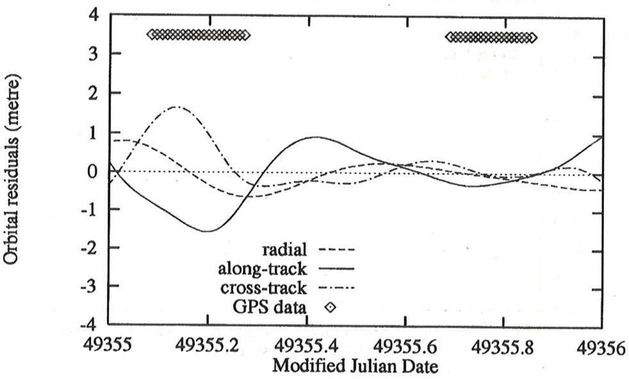

Figure 7 - Orbital residuals for PRN 15 - the whole day.

Figura 7 - *Resíduos orbitais para o satélite PRN 15, para o período de 1 dia.*

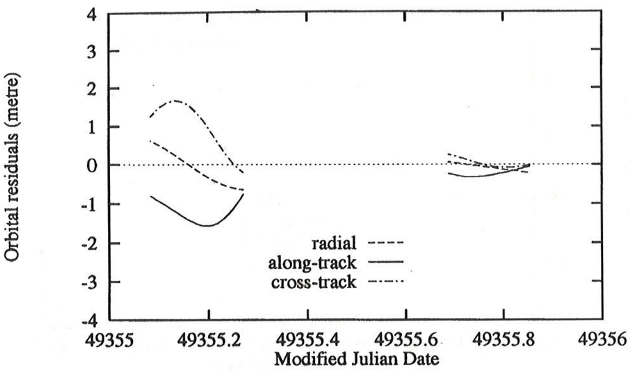

Figure 8 - Orbital residuals for PRN 15 - data coverage only.

Figura 8 *- Residuos orbitais para o satélite PRN 15, para o período coberto por observações.*

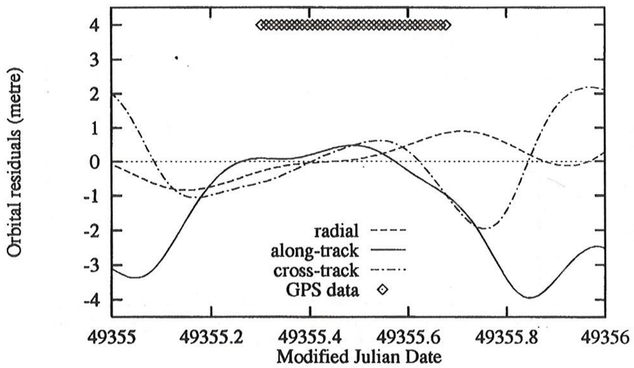

Figure 9 - Orbital residuals for PRN 28 - the whole day.

Figura 9 *- Residuos orbitais para o satélite PRN 28, para o período de 1 dia.*

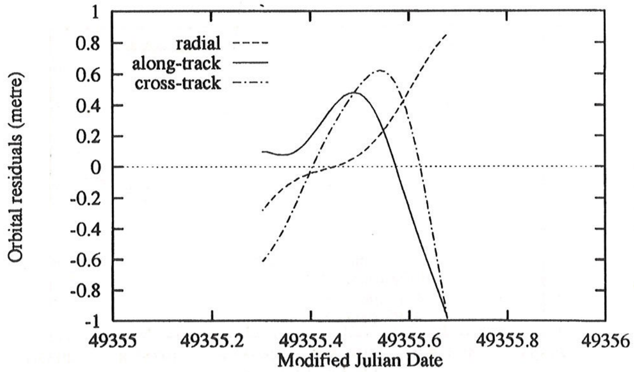

Figure 10 - Orbital residuals for PRN 28 - data coverage only.

Figura 10 *- Residuos orbitais para o satélite PRN 28, para o período coberto por observações.*

whereas the orbits we have generated, come from a regional network. In the case of regional orbits, the orbit trajectories tend to adjust themselves to the observations, distorting somewhat those parts of the orbit with no coverage. Therefore, we can conclude that regional ephemerides are of "good" quality only for the period of time the GPS satellites are observed.

CONCLUSIONS

The technique of orbit improvement with associated generation of ephemerides for GPS satellites has been described. It has been shown that without orbit improvement, predicted GPS orbits tend to degenerate in accuracy quickly, reaching errors of tens of metres after the integration of two orbital arcs (24 hours). A test orbit improvement was carried out based on a regional network composed of 8 North American IGS stations. The results of this combined adjustment of stations and orbits was assessed by using the published ITRF92 coordinates and the IGS orbits as benchmarks. Baselines with relative error of the order of 2×10^{-8} were obtained. The generated regional ephemerides agree with those of the IGS at or below one metre level. A comparison with the IGS products also shows that regional orbits can be of sufficiently good quality for the period of time when observations are collected.

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PRINCÍPIOS DE REFINAMENTO DE ÓRBITAS E GERAÇÃO DE EFEMÉRIDES PARA OS SATÉLITES DO SISTEMA DE POSICIONAMENTO GLOBAL

As efemérides que descrevem a órbita dos satélites GPS representam uma solução particular das equações de movimento. A solução destas equações requer condições iniciais (posição e velocidade, agrupados no chamado vetor de estado inicial) e um modelo que descreva as forças que governam o movimento dos satélites. As condições iniciais têm que ser consistentes com a órbita a ser gerada. Uma pequena diferença nas condições iniciais pode acarretar erros na posição dos satélites da ordem de quilômetros após alguns dias de integração. Para minimizar este problema, as condições iniciais, bem como alguns parâmetros do modelo de força (por exemplo, os parâmetros da pressão da radiação solar) devem ser ajustados dentro de um processo aqui chamado de refinamento de órbitas. Neste contexto, as condições iniciais e os parâmetros do modelo de força constituem os parâmetros orbitais. No procedimento de refinamento de órbitas, os parâmetros orbitais são estimados usando observações coletadas por estações cujas coordenadas sejam conhecidas, ou que sejam estimadas junto com os parâmetros orbitais. As condições iniciais ajustadas podem então ser utilizadas para a geração das órbitas dos satelites GPS. Este artigo se ocupa do refinamento e geração de órbitas para os satelites GPS. O modelo de força adotado utiliza: a representação do geopotencial pelo modelo GEM-T3, até grau e ordem 8; considera o sol e a Lua como elementos puntuais; leva em consideração os efeitos diretos e ao longo do painel solar dos satelites GPS oriundos da pressão da radiação solar; modela as marés terrestres e os efeitos relativísticos. Os efeitos provocados pela refletividade terrestre, marés oceânicas, arrasto atmosférico, manobras dos satelites e campo gravitacional dos planetas foi desconsiderado. Este modelo foi implementado no program PREDICT. Este programa permite ao usuário a escolha de 3 modelos para a pressão da radiação solar. Ele tambem modela a passagem do satelite pela zona de penumbra terrestre. O sistema inercial adotado é o sistema de ascenção reta na época de referência. A integração numérica pode ser efetuada pelos metodos de Adams-Moulton ou Störmer-Cowell, à escolha do usuário. O programa PREDICT permite a geração de efemérides. Para o refinamento das condições iniciais envolvendo observações GPS, etapa que precede a geração das efemérides para os satélites deste sistema, utilizou-se o programa DIPOP, desenvolvido na Universidade de New Brunswick. Este programa permite o pré-processamento e ajustamento pelo método dos mínimos quadrados (usando-se o modelo paramétrico) das observações GPS. Para o nosso propósito, as derivadas parciais correspondendo a solução das "equações variacionais" foram implementadas. Outras implementações efetuadas no DIPOP foram a capacidade de se levar em consideração as correlações matemáticas entre bases ocupadas simultaneamente, bem como a de se estimar parâmetros troposféricos. As órbitas foram determinadas usando-se dados de uma subrede do Serviço Internacional GPS para a Geodinâmica (IGS), composta por estações no Canadá e EUA. A partir desta solução, órbitas regionais são geradas, e comparadas com as órbitas do IGS, que são órbitas globais. Desta comparação se conclui que órbitas de boa qualidade podem ser geradas para os satélites dentro do perído em que os mesmos são observados. Uma avaliação da precisão externa do ajustamento desta rede é possivel através da comparação das bases ajustadas com os valores publicados do referencial ITRF92 (IERS International Terrestrial Reference Frame). Desta avaliação conclui-se que bases com erro relativo da ordem de 2×10^{-8} podem ser determinadas, usando a técnica descrita.