A QUICK REVIEW OF 2D TOPOGRAPHIC TRAVELTIMES

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ABSTRACT. The Common-Reflection-Surface (CRS) stacking method was originally introduced as a data-driven method to simulate zero-offset sections from 2-D reflection pre-stack data acquired along a straight line. This approach is based on a second-order hyperbolic traveltime approximation parameterized with three kinematic wavefield attributes. In land data, topographic effects play an important role in seismic data processing and imaging. Thus, this feature has been recently considered by the CRS method. In this work we review the CRS traveltime approximations that consider the smooth and rugged topography. In addition, we also review the Multifocusing traveltime for a rugged topography. By means of a simple synthetic example, we finally provide first comparisons between the various traveltime expressions.

Keywords: Traveltime, CRS, Multifocus, Topography.

RESUMO. O método de empilhamento Superfície de Reflexão Comum (SRC) foi originalmente introduzido como um método data-driven para simular seções afastamento-nulo a partir de dados sísmicos de reflexão pré-empilhados 2-D adquiridos ao longo de uma linha de aquisição reta. Este método está baseado em uma aproximação de tempos de trânsito hiperbólica de segunda ordem parametrizada com três atributos cinemáticos do campo de onda. Em dados terrestres, os efeitos topográficos desempenham um papel importante no processamento e imagemamento de dados sísmicos. Assim, esta característica tem sido considerada recentemente pelo método SRC. Neste trabalho apresentamos uma revisão das aproximações de tempos de trânsito SRC que consideram topografia suave e rugosa. Adicionalmente, nós revemos também a aproximação de tempos de trânsito Multifoco para o caso da topografia rugosa. Por meio de um exemplo sintético simples, nós fornecemos finalmente as primeiras comparações entre as diferentes expressões de tempos de trânsito.

Palavras-chave: Tempos de trânsito SRC, Multifoco, Topografia.

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INTRODUCTION

The Common-Reflection-Surface (CRS) and the Multifocus (MF) methods are designed to produce clear stacked sections and useful time-domain attributes by means of coherence analysis methods directly applied to multicovery reflection data. In this way, both methods, that have a similar purpose and approach as the well-established common-midpoint (CMP) method, represent powerful extensions of the latter. As opposed to the CMP method that is applied to CMP gathers and extracts a single parameter (the normal-moveout (NMO) velocity) on manually picked events, the CRS and MF methods (a) make full use of the available multicovery data by applying the stacking procedure on supergathers that are free from the CMP restriction; (b) extract more parameters (three in the 2D situation) at each time sample of the stacked section to be constructed and (c) the procedure is applied to all time samples without the need of event selection. The CRS and MF methods fall into the category of the so-called macro-model independent or data-driven time-domain methods (see, e.g., Hubral (1999) for a general account and discussion).

A distinctive feature of the CRS and MF methods is the use of specific multiparameter traveltime moveouts. The parameters of these moveout expressions are directly inverted from the data by means of coherence analysis (semblance) procedures. With the help of the inverted parameters, the obtained moveouts are employed to stack the data. Note that this is exactly what the conventional velocity analysis method does under the restriction of the one-parameter NMO traveltimes applied to CMP gathers.

Originally, both the CRS and the MF have been derived under the following assumptions: (a) 2D propagation; (b) locally-constant and supposedly known near-surface velocity and (c) no topographic effects along the seismic line. The latter condition means that the data have been acquired on a horizontal seismic measurement surface. Under these considerations, the CRS and MF moveouts are expressed as functions of three parameters, namely the emergence angle, $\beta_N$, of the zero-offset (ZO) ray with respect to the (planar) surface normal, and two wavefront curvatures, $K_{NIP}$ and $K_N$, of the so-called NIP- and N-waves that relate to the ZO ray at its emergence point. We recall that the abbreviation NIP stands for normal-incident-point, namely the point where the ZO ray hits the reflector and that the NIP-wave is a fictitious wave that starts as a wavefront with the shape of the reflector at NIP and progresses upwards with half of the medium velocity. For detailed description and discussion of the NIP- and N-waves, the reader is referred to Hubral (1983).

It is to be observed that the CRS and MF methods can also be normally applied in the case the above requirements are not met. In that case, the three-parameter CRS and MF represent simply stacking parameters that provide a best fit to the reflection events, but cannot be directly attached to the above-mentioned seismic propagation attributes (angles and curvatures). For example, the classical CRS and MF expressions consider the condition of a locally constant near-surface velocity. This means that that the near-surface velocity is considered constant in the vicinity of each central point but can vary as we change from each central point to another. To be consistent with the second-order formulation, the traveltime expression has to consider, not only the velocity, but also its gradient at each central point. As a consequence, wrong near-surface velocities will give rise to wrong emerging angles, even though correct ray parameters can be extracted. In the same way, topographic effects, if not correctly taken into account, will contaminate the CRS attributes affecting their interpretation.

For a planar or smoothly curved measurement surface, the 2D CRS (hyperbolic) traveltime that takes full consideration of the velocity gradient at the central point is presented in Chira et al. (2001). In the MF traveltime expressions, velocity gradients at the central points are not considered. To our knowledge, examples of the influence of the velocity gradients have not yet been provided in the literature. In this paper, the usual assumption of a locally-constant near-surface velocity will be also adopted, our attention being concentrated on the incorporation of topography into the traveltime expressions.

2D CRS AND MF TRAVELTIMES FOR A PLANAR MEASUREMENT SURFACE

We recall the “classical” 2D CRS and MF moveout expressions in the simple case of a planar measurement surface. More explicitly, we suppose that all source and receiver pairs are located on a horizontal seismic line. On the seismic line, we assume a fixed point, called central point, on which a simulated ZO trace is to be constructed by stacking along traces that belong to (generally non-symmetric) nearby source-receiver. As explained above, we also consider a constant velocity in the vicinity of the central point. We adopt midpoint and half-offset coordinates $(x_m, h)$ for the location of a source-receiver in the vicinity of the central point, of the zero-offset (ZO) ray with respect to the (planar) surface normal, and two wavefront curvatures, $K_{NIP}$ and $K_N$, of the so-called NIP- and N-waves that relate to the ZO ray at its emergence point. We recall that the abbreviation NIP stands for normal-incident-point, namely the point where the ZO ray hits the reflector and that the NIP-wave is a fictitious wave that starts as a point source at NIP and progresses upwards to the measurement surface with half the medium velocity. In the same way, the abbreviation N-wave, stands for the normal wave, which is also
\( X_0 = (x_0, 0) \). We denote \( m = x_m - x_0 \), by \( T_0 \) the (two-way) reflection time along the ZO (central) ray and by \( v_0 \) the velocity at the central point. As explained above, we assume that the velocity remains constant for all source-receiver locations where the traveltime is to be computed. Finally, \( \beta_0 \), \( K_{NIP} \) and \( K_N \) denote the emergence angle and the curvatures of the NIP and N waves that refer to the ZO ray as observed at \( X_0 \). Under these considerations, the CRS traveltime reads (see, e.g., Jäger et al. (2001))

\[
T_{CRS}^2(m, h) = \left[ T_0 + \frac{2 \sin \beta_0}{v_0} m \right]^2 + \frac{2 T_0 \cos^2 \beta_0}{v_0} \left[ K_N m^2 + K_{NIP} h^2 \right].
\]

For the same situation of a planar measurement surface, the corresponding 2D MF traveltime is given by (see, e.g., Gelchinsky et al. (1999) with a different notation)

\[
T_{MF}(m, h) = T_0 + \frac{1}{v_0} K_S \left[ \sqrt{[K_S (m-h) + \sin \beta_0]^2 + \cos^2 \beta_0 - 1} \right] + \frac{1}{v_0} K_G \left[ \sqrt{[K_G (m+h) + \sin \beta_0]^2 + \cos^2 \beta_0 - 1} \right],
\]

where

\[
K_S = \frac{K_N - \sigma K_{NIP}}{1-\sigma}, \quad K_G = \frac{K_N + \sigma K_{NIP}}{1+\sigma},
\]

and

\[
\sigma = \frac{h}{m + (m^2 - h^2) K_{NIP} \sin \beta_0}.
\]

**CRS APPROXIMATION FOR SMOOTH TOPOGRAPHY**

The CRS traveltime expression (1) admits a natural extension to the case of a smoothly curved measurement surface. More specifically, let us assume that, at the central point and with respect to a horizontal datum, the seismic line has curvature, \( K_0 \) and dip \( \alpha_0 \). See Figure 1(a). Moreover, let \( \gamma_0 \) denote the emergence angle of the ZO central ray with respect to the normal to the curved seismic line at the central point. As shown in Chira (2003) (with a different notation), the CRS traveltime for a source-receiver pair located by \((m, h)\) is given by

\[
T_{CRS}^2(m, h) = \left[ T_0 + \frac{2 \sin \gamma_0}{v_0 \cos \alpha_0} m \right]^2 + \frac{2 T_0 \cos \gamma_0}{v_0 \cos^2 \alpha_0} \left[ K_N \cos \gamma_0 - K_0 \right] m^2 + \frac{2 T_0 \cos \gamma_0}{v_0 \cos^2 \alpha_0} \left[ K_{NIP} \cos \gamma_0 - K_0 \right] h^2.
\]

We ready verify that, in the case of a horizontal seismic line, we have \( K_0 = \alpha_0 = 0 \) and \( \gamma_0 = \beta_0 \), so that equation (5) reduces to its previous counterpart equation (1), as expected.

As reported in Chira (2003), the traveltime moveout (5) were tested for a synthetic model of smooth topography with good results. The formulation breaks down, however, as the topography becomes more pronounced.

**CRS AND MF APPROXIMATIONS FOR RUGGED TOPOGRAPHY**

We now consider the extension of the CRS traveltime to the case of arbitrary (rugged) topography. Following Zhang et al. (2002), we find useful to consider vector midpoint, \( \mathbf{m} \), and half-offset, \( \mathbf{h} \), coordinates with respect to the Cartesian system of horizontal datum and downward vertical. See Figure 1(b). More specifically, we set \( \mathbf{m} = (m_x, m_z) \) and \( \mathbf{h} = (h_x, h_z) \) which locate the corresponding source and receiver pair as \( \mathbf{S} = \mathbf{m} - \mathbf{h} \) and \( \mathbf{G} = \mathbf{m} + \mathbf{h} \), respectively. As shown in Zhang et al. (2002), we have

\[
T_{CRS}^2(\mathbf{m}, \mathbf{h}) = \left[ T_0 - \frac{2}{v_0} (m_x \sin \beta_0 + m_z \cos \beta_0) \right]^2 + \frac{2 T_0}{v_0} K_N \left( m_x \cos \beta_0 - m_z \sin \beta_0 \right)^2 + \frac{2 T_0}{v_0} K_{NIP} \left( h_x \cos \beta_0 - h_z \sin \beta_0 \right)^2.
\]

The corresponding MF traveltime for rugged topography has been described in Gurevich et al. (2002). In our notation, the MF is given by
NUMERICAL EXPERIMENTS

In order to check the accuracy of the traveltime formulas discussed in this work, we consider the simple 2D model depicted in Figure 2. It consists of two homogeneous acoustic layers, with velocities 1.75 km/s and 2.5 km/s, respectively, separated by a curved interface. The measurement line has a rugged (nonsmooth) topography. We have used the ray tracing package SEIS88 (Červený & Psánsik, 1988) to model the reflection traveltimes for the reflecting interface. We have simulated a multiple coverage around the central point with $x_0 = 2$ km.

Figures 3–5 show the modelled reflection traveltimes for three different configurations, Common-Source (CS), Common-Offset (CO) and Common-Midpoint (CMP), and the respective approximation formulas, CRS smooth [CRS-S], equation (5), CRS rugged [CRS-R], equation (6), and Multifocus, equation (7). Also depicted are the relative errors for the three approximations.

As readily observed, the CRS-S formula gives poor approximations, whereas CRS-R and Multifocus present good results with relative errors of the same order.

We have also compared the exact reflection traveltimes with the corresponding diffraction traveltimes approximations, i.e., taking $K_{NIP} = K_N$ in CRS and Multifocus formulas. The objective of this experiment is to check the possibility of its use as a two-parameter search in the application of the CRS method. We note that such a strategy has been already implemented by

\[
T_{MF}(m, h) = t_0 + \frac{1}{v_0 K_S} \left[ \sqrt{[K_S (m_x - h_x) + \sin \beta_0]^2 + [K_S (m_z - h_z) + \cos \beta_0]^2} - 1 \right] \\
+ \frac{1}{v_0 K_G} \left[ \sqrt{[K_G (m_x + h_x) + \sin \beta_0]^2 + [K_G (m_z + h_z) + \cos \beta_0]^2} - 1 \right],
\]

where

\[
K_S = \frac{K_N - \sigma K_{NIP}}{1 - \sigma}, \quad K_G = \frac{K_N + \sigma K_{NIP}}{1 + \sigma},
\]

\[
\sigma = \frac{h_x - h_z \tan \beta_0}{m_x - m_z \tan \beta_0 + Q K_{NIP} \sin \beta_0},
\]

and

\[
Q = (m_x^2 - h_x^2 - m_z^2 + h_z^2) + (m_x m_z - h_x h_z) \cos 2\beta_0.
\]
**Figure 2** – 2D acoustic model for the numerical experiments. The ZO ray is plotted in red, the reflection ray in blue and the diffraction ray in dashed blue. Also indicated are the normal-incidence point (NIP) and the reflection point (R).

**Figure 3** – Reflection traveltime approximations for a CS configuration with \( x_s = 2.0 \) km.
Figure 4 – Reflection traveltime approximations for a CO configuration with $h = 0.5$ km.

Figure 5 – Reflection traveltime approximations for a CMP configuration centered at $x_m = 2.0$ km.
Figure 6 – Diffraction traveltime approximations for a CS configuration with $x_S = 2.0 \text{ km}$.

Figure 7 – Diffraction traveltime approximations for a CO configuration with $h = 0.5 \text{ km}$. 
Garabito et al. (2003) in the case of Marmousi data with good results. Figures 6 and 7 depict the results obtained in the present experiment. Observe that the CMP configuration is not shown since the travelt ime expressions are the same as in the previous case.

As expected, the relative errors were now increased. Once again, the CRS-S formula has the worst behaviour whereas CRS-R and Multifocus formulas behave similarly. We verify that, for small aperture the relative errors remain very reasonable, so that the CRS-R and Multifocus formulas for diffraction traveltimes are able to well approximate the exact (modelled) reflection traveltimes. As a conclusion, a two-parameter search in the CRS method using $K_{NLP} = K_N$ in the formulas, have the potential of producing good initial approximations for the CRS attributes.

CONCLUSIONS

We quickly revisited the Common-Reflection-Surface (CRS) and Multifocus travelt ime moves out in the case of a topographic measurement surface. By means of a simple synthetic example, we provided first comparisons on their accuracy and validity. Our results show that the approximation formulas for rugged topography yield good results, not only to approximate the exact (modelled) reflection traveltimes, but also as a two-parameter formula in the search for initial approximations for the CRS attributes.

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