



COMPARISON OF OBJECTIVE FUNCTION TOPOLOGY AND ACCURACY BETWEEN DIFFERENT TRAVEL-TIME APPROACHES FOR CONVERTED WAVE EVENTS

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ABSTRACT. Many non-hyperbolic approximations were proposed to perform the velocity analysis for several effects which cause the non-hyperbolic behaviour in travel-time curves from PP and PS reflection events. With the application of the OBN (Ocean Bottom Nodes) technology, even the approximations that present good results in previous works lack good results when there was a large difference of datum between source and receivers. Recently, a converted wave moveout approach to describe the combined behaviour of many causes which generate the non-hyperbolicity, including the difference of datum, was proposed. In this work, we tested this approach to compare it to the best converted wave moveout approach presented in the last decades. Residual function maps, to study the complexity of the objective function topology and the comparison of travel-time curves, were tested to find which approach is the best option to perform the velocity estimation through an inversion procedure to recover velocity information of non-hyperbolic reflection events of a pre-salt structure obtained with OBN technology.

Keywords: converted waves, OBN, non-hyperbolic travel-time.

RESUMO. Muitas aproximações não-hiperbólicas foram propostas para realizar a análise de velocidade para vários efeitos que causam o comportamento não-hiperbólico na curva do tempo de trânsito a partir de eventos de reflexão *PP* e *PS*. Com a aplicação da tecnologia *OBN* (Ocean Bottom Nodes), mesmo as aproximações que apresentam bons resultados em trabalhos anteriores carecem de bons resultados quando havia grandes diferenças de profundidade entre fonte e receptores. Recentemente, foi proposta uma abordagem para descrever o comportamento combinado de várias causas que geram a não-hiperbolicidade, incluindo a diferença de profundidade. Esta abordagem foi testada para ser comparada com a melhor abordagem apresentada nas últimas décadas. Os mapas residuais de funções objetivo, para estudar a complexidade da topologia da função objetivo e a comparação das curvas de tempos de trânsito, foram testados para descobrir qual abordagem é a melhor opção para realizar a estimativa de velocidades por um procedimento de inversão para eventos de reflexão de estruturas do pré-sal.

Palavras-chave: ondas convertidas, OBN, tempo de trânsito não-hiperbólico.

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INTRODUCTION

In recent decades, several non-hyperbolic approximations have been proposed for performing the velocity analysis in seismic data processing, and many of them were multiparametric, taking into account not only the time for zero offset and the *RMS* (root mean square) velocity, but also an additional parameter that allows us to describe the non-hyperbolicity for a specific reason; for instance, large offsets in a layered medium (e.g., Malovichko, 1978; Ursin and Stovas, 2006; Blias, 2009), anellipticity (e.g., Muir and Dellinger, 1985), wave conversion (e.g., Slotboom, 1990), and anisotropy (e.g., Alkhalifah and Tsvankin, 1995). Li and Yuan (2003) proposed an approximation that describes the non-hyperbolicity due to the effects of long-offset, layered media and wave conversion. This approximation has shown quite good results in previous and recent works which tested this equation for converted events and/or OBN data (e.g., Wang and Pham, 2001; Bokhonok, 2011; Wang et al., 2014; Hao and Stovas, 2015; Tseng et al., 2016; Zuniga, 2017; Zuniga et al., 2017; 2019c; Lu et al., 2018; Farra and Pšenčík, 2018; Xu and Stovas, 2018 and 2019; Abedi and Stovas, 2019a).

Recently, complex and deeper reservoirs have been discovered and, with an increase in the complexity of the technology and geometry acquisition, it became necessary to take into account the non-hyperbolicity due to another effect, the use of *OBN* (ocean bottom nodes) technology. This kind of technology modifies the acquisition geometry due to the fact that the source is on the ocean surface while the receivers are coupled on the ocean bottom. For this reason, Wang and Pham (2001) have proposed an approximation which generalized the one proposed by Li and Yuan (2003), which can consider the difference of datum between the source and the receivers. However, for this approximation, it is difficult to describe this effect for travel-time reflection events of ultra-deep reservoirs, since it was proposed (Wang and

Pham, 2001) and tested (Wang et al., 2014) for a water depth of around 1 km. For this reason, Zuniga (2021) has proposed an approximation that is capable of describing the non-hyperbolic effects from the difference of datum between source and receiver for ultra-deep reservoirs. This generalization of Li and Yuan (2003) approximation and the parameter proposed by Wang and Pham (2001) use an additional parameter to describe the effects for which a combination of the two approximations is proposed. The additional parameter of this approximation has only the function of performing a better curve fitting to reach more accurate information concerning the *RMS* velocity.

In this work, the study of the complexity of the objective function topology was carried out to understand the behavior concerning the uniqueness, stability, and sensitivity of the approximations used and their variables. So, it was possible to understand which kind of information can be recovered more accurately. The residual error was also computed and analyzed to determine which approximation tested in this work is the best option for performing the velocity analysis of *PP* and *PS* reflection events for this type of structure (i.e., a pre-salt ultra-deep carbonate reservoir).

Even though Li and Yuan (2003) approximation has presented very good results in previous works for several geological models, it was not proposed to be used for the kind of scenario tested in many of them. For this reason, it is very desirable to test and to compare its results to the ones related to the approximation proposed by Zuniga (2021) in order to determine if this recently proposed approach is a more appropriate approximation to be used in this kind of scenario. This comparison is essential for complex structures with data obtained with the use of *OBN* technology, since this approximation (i.e., Zuniga, 2021) was only recently proposed; and, therefore, there are not enough previous results concerning its accuracy in comparison to other approximations.

Used model

Table 1 shows the characteristics of the model used for this study, an offshore layered model with a carbonate reservoir ($V_P = 4010$ m/s and $V_S = 2012$ m/s). It is sealed by a salt structure consisting of the 3rd, 4th and 5th layers. The travel-time curves were raytraced for *PP* and *PS* reflection events (Margrave, 2000 and 2003; Thorbecke and Draganov, 2011). The reflection events were generated for a maximum offset between source and receiver of 15 km and with the *OBV* technology, where the source is on the surface and the receivers are on the ocean floor, resulting in a difference of datum between the source and the receivers.

Table 1 - The parameters of the Model: Layer thickness (Δz), P-wave velocity (V_P), RMS S-wave velocity (V_S) and V_P/V_S ratio.

Layer	Δz (m)	V_P (m/s)	V_S (m/s)	V_P/V_S
Water	2157	1500	0	-
1	496	2875	1200	2.40
2	108	3505	1628	2.15
3	664	4030	2190	1.84
4	262	5005	2662	1.88
5	1485	4220	2210	1.91

This model was previously used by Zuniga (2017 and 2021) and Zuniga et al. (2019a), and has its parameters based on well log data from pre-salt from Santos Basin. It is a fairly common structure in Santos Basin, making it an important model and benchmark for seismic processing tests for this type of reservoir.

Converted wave travel-time approaches

The approximation proposed by Li and Yuan (2003) uses a non-hyperbolic parameter denoted as γ . It was previously studied by Li and Yuan (1999 and 2001), and it is based on the anisotropic parameters of Thomsen (1986). Eq. 1 considers the *CP* (Conversion Point) aiming to control the non-hyperbolic effects associated with the wave conversion joint to large offsets for layered media.

$$t = \sqrt{t_0^2 + \frac{x^2}{V^2} + \frac{(1-\gamma)}{\gamma V^2} \cdot \frac{-(\gamma-1)x^4}{4t_0^2 V^2 + (\gamma-1)x^2}} \quad (1)$$

In Equation 1, γ is the ratio between the squared *P*-wave stacking velocity V_{P2} and the squared converted wave stacking velocity V_{C2} , and defined by the expression

$$\gamma = \frac{v_{P2}^2}{v_{C2}^2} = \frac{\gamma_{eff}(1+\gamma_0)}{(1+\gamma_{eff})} \quad (2)$$

Here, $\gamma_{eff} = \gamma_2^2/\gamma_0$, where γ_2 is the ratio between the stacking *P*-wave and stacking *S*-wave, and γ_0 is the ratio between the *P*-wave and *S*-wave velocities, which travel along the normal component. Equation (1) was lately studied by Li (2003) and then compared with other approximations by Zuniga et al. (2017, 2018, 2019c).

Recently, Zuniga (2021) proposed the following approximation with the same parameter γ

$$t = \sqrt{t_0^2 + \frac{x^2}{V^2} - \frac{(\gamma-1)^2 x_{LS}^4}{\gamma V^2 [4t_0^2 V^2 - (\gamma-1)x_{LS}^2]}} \quad (3)$$

where x_{LS} is defined as follows

$$x_{LS} = x \left(1 + \frac{z_{WD} V_{WD}}{t_0 V_{C2}^2} \right) \quad (4)$$

Here, z_{WD} is the thickness of the water layer, and V_{WD} is the *P*-wave velocity in it. Both parameters are a priori information and are not restored during the inversion procedure.

However, this approximation uses a relation concerning the offset similar to that proposed by Wang and Pham (2001). When this relation was originally proposed for models with limited difference of datum between source and receivers, it could not control this effect for ultra-deep reservoirs. Thus, Equation 3 considers an apparent offset concerning the difference of ray inclination between the *P*-waves in the water and in the solid medium, which is not significantly affected by depth.

Complexity analysis of the objective function for PP and PS events with both approaches

Residual function maps (*RMF*) are effective tools to study the complexity of the topology of an objective function (Larsen, 1999; Kurt, 2007). The analysis of the objective functions by an *RFM* can provide important information concerning the stability and uniqueness of the problem, and also concerning the sensibility of each parameter to be recovered during the inversion (e.g., Larsen, 1999; Li and Yuan, 2003; Bokhonok, 2011; Du and Yan, 2013; Lu et al., 2015; Aleardi et al., 2017; Zuniga, 2017; Zuniga et al. 2018; 2019b).

In this work, the *RMF* was used to correlate the *RMS* (Root mean squared) velocity and the additional parameter, which is γ . The third dimension in the hyperplane represents the minimum values, the residual error between the calculated and observed travel-time curves.

Figure 1A represents a complex structure of the objective function, with both global and local minimum regions. The same can be observed in Figure 1B. However, in this case, it is possible to observe the displacement of the structure due to the difference of parameters that satisfies the problem, once it is a converted event and not a *PP* reflection event. The Li and Yuan (2003) equation showed a very homogenous relation between the sensibilities of the two parameters, which helps to recover them with a homogeneous relative error.

Figure 2A, which represents the *PP* event for Equation 3 and Figure 2B (*PS* event), showed almost the same behaviour as in Figure 1, with a very similar multimodal behaviour and structural characteristics regarding the minimum regions and differences between the *PP* and *PS* events. However, in this case, parameter γ turned out to be more sensitive than the velocity.

The structure presents no results for a γ value of slightly less than 0.55. For this value of the parameter γ , there is a local minimum solution associated with the velocity value slightly exceeding the real one, which is possibly related to the behaviour of the *PP* event. For $\gamma = 1$, we can indicate a region between the global

and local minimum regions associated with the hyperbolic part of Eq. 1 and Eq. 3 once, when the parameter γ tends to 1, both equations tend to the hyperbola equation. Since both equations have the same origin, some behaviours were expected to be very similar. However, the large variation in sensitivity was higher than expected.

Comparison of accuracy between both approaches

The comparison of the accuracy of the approximations is done by comparing the difference between the observed curve and the curve calculated for each approach. The one that represents the residual travel-time error closest to zero, together with the offsets, represents the approximation that more accurately reconstructed the parameters.

This method was used to compare the accuracy of several non-hyperbolic travel-time approximations and showed to be an efficient manner to determine the best approximation to be used in a type of model, such as q-P reflection events in VTI media (e.g., Aleixo and Schleicher, 2010; Golikov and Stovas, 2012), converted wave events in near-surface structures (e.g., Bokhonok, 2011; Lu et al., 2018), converted waves in VTI media (e.g., Hao and Stovas, 2015; Tseng et al., 2016), OBN data (e.g., Wang and Pham, 2001; Wang et al., 2014), converted waves and OBN data (e.g., Zuniga, 2017; Zuniga et al., 2017 and 2019c), orthorhombic media (e.g., Xu and Stovas, 2018 and 2019), and anisotropic media (e.g., Farra and Pšenčík, 2018; Abedi and Stovas, 2019b).

Li and Yuan (2003) approximation presented a very good result with an error of less than 0.05% for the *PP* event and slightly above 0.08% for the *PS* reflection event (Fig. 3).

The approximation proposed by Zuniga (2021) showed an error of less than 0.02% for the *PP* reflection event and an error of less than 0.05% for the *PS* reflection event, which is extremely accurate, even for this controlled situation (Fig. 3).

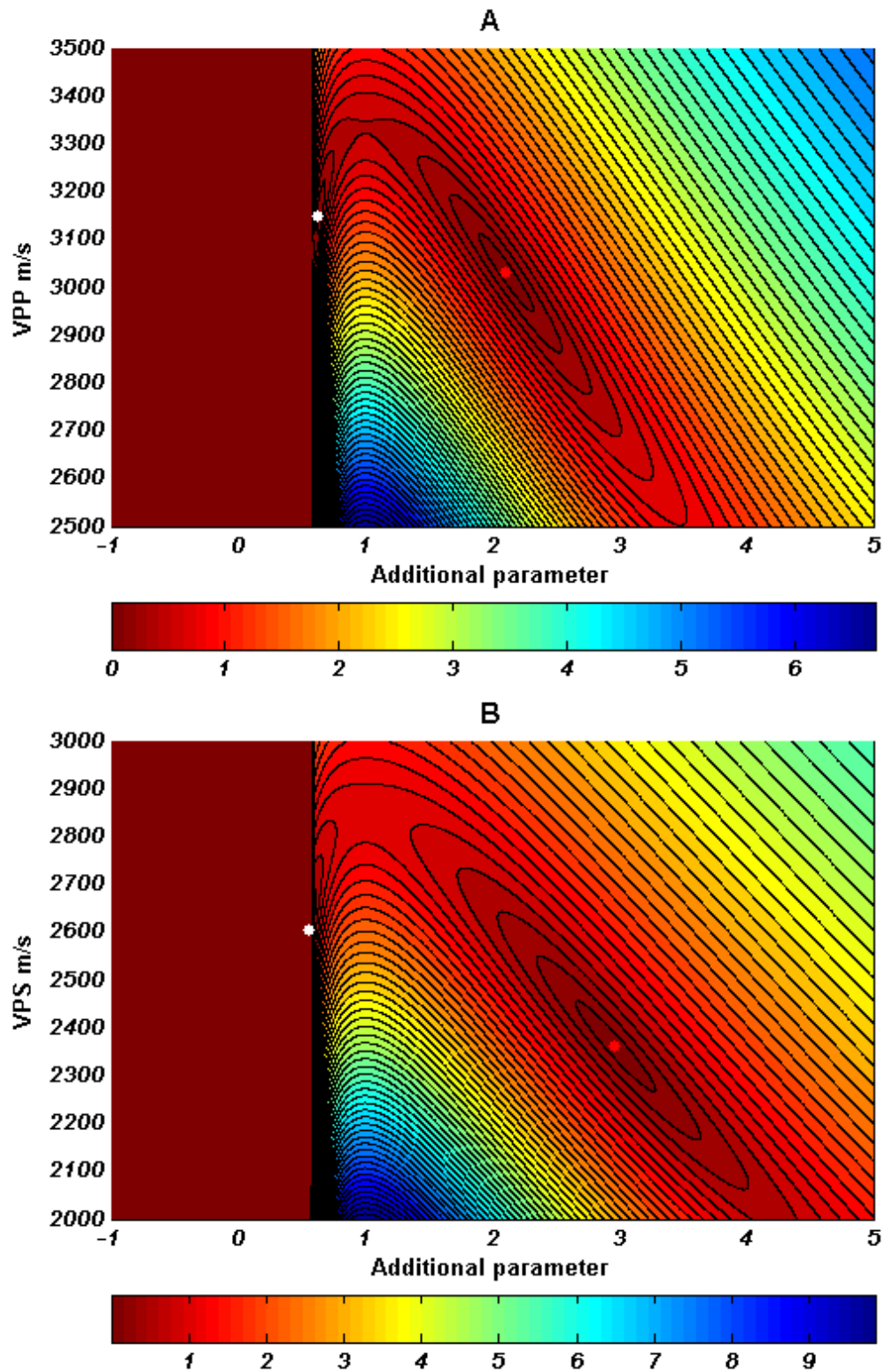


Figure 1 - The residual function maps demonstrate the complexity of the topology of the approximation proposed by Li and Yuan (2003) by relating parameter γ (additional parameter) and RMS velocity for (A) PP reflection event and (B) PS reflection event. The red dot represents the global minimum region and the white dot represents the local minimum region.

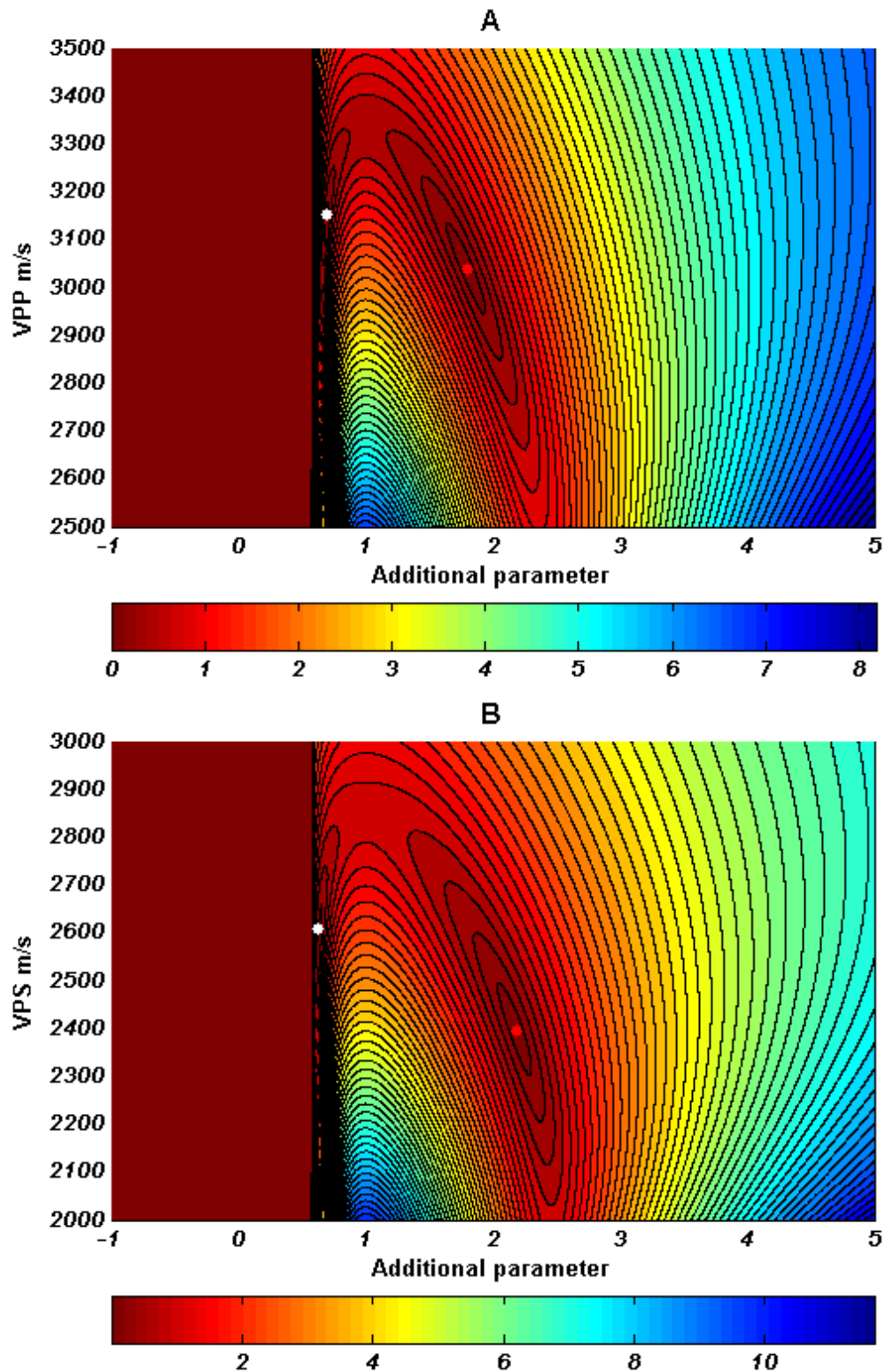


Figure 2 - The residual function maps demonstrate the complexity of the topology of the approximation proposed by Zuniga (2021) by relating parameter γ (additional parameter) and RMS velocity for (A) PP reflection event and (B) PS reflection event. The red dot represents the global minimum region and the white dot represents the local minimum region.

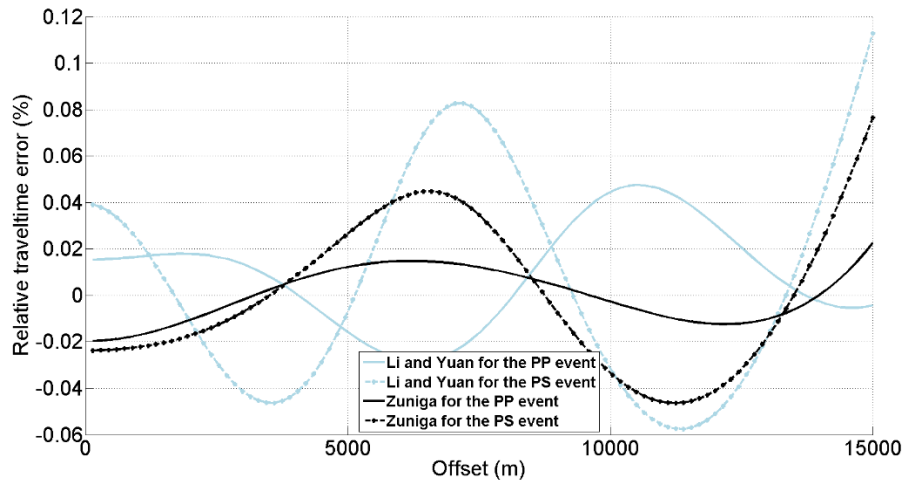


Figure 3 - Relative errors in travel-time between the observed curve and the calculated curve of the Li and Yuan (2003) approximation and Zuniga (2021) approximation for PP and PS reflection event.

The relative average processing time to perform the inversion with the Li and Yuan (2003) approximation was almost 17% higher for the PS event than for the PP event, in contrast to Zuniga (2021) equation, which showed an increase of 11% in the relative average processing time of the PS event when compared to the PP event. However, the Zuniga (2021) approximation presented a relative average processing time 9% higher than the relative average processing time presented for the PP event using the Li and Yuan (2003) approximation; the relative average processing time was 4% higher for the PS event.

CONCLUSIONS

The approximation proposed by Zuniga (2021) presented a very peculiar behavior concerning its topology. However, the higher sensitivity of parameter γ for this equation (with similar behaviour of the RMS velocity), in comparison to the other approach, is important to perform the inversion procedure, once the velocity, which has a physical meaning, still behaves in a known manner, and γ becomes a much more effective fitting parameter for recovering velocity information in a more accurate manner.

For this reason, the approximation proposed by Zuniga (2021) clearly showed a better set of results during the inversion procedure, allowing a more accurate recovering of the RMS velocity information for this type of scenario. Even with a slightly higher processing time, this approximation presented significantly more accurate results than any other approximation tested so far for this type of model, which makes it the most appropriate non-hyperbolic multiparametric approximation to be used in this type of scenario.

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