

VELOCITY INVERSION OF PSP-WAVES ON MARINE SEISMIC DATA USING OBN TECHNOLOGY

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ABSTRACT. Performing the velocity analysis, during the seismic processing, is very complex for converted waves. The problem becomes even more challenging when there is more than one wave conversion. The use of PSP-wave reflection events allows obtaining a more detailed set of information, which is interesting to other steps of seismic processing and other kinds of processing routines, which enhance the structural characterization of a model. Using PSP reflection events presents much more difficulty to recover velocity information, since it involves the nonhyperbolicity generated by the difference of datum between source and receivers, layered media with large offsets, and wave conversion. It is much more complicated to characterize the wave conversion in a PSP reflection than in PS reflection events because the wave conversion happens twice during the wave propagation. Since there is no specific mathematical description for this kind of travelttime event, we propose to treat the velocity analysis as an inverse problem by calculating the travelttime event with a general nonhyperbolic travelttime approximation to fit the observed events. For our tests, we modelled a Pre-Salt structure using well log data from Santos Basin. Thus, it is possible to recover the velocity information of converted PSP event with the proposed inversion procedure; and, therefore, to obtain a better velocity estimation to be used for KMAH (Keller–Maslov–Arnol'd–Hörmander) index.

Keywords: converted waves, inversion, PSP event, OBN, nonhyperbolic.

RESUMO. Realizar a análise de velocidades, durante o processamento sísmico, é muito complexo para ondas convertidas. O problema se torna ainda mais desafiador em uma condição na qual há mais de uma conversão. O uso de eventos de reflexão de ondas PSP permite obter um conjunto mais detalhado de informações, o que é interessante para outras etapas do processamento sísmico e outros tipos de rotinas de processamento, melhorando a caracterização estrutural. Usar eventos de reflexão PSP, nas aquisições marítimas, com tecnologia OBN, apresenta muito mais dificuldade para recuperar informações de velocidade, já que envolve a não-hiperbolicidade originada da diferença de dados entre fonte e receptor, meios estratificados e com longos afastamentos na linha de aquisição e conversão de ondas. A última, neste caso, é muito mais complexa do que para os eventos PS devido à conversão que ocorre duas vezes durante a propagação. Uma vez que não há uma descrição matemática específica para este tipo de evento de tempo de trânsito, é proposto tratar a análise de velocidades como um problema inverso para calcular o evento de tempo de trânsito com uma aproximação não-hiperbólica, para ajustar os eventos observados PP, PS, PSS e PSP. Com esses dados, é possível recuperar as informações de velocidade de eventos de ondas convertidas PSP e entender quão mais difícil é esta etapa para este tipo de evento de reflexão e, por conseguinte, obter uma melhor estimativa de velocidades para ser usada para o índice KMAH (Keller–Maslov–Arnol'd–Hörmander).

Palavras-chave: ondas convertidas, inversão, evento PSP, OBN, não-hiperbólica.

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INTRODUCTION

To perform a velocity analysis according an inversion criterion, it is necessary to characterize the traveltimes curve events. For nonhyperbolic cases, it is significantly harder; however, the information found in converted wave events are quite enriching. For this reason, several nonhyperbolic approximations were proposed for different kinds of nonhyperbolicity, such as, for instance: the cases in which there are large offsets with a layered medium, which result in a difference between the RMS (root mean square) velocity and the real velocity (e.g., Malovichko, 1978; Ursin and Stovas, 2006; Blias, 2009); anellipticity (e.g., Muir and Dellinger, 1985), which distorts the wave front; wave conversion (e.g., Slotboom, 1990), where there is an asymmetry between the incident and reflected angles; OBN (ocean bottom nodes) data (e.g., Zuniga, 2021), where there is an asymmetry between the incident and reflected ray paths; anisotropy (e.g., Alkhalifah and Tsvankin, 1995), where there is a significant variation of the behaviour of the wave propagation depending on the axis of the propagation; and the combination of the influence of layered media with large offsets and wave conversion (e.g., Li and Yuan, 2001 and 2003).

In the last years, Li and Yuan (2003) approximation was found to be the most general and the most efficient for offshore conditions by presenting to be very effective for both PP and PS reflection events (Hao and Stovas, 2015; Tseng et al., 2016; Zuniga et al., 2017; 2018; Lu et al., 2018; Farra and Pšenčík, 2018; Xu and Stovas, 2018 and 2019), and also for both streamer technology and OBN technology (Wang and Pham, 2001; Wang et al., 2014; Zuniga et al., 2019; Zuniga, 2021).

Differently from the PS converted wave event, for offshore acquisitions, the PSP event has two conversion points: one between the water and the ocean bottom, from an incident P-wave to an S-wave; and another on the reflector interface, from an incident S-wave to a reflected P-wave. This kind of event presents much more complexity concerning the ray tracing, which brings much more difficulty to perform the inversion due to the

stronger nonhyperbolicity. Even though the curve fitting between the observed and the calculated curves being considerably harder, the information obtained from a PSP event is required to refine a model regarding some areas of a medium with specific features, such as for time lapse modeling and more detailed seismic modeling (Mitrofanov and Priimenko, 2018). Once the PSP waves can be obtained from an acquisition using the OBN technology, and not only from an acquisition using the streamer technology, it is possible to apply all the benefits showed by Mitrofanov and Priimenko (2018) of the data obtained with the PSP event information, such as a better use of the AVO (amplitude versus offset) inversion and FWI (Full Waveform Inversion).

In previous works, the complexity of the topology of the objective function (Larsen, 1999; Kurt, 2007) by the analysis of RMF (residual function maps) was performed to understand the behaviour of many hyperbolic and nonhyperbolic functions for different offshore reflection events (Aleari et al., 2017; Zuniga, 2017); converted PS reflection events (Larsen, 1999; Li and Yuan, 2003; Du and Yan, 2013; Lu et al., 2015; Zuniga et al., 2017; 2019); L2- and L1-norm (Loris et al., 2007; Santos and Figueiró, 2011; Zuniga et al., 2019; Costa et al., 2020; Zuniga, 2021); and different kinds of optimization algorithms (Tawarmalani and Sahinidis, 2004; Rios and Sahinidis, 2013; Zuniga, 2021). However, this kind of analysis was not necessary in this case, once the variation of the events (PP, PS, PSS or PSP), for Li and Yuan (2003) approximation, presented only displacements concerning the variation of the value of the parameters (Zuniga, 2017).

For another kind of multimodal approximation, described by Zuniga et al. (2018), there is an exchange of positions between the global and local minimum regions in its topology, when the PP and PS events are compared (e.g., Muir and Dellinger, 1985; Ursin and Stovas, 2006; Blias, 2009). In the case of using this kind of approximation, it would be extremely necessary to perform a complexity analysis of the topology of the objective function by using the RMF, since it is not known in which minimum region the global minimum would be located in a PSP event.

In this work, we proposed to treat the velocity analysis as an inverse problem, in which the traveltimes curve calculated with Li and Yuan (2003) approximation fits to the observed PP, PS, PSS, and PSP traveltimes events. This makes it possible to obtain the RMS (Root Mean Squared) velocity of PSP events, an essential information to perform some non-conventional analysis in seismic processing, allowing to apply this information for problems in which Keller–Maslov–Arnol'd–Hörmander (KMAH) index is used as described by Mitrofanov and Priimenko (2018). With these kinds of results, it is also possible to determine how accurate and how more complex is to obtain velocity information of the PSP events, in comparison to other events, which provide essential information to select a more appropriate nonhyperbolic approximation for each type of reflection event. Once the use of KMAH index demands a higher amount of velocity information, an approach able to recover velocity information of PSP events becomes even more interesting.

Model

The model used in this work is the same used previously by Zuniga (2017) and Zuniga et al. (2019) in order to perform the velocity analysis treating it as an inverse problem. The parameters of the model (Table 1) are based on well log data from pre-salt from Santos Basin. It is a very common structure found in the Santos Basin, and an offshore layered model presenting a carbonate reservoir, with $V_P = 3599$ m/s and $V_S = 1805$ m/s, sealed by salt (4th, 5th and 6th layers), composed by a layer of anhydrite between two layers of halite (Table 1). Figure 1 shows the velocity profile of the structure.

Table 1 - The parameters of the Model: Layer thickness (Δz), P-wave velocity (V_P), S-wave velocity (V_S) and V_P/V_S ratio.

Layer	Lithology	Δz (m)	V_P (m/s)	V_S (m/s)	V_P/V_S
0	Water	2101	1500	0	-
1	Sandstone	431	2852	1190	2.40
2	Shale	82	3390	1512	2.24
3	Sandstone	525	3461	1590	2.18
4	Halite	212	3801	1885	2.02
5	Anhydrite	1151	4321	2219	1.95
6	Halite	503	3820	1899	2.01

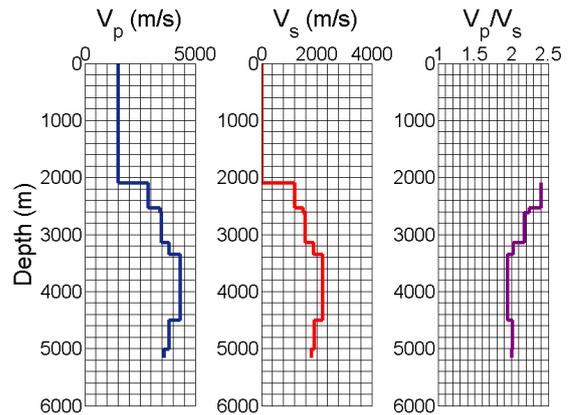


Figure 1 - P-wave velocity (V_P), S-wave velocity (V_S) and V_P/V_S ratio profiles of the Model.

The ray tracing of the reflection seismic events was initially constructed using an adapted version of the MATLAB toolbox proposed by Margrave (2000; 2003). However, to perform a better characterization, especially for the PSP-wave event, a more complex software is needed for the case we test in this work. To overcome this problem, the 2D wave field modelling by a finite difference scheme proposed by Thorbecke and Draganov (2011) was used to generate a 2D model (Fig. 2). Both acoustic and elastic wave equations were used considering compressional-wave velocity, shear-wave velocity, and density, with which the bulk modulus and shear modulus can be previously calculated for the modelling. For the geometry of acquisition, a difference of datum between source and receivers was used. It was possible considering the OBN technology, in which the receivers are on the bottom of the ocean, while the source is on the surface. With this condition, there is a more nonhyperbolic reflection for the converted PS-waves, and an even more complex one for a PSP-wave reflection. For the PS-wave, there is an incident ray of the P-wave until it reaches the reflection point, while the reflected ray travels as an S-wave until it reaches the receiver. For the PSP reflection, a much more complex ray tracing is expected, once the incident P-wave ray is converted in an S-wave at the point in which it reaches the ocean bottom, and then, it reflects as a P-wave again. Another test performed in this work, but also with no previously real data tested, was to consider a PSS-wave reflection event, in which, similarly to the PSP event, the incident ray

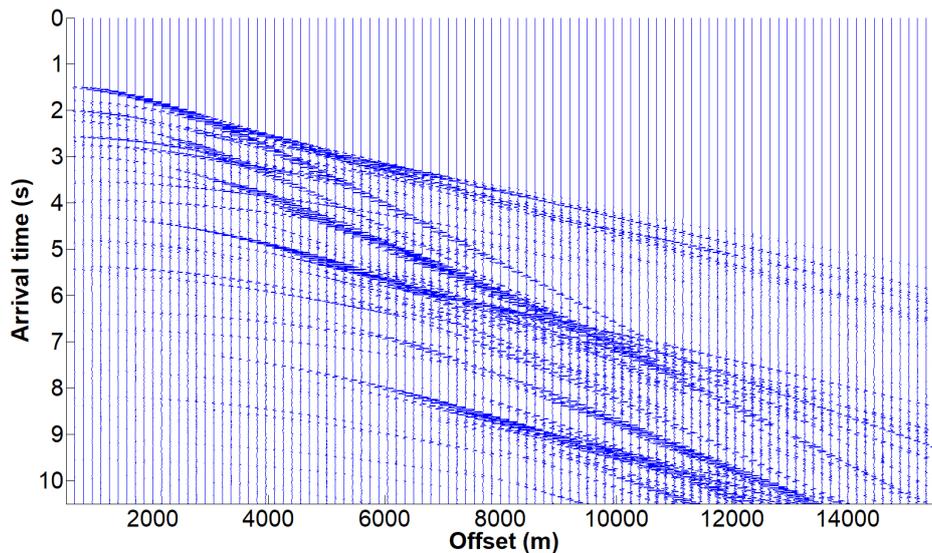


Figure 2 - Seismogram of the 2D model generated with well log data described in Table 1.

on the water is a P-wave, while the incident ray on the geological structure is an S-wave; however, unlikely the PSP event, the reflected ray is an S-wave. The difference among each kind of ray tracings (PP, PS, PSP and PSS) is shown in Figure 3.

PSP-wave separation and Keller–Maslov–Arnol’d–Hörmander (KMAH) index

Mitrofanov and Priimenko (2018) proposed a method to obtain PSP-wave reflection events by using an algorithm which uses *a priori* information regarding the medium. This allows the application of the ray tracing procedure to identify this kind of reflection events. The time intervals can be composed by reflected signals from the aimed interface; and these time intervals are applied to pick the times to select the PSP-wave by being determined with the application of the ray tracing procedure (Dankbaar, 1985; Hubral et al., 1996; Mitrofanov and Priimenko, 2013).

This method considers a group of algorithms based on using the maximum of the *a priori* information available, such as the velocity of the wave propagation for the medium (Mitrofanov and Priimenko, 2013); for instance, LSMF (short for "Learn, Select, Mutate and Forget") algorithm (Nemeth, 1996), true amplitude migration algorithms and CFP (Common Focal Point) operators (Hubral et al., 1996; Bolte, 2003).

The algorithm must be operated under some conditions which are formulated according to the results of studies based on the analysis of existing algorithms proposed to select PS-waves: strong background noise events, in which their properties are close to the PSP-wave; purpose of identifying the regions with the strongest excessive increasing of the amplitudes of the converted waves, which are able to produce a maximum signal-to-noise ratio in the set of selections; purpose of determining the regions which present caustic singularities for converted waves, aiming to extirpate the effect of this kind of features (Nefedkina *et al.*, 1980; Dankbaar, 1985; Lou *et al.*, 2006).

The algorithm uses the second derivative of the hodograph of the PSP-wave, which can provide a faster solution for the direct problem and can also perform the identification of regions with caustic singularities for converted waves (Kuedyukova, 1993; Mitrofanov and Priimenko, 2018).

The method proposed by Mitrofanov and Priimenko (2018) is based on the identification of the regions with caustic singularities that can be performed by using the KMAH index, which is associated to the Jacobean of the transformation of the coordinates of a ray into Cartesian coordinates. Primarily, a set of reference rays for a static boundary is created, and then, the same condition is performed for a specified kind of wave. Then, two rays, which are the closest reflected rays to the surface, are selected and, by using the

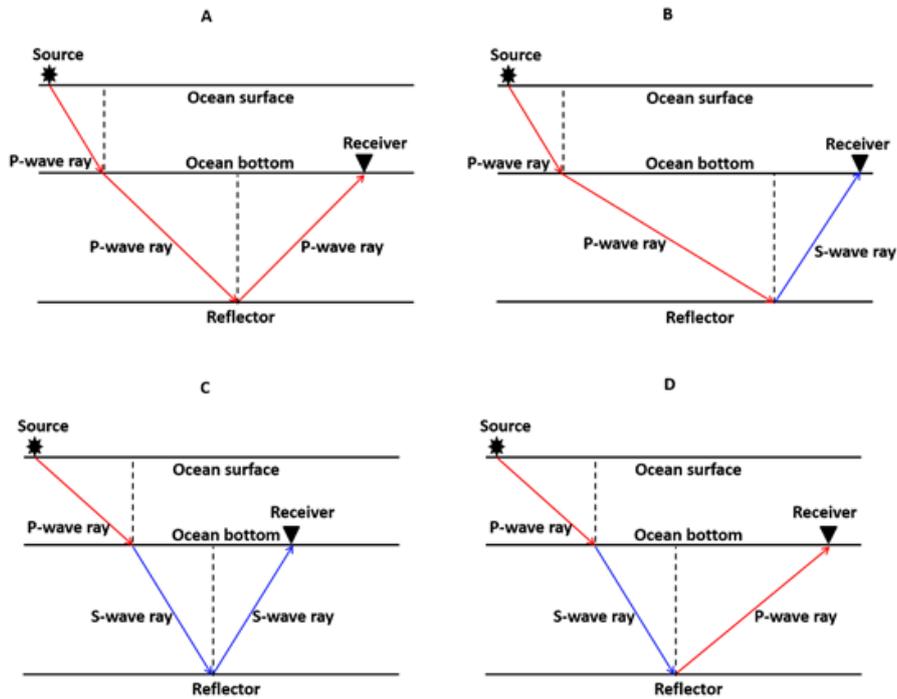


Figure 3 - Ray tracing of an offshore simplified subsurface model using OBN technology for (A) a PP event, (B) a PS event, (C) a PSS event, and (D) a PSP event.

standard shooting procedure, it is possible to determine the two reflected rays directed to the surface. A linear interpolation, to find the angle related to a receiver point, between the selected rays is performed and this angle, which was previously obtained, is used to determine a new ray, defined by its ray parameter, as an initial approximation.

Since broader velocity information is essential to perform a reliable KMAH index analysis, an approach able to recover PSP information is strongly desirable.

Nonhyperbolic travelttime approach

The approximation selected to perform the velocity analysis was the one proposed by Li and Yuan (2003), once it was previously tested for different kinds of models and showed the best set of results in previous works (e.g., Wang and Pham, 2001; Wang et al., 2014; Hao and Stovas, 2015; Tseng et al., 2016; Zuniga, 2017; Zuniga et al., 2017 and 2019; Lu et al., 2018; Farra and Pšenčík, 2018; Xu and Stovas, 2018, 2019). This approximation uses the γ parameter to control nonhyperbolicity generated by the wave conversion of a PS-wave that happens at the

conversion point. The additional parameter γ was based on parameters proposed by Thomsen (1986), when he described the anisotropic parameters proposed by him. Li and Yuan (1999) studied how γ – which is Thomsen’s γ parameter adapted by Li and Yuan (2001) – could be applied to describe nonhyperbolicity, and how it behaves in inhomogeneous and anisotropic media (Li and Yuan, 2001; Li, 2003), leading them to develop a converted-wave moveout and conversion-point equation in layered media (Li and Yuan, 2003), to understand in which conditions the proposed approximation works in a better manner.

$$t = \sqrt{t_0^2 + \frac{x^2}{v^2} - \frac{(\gamma - 1)}{\gamma v^2} \frac{(\gamma - 1)x^4}{4t_0^2 v^2 + (\gamma - 1)x^2}}, \tag{1}$$

where x is the offset, t_0 is the travelttime for zero offset, v is the RMS velocity, γ is the ratio between the squared P-wave stacking velocity v_{P2} and the squared converted wave stacking velocity v_{C2} , as it can be observed in Equation 2

$$\gamma = \frac{v_{P2}^2}{v_{C2}^2} = \frac{\gamma_{eff}(1 + \gamma_0)}{(1 + \gamma_{eff})}, \tag{2}$$

in which $\gamma_{eff} = \gamma_2^2/\gamma_0$, where γ_2 is the ratio between the stacking P- and S-waves, γ_0 is the ratio between P-wave velocity and S-wave velocity which travel along the normal component.

Comparison of accuracies to recover the information of each event

To compare the observed curve on the seismic record and the calculated one with the Li and Yuan (2003) approach for each reflection event (PP, PS, PSS and PSP) is the best manner to find how difficult is to recover the velocity information for a PSP event, with difference of datum between source and receiver, in comparison to the other events. So, it is necessary to compare the residual traveltimes between the observed traveltime curves (*i.e.*, PP, PS, PSS, and PSP events) in the seismic record and the curve calculated with the nonhyperbolic traveltime approximation. Since it was treated as an inversion procedure according an optimization criterion, the selected optimization algorithm used was the one proposed by Nelder and Mead (1965) selecting Least Squares (L2-norm) as the minimization method. The inversion is performed by fitting the traveltime calculated with the nonhyperbolic approximation to the observed traveltime curves along the offset axis, and by minimizing the Least Squares errors between them. With the optimization criterion, each iteration delivers a more accurate velocity estimation until it reaches its minimum relative traveltime error. Each starting point is randomly selected to overcome the possibility of reaching only a local minimum region. Then, it is possible to observe that the curve which presents more points closer to zero has a lesser error, and therefore, it is easier and more accurate to recover the information regarding the velocity.

Zuniga (2021) observed that this approach strongly depends on an accurate traveltime picking, and proposed a specific traveltime extraction for this kind of problem, which is based on recovering the seismic spectrum of each wavelet in an event to find its position in each trace. However, it was also observed that this technique is valid only to extract events related to interfaces which present a strong difference of physical properties between layers and present a very thick layer overlying the target

interface (*e.g.*, a two-kilometer salt structure which is sealing a pre-salt carbonate reservoir). Another important observation was that the number of OBN receivers used in an acquisition is relevant for this kind of event; however, it was observed that if it is possible to perform this kind of traveltime picking and velocity analysis for a number of receivers, the accuracy tends to increase when both approaches are performed for a lower number of receivers. This happens because the number of points to be fit in a traveltime event is lower, and, therefore, easier to perform the inversion procedure, since it demands less iterations.

This comparison method was used previously to compare different approaches (Aleixo and Schleicher, 2010; Golikov and Stovas, 2012; Zuniga, 2017; Zuniga et al., 2017 and 2019) and, once it brought good results on determining the best approximation to recover the velocity information, it is also applicable to determine in which event is the most difficult to have its velocity information recovered, using the same approach.

In Figure 4, it is possible to observe the relative errors in traveltime for each layer, and how they increases with depth because of the increasing in the discrepancy between the real interval velocities and the RMS velocity. The apparent sinusoidal behaviour of the error is related to the very low error in fitting traveltimes close to the t_0 ; and it makes the calculated traveltime curve crosses the observed traveltime curve in further offsets even when they are very well fit, as described by Zuniga (2017 and 2021). For the first interface, the error is very low because the RMS velocity is very similar to the real interval velocity, since there are only two different rock layers. The error is increased drastically with the increase of the quantity of layers. The error of PP events (*i.e.*, events without wave conversion) increases with depth in a more subtle manner, while the errors of PS and PSS events (*i.e.*, events which the wave is converted only once) increase in a less subtle manner and also vary very similarly between each other. Differently from them, the traveltime error of PSP events (*i.e.*, events which the wave is converted twice) increases in a more abrupt manner in comparison to the other events.

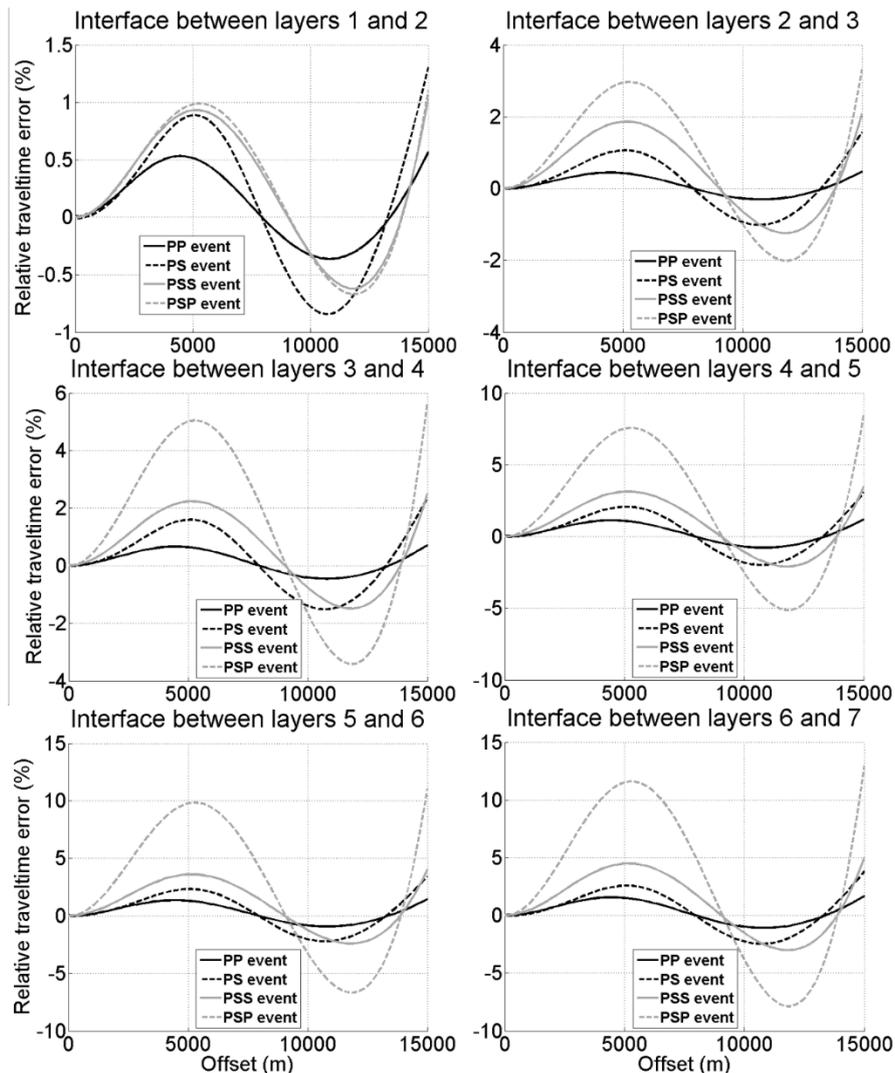


Figure 4 - Relative errors in traveltimes, for each reflection event related to each interface, between the observed curve and the calculated one with Li and Yuan (2003) approximation of the target events for PP, PS, PSS, and PSP reflection events.

The stronger variation of the error for events which present more wave conversions might be related to the higher influence of the discrepancy between the RMS velocity and the real interval velocity, once with more wave conversions there are more accumulated error between the real and the RMS velocity. The first interface considered in this work is the one between layers 1 (unconsolidated sandstone) and 2 (shale); the second one is the one between layers 2 (shale) and 3 (sandstone); the third one is between layers 3 (sandstone) and 4 (halite); the fourth one is between layers 4 (halite) and 5 (anhydrite); the fifth one is between layers 5 (anhydrite) and 6 (halite); and the sixth interface is the one between the sixth layer (the bottom of the salt structure) and the seventh layer (carbonate reservoir).

In Figure 5, we show the average relative error in traveltimes among each interface found for each type of event. The maximum average relative errors are found for each event, and can be observed in Figure 5. For the PP reflection event, the simplest one, there is a maximum average relative error of around 0.91%; for the PS reflection event, there is a maximum average relative error of around 1.87%; for the PSS reflection event, it was observed a maximum average relative error of around 2.32%; and for the PSP event, the highest average relative error observed was of around 4.98%. The spikes of errors after 13500 meters of offset were disregarded, once the effects of the RMS velocity increase the error in a nonhomogeneous way; and it could be harmful for the analysis.

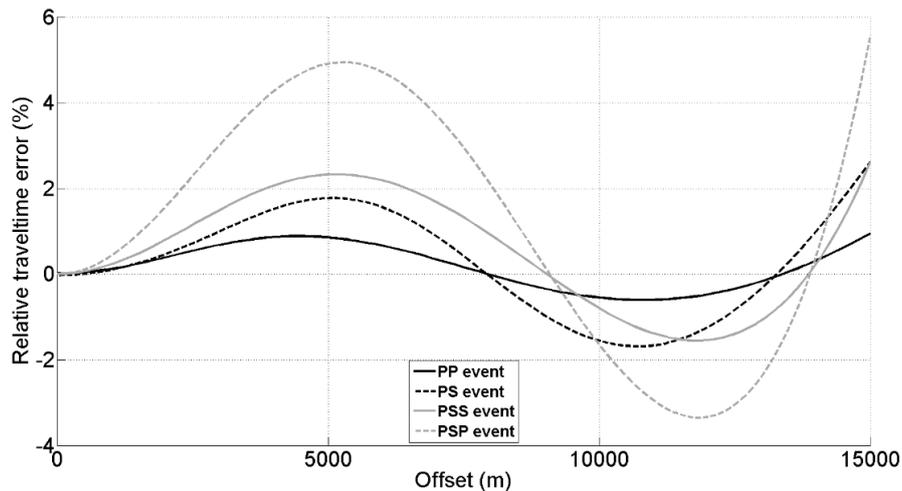


Figure 5 - Average relative errors in traveltimes between the observed curve and the calculated one with Li and Yuan (2003) approximation of the target events for PP, PS, PSS, and PSP reflection events found in Figure 4.

The shape of the variation of the residual traveltimes error curve of each event presented a very stable condition, varying only the magnitude of the error, but not how it is distributed along the offset (Fig. 5).

The error, when it is close to the t_0 , tends to zero, being important to obtain a good pair of information – t_0 and RMS velocity (Fig. 5). For this reason, it is possible to perform a better structural characterization not depending on the kind of the event.

CONCLUSIONS

As it was previously observed, the greater the influence of the S-waves on the event, the less accurate the obtained information is. This happens because the topology of the objective function of the traveltimes event becomes less abrupt due to the lesser difference of velocity, once the event is, in general, slower. In addition, there is, of course, the influence of increasing the nonhyperbolicity of the event due to the wave conversion on the target layer (for the PS event) and on the ocean bottom (for the PSS event). For this reason, the PS event presented an error higher than the PP event and, therefore, the PSS event presented an error higher than the PS event. However, the fact that the PSP event presented a much higher error than all the other events showed

that the influence of the nonhyperbolicity is much higher, for this event, than the influence of the less strong gradient concerning the complexity of the topology of the objective function, since the PSP event has a shorter S-wave part, regarding the ray tracing, in comparison to the PSS event, and has an S-wave part, regarding the ray tracing, comparable to the PS event.

The proposed inversion procedure and method of comparison presented to be a very efficient manner to recover the velocity information related to PSP reflection event for this kind of model. It also allows understanding how complex is to perform the inversion and recover the information regarding the RMS velocity for each kind of event, which also provides information to determine the most appropriate nonhyperbolic traveltimes approximation to perform the inversion procedure of each kind of reflection event. Despite of the relative errors of the PSP events presented to be significantly higher than the ones of the other events, the increase of the relative errors, for this kind of event, is not higher enough to preclude obtaining an acceptable accuracy in order to recover the RMS velocity information of this kind of event, during the inversion. However, since it is less accurate to obtain the RMS velocity of a PSP event in comparison to a PS event, it is mandatory to apply the inversion while performing the velocity analysis of all kinds of events available,

aiming to minimize the uncertainties of the information obtained from the PSP event, in order to enrich velocity information that will be used in KMAH index.

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