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# COMPARISON BETWEEN L2- AND L1-NORM AND AMONG OPTIMIZATION ALGORITHMS FOR A NONHYPERBOLIC MULTIPARAMETRIC APPROACH FOR CONVERTED WAVE AND OBN DATA

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**ABSTRACT.** As velocity analysis is an important step in seismic processing, several nonhyperbolic traveltime approximations have been proposed during the last decades, and each nonhyperbolic approximation was developed for different conditions and with different proposals. However, none of them was proposed to consider the combined effect of the nonhyperbolicity coming from layered media with large offsets, wave conversion and difference of datum between source and receiver. For this, a nonhyperbolic multiparametric traveltime approximation, which is capable of describing this combination of effects, was recently proposed. As this approximation was developed to characterize ultra-deep reservoirs, the understanding of its behavior is necessary for an offshore reservoir concerning the objective function topology complexity, as it is important for a better understanding of its behavior during the inversion procedure, and also important to determine the kind of optimization algorithm to be used. It is proposed performing the inversion procedure with different optimization algorithms and norms. It is also proposed the complexity analysis of the objective function. Then, a comparison between each norm and among each algorithm concerning their accuracy and efficiency is proposed to find which combination is the most effective to recover the RMS velocity information for this kind of scenario.

**Keywords:** nonhyperbolic equation; optimization algorithm; topological analysis; inverse problem.

## INTRODUCTION

In the middle of the last century, Dix (1955) proposed the hyperbola equation to perform the velocity analysis, an important step for the reflection seismic data processing. However, the hyperbolic approximation is not suitable for layered media with large offsets, wave conversion and difference of datum between source and receiver. For these reasons, in the last decades, several nonhyperbolic approximations able to describe effects of the nonhyperbolicity in a traveltime event were developed (*e.g.,* Malovichko, 1978; Muir and Dellinger, 1985; Slotboom, 1990; Alkhalifah and Tsvankin, 1995; Wang and Pham, 2001; Li and Yuan, 2003; Ursin and Stovas, 2006; Blias, 2009).

Several nonhyperbolic approximations were tested in the last years, aiming to find the best one for a specific situation or aiming a general behavior (*e.g.*, Wang and

Pham, 2001; Bokhonok, 2011; Wang et al., 2014; Hao and Stovas, 2015; Tseng et al., 2016; Zuniga, 2017; Zuniga et al., 2017, 2019; Farra and Pšenčík, 2018; Lu et al., 2018; Xu and Stovas, 2018, 2019; Abedi and Stovas, 2019a, 2019b). However, even though the approximation proposed by Li and Yuan (2003) showed the best results in a general manner in previous works, more accurate results could be reached if the use of the OBN (Ocean Bottom Nodes) technology would be considered. This enhancement concerning the results would be possible by considering the difference of datum between source and receiver, and not only the nonhyperbolicity coming from the wave conversion and the RMS (root mean square) velocity relation. Wang and Pham (2001) tried to apply the effects of the OBN technology generalizing the approximation proposed by Li and Yuan (2003). However, it did not present good results for ultra-deep reservoirs (Wang et al., 2014; Zuniga, 2021). For this reason, Zuniga (2021) proposed a nonhyperbolic multiparametric traveltime approximation capable of describing nonhyperbolic effects from the relation of large offsets with layered media, wave conversion, and the difference of datum between source and receivers.

In this work, it is proposed testing the approximation developed by Zuniga (2021) concerning the complexity of its topology of the objective function to understand its stability and unicity, an essential step to determine what kind of optimization algorithm is more appropriate to be used and also to evaluate the influence of L2- and L1-norm. The sensitivity analysis of the parameters for L2- and L1-norm is also proposed, aiming to find out whether there is a significant variation in the topological behavior, in order to determine which norm is the most appropriate for this kind of inversion procedure. These analyses are important to predict some possible limitations during the application of the inverse problem. Likewise, it is proposed to perform the inversion of PP and the PS reflection events with five different optimization algorithms in order to determine which one presented the best processing time and accuracy when combined with the approximation tested. It was also tested whether the L1-norm application during the inversion really finds a better result with this approximation, when compared to the L2-norm. This proposed combination of analyses allows determining the best combination of optimization algorithm and norm to be used with the approximation proposed by Zuniga (2021) for reflection events from ultra-deep reservoirs acquired with the OBN technology.

## Nonhyperbolic multiparametric traveltime approximation for converted wave and OBN

A general nonhyperbolic multiparametric traveltime approximation started to be proposed by Li and Yuan (1999) with the $\gamma$ parameter based on the anisotropic parameter of Thomsen (1986). Later, Li and Yuan (2001, 2003) proposed the approximation which was able to control the effects of wave conversion; it was lately studied by Li (2003) and compared with several other approximations (*e.g.*, Wang and Pham, 2001; Wang et al., 2014; Hao and Stovas, 2015; Tseng et al., 2016; Zuniga, 2017; Zuniga et al., 2017, 2019; Farra and Pšenčík, 2018; Lu et al., 2018; Xu and Stovas, 2018, 2019).

Zuniga (2021) developed an approximation based on the one proposed by Li and Yuan (2003), which also uses the $\gamma$ parameter to perform the curve fitting; it makes the approximation able to recover the RMS velocity ($V$) and the time for zero offset ($t_0$). However, as Wang and Pham (2001) and Wang et al. (2014) proposed to control the datum difference effect between source and receivers, it was necessary to correct the behaviour of the vector of offset ($x$) to an apparent offset ($x_{LS}$). Even though this approximation is efficient for some situations, it lacks good result for ultra-deep reservoirs, which led to the development of an approximation that considers not only the difference of datum in a more general way but also the same behaviours proposed by Li and Yuan (2003) with the $\gamma$ parameter. So, the approximation proposed by Zuniga (2021) which is able to describe the effects of layered media with large offsets, wave conversion and difference of datum between source and receivers is given by:

$$t = \sqrt{t_0^2 + \frac{x^2}{V^2} + \frac{-(\gamma - 1)^2 \, x_{LS}^4}{\gamma \, V^2 [4t_0^2 \, V^2 + (1 - \gamma) x_{LS}^2]}} \qquad (1)$$

The apparent offset, $x_{LS}$, considers the difference of ray inclination between the P-wave in the water and in the solid medium for the down-going ray and considers that the up-going ray stops at the bottom of the ocean, the interface between the water and sediments. The $x_{LS}$ is given by:

$$x_{LS} = x\left(1 + \frac{z_{WD} \, V_{WD}}{t_0 \, V_{C2}^2}\right), \qquad (2)$$

where $z_{WD}$ is the water depth and $V_{WD}$ is the P-wave velocity in the water. Both are *a priori* parameters and shall not be recovered during the inversion.

## Model used to perform the tests

To perform complexity tests, it is necessary to use a model already known to compare the results with the previous ones. The offshore model used by Zuniga (2017, 2021) and Zuniga et al. (2019a) is a structure usually found in the Santos Basin. This structure was modelled using the parameter in Table 1, which was extracted from well logs from a pre-salt structure from the Santos Basin. In Table 1, it is possible to observe the characteristics of the offshore layered Model with a carbonate reservoir ($V_P$ = 4010 m/s and $V_S$ = 2012 m/s) sealed by a salt structure composed by the 3rd, 4th and 5th layers. The depth of the bottom of the carbonate layer is unknown. The traveltime curves were generated by the raytracing method for the PP and PS reflection events (Margrave, 2000, 2003) and by a 2D finite difference modelling scheme for the wave propagation (Thorbecke and Draganov, 2011), considering the use of the OBN technology and a maximum offset between source and receivers of 15 km. A total of 100 receivers were used with spacing of 150 meters between each one, the same spacing between source and receiver.

Table 1: The parameters of the Model: Layer thickness (Δz), P-wave velocity ($V_P$), S-wave velocity ($V_S$) and $V_P/V_S$ ratio.

| Layer | Δz (m) | $V_P$ (m/s) | $V_S$ (m/s) | $V_P/V_S$ |
|---|---|---|---|---|
| Water | 2157 | 1500 | 0 | - |
| 1 | 496 | 2875 | 1200 | 2.40 |
| 2 | 108 | 3505 | 1628 | 2.15 |
| 3 | 664 | 4030 | 2190 | 1.84 |
| 4 | 262 | 5005 | 2662 | 1.88 |
| 5 | 1485 | 4220 | 2210 | 1.91 |
| 6 | - | 4010 | 2012 | 1.99 |

## Complexity analysis of the topology of the objective function for L2- and L1-norm

RMF (residual function maps) is an important tool to perform the topology complexity study of an objective function as shown by Larsen (1999) and Kurt (2007). The use of RMF allows obtaining important information about the stability and unicity of the function, and also the information about the sensibility of each aimed parameter (*e.g.*, Larsen, 1999; Li and Yuan, 2003; Bokhonok, 2011; Du and Yan, 2013; Lu et al., 2015; Aleardi et al., 2017; Zuniga et al., 2018, 2019c).

For this work, the RMF was a two-dimensional correlation between the RMS (Root mean squared) velocity and the $\gamma$ parameter, with the third dimension in the hyperplane representing the minimum values as the residual between the observed and the calculated curve.

Considering the comparison between L2- and L1-norm, it is important to describe that the least squares (i.e., L2-norm) error approximate solution is preferred for several problems in signal processing. However, the least absolute deviation (i.e., L1-norm) can be preferable in several situations, as a complex topology of objective function with small distributions can be attenuated with this norm (Khaleelulla, 1982; Bourbaki, 1987; Zuniga et al., 2019b; Costa et al., 2020).

In Figure 1, for the PP reflection event, the L2-norm (Fig. 1A) presents a more stable structure than the one presented by Figure 1B. However, both structures presented a multimodal behavior with a higher sensibility of the $\gamma$ parameter than the RMS velocity did. The main difference between the two norms concerns the minimum regions, considering that, even with local and global regions more connected between them with the L1-norm, a little narrower structure for the global minimum region can be observed, providing a more accurate result during the inversion.

The same characteristics and behaviours can be observed in Figure 2 (PS event). However, there is the displacement of the structure due to the different set of values of parameters as it was already discussed by Zuniga (2017) and Zuniga et al. (2017).

As Zuniga (2017) and Zuniga et al. (2019a) described, concerning the behaviour of this approximation, the $\gamma$ parameter does not exist for values lower than around 0.55, which is closely associated to the local minimum region and possibly related to the part of the P wave event, since this is a part of a solution for a higher RMS velocity. Another behaviour observed is that the L1-norm starts connecting more the local and the global minimum regions, which happens exactly in the $\gamma$ value of 1. It happens due to the fact that this approximation tends to the hyperbolic behaviour when the $\gamma$ parameter tends to 1. So, if the hyperbola equation was used (Dix, 1955), it would reach a higher RMS velocity.

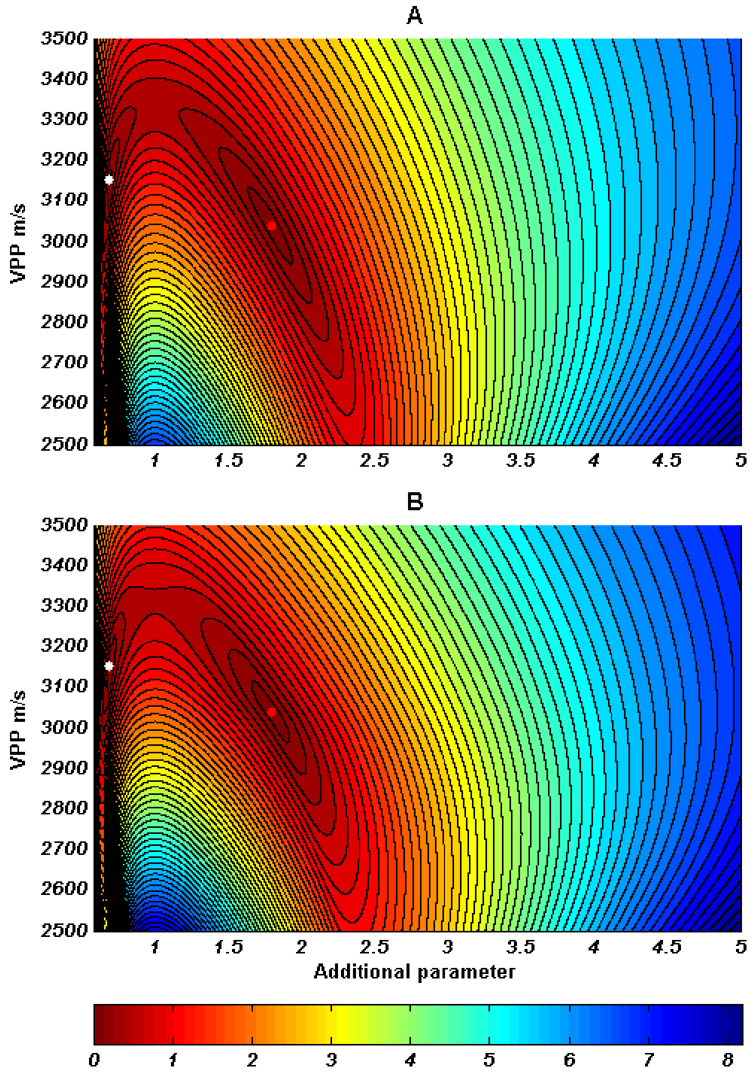

Figure 1: Residual function maps demonstrate the complexity of the topology of the approximation proposed by Zuniga (2021) by relating $\gamma$ parameter (additional parameter) and RMS velocity for the PP reflection event with (A) L2-norm and (B) L1-norm. The red dispersions represent the global minimum region, and the white dispersions represent the local minimum region.

## Accuracy analysis with different optimization algorithms for L2- and L1-norm

Accomplishing the comparison of the optimization algorithms is an important step to find out which algorithm is the most accurate for nonhyperbolic multiparametric approximation and which is the most efficient manner to perform the inversion; therefore, the aimed parameters can be recovered for the analysed scenario in a more reliable way. This comparison can be applied for several types of reflection events, such as for q-P reflection events in VTI media (*e.g.*, Aleixo and Schleicher, 2010; Golikov and Stovas, 2012), converted wave events in near-surface structures (*e.g.*, Bokhonok, 2011; Lu et al., 2018), converted waves in VTI media (*e.g.*, Hao and Stovas, 2015; Tseng et al., 2016), OBN data (*e.g.*, Wang and Pham, 2001; Wang et al., 2014), converted waves and OBN data (*e.g.*, Zuniga, 2017; Zuniga et al., 2017, 2019a), orthorhombic media (*e.g.*, Xu and Stovas, 2018, 2019), and anisotropic media (*e.g.*, Farra and Pšenčík, 2018; Abedi and Stovas, 2019b). In this work, it was computed the residual error between the observed curve and the calculated one, for each optimization algorithm used with L2- and L1-norm.

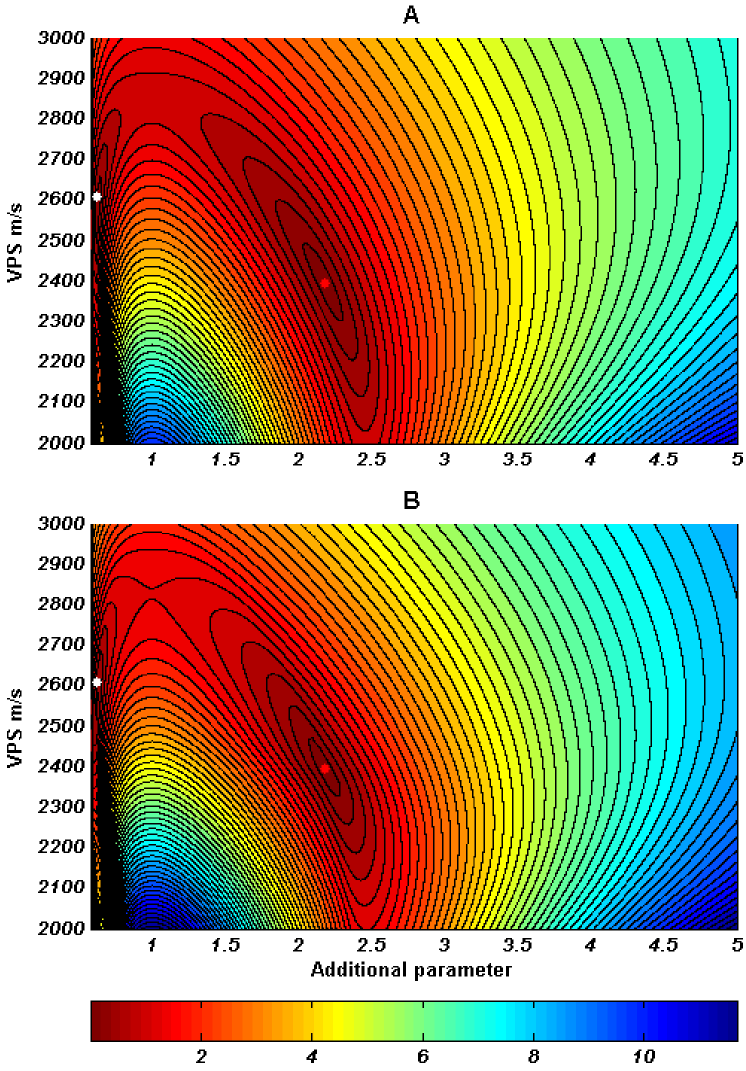

Figure 2: Residual function maps demonstrate the complexity of the topology of the approximation proposed by Zuniga (2021) by relating $\gamma$ parameter (additional parameter) and RMS velocity for the PS reflection event with (A) L2-norm and (B) L1-norm. The red dispersions represent the global minimum region, and the white dispersions represent the local minimum region.

The main objective of the optimization is to find the best option among a set of options (Horst et al., 2000). As this work is aimed to find the minimum value of error, several optimization algorithms must be tested to find out which one presents the best solution. Many optimization algorithms were tested for several problems (Rios and Sahinidis, 2013), but there are only few works that compare them for the problem approached here. Many optimization algorithms presented very good results in many areas of science and engineering, such as the implicit filtering algorithms (Winslow et al., 1991; Gilmore and Kelley, 1995; Kelley, 2011), Powell's model-based algorithms (Powell, 2006, 2008), triple approach algorithms based on extreme barrier, filter and progressive barrier (Abramson, 2002; Audet and Dennis, 2006, 2009; Abramson et al., 2009), simplex-based algorithms (Coxeter, 1948; Spendley et al., 1962; Nelder and Mead, 1965; Lagarias et al., 1998; Lewis et al., 2000) and pattern search method algorithms (Custódio and Vicente, 2007, 2008; Custódio et al., 2010). However, these kinds of algorithms are proposed to local search optimization, efficient for unimodal problems. This generates the necessity of using the multi-start procedure to solve multimodal problems (Kan and Timmer, 1978a, 1978b; Terlaky and Sotirov, 2010; Sotirov and Terlaky, 2013). Some global search optimization algorithms based on particle swarm motion, evolution strategy and multilevel coordinate search (Hansen, 2006; Vaz and Vicente, 2007; Huyer and Neumaier, 2008) were also considered. However, five optimization algorithms with a significant difference in their ways of functioning were selected.

An optimization algorithm proposed to perform global search is the ASA (Adaptive Simulated Annealing) that depends on the magnitude of the parameters (Metropolis et al., 1953). It was initially proposed to solve combinatorial problems (Kirkpatrick et al., 1983) and then to solve continuous and other kinds of problems (Aarts and van Laarhoven, 1985; Romeo and Sangiovanni-Vincentelli, 1991; Bélisle et al., 1993).

The RMS (Response Surface Method) is based on approximating an unknown function by a response called metamodel (Matheron, 1967; Barton, 1994). If there is a difference between the function and the metamodel, this difference is assumed to be caused by a model error (Jones, 2001; Barros et al., 2004).

TOMLAB/EGO is an algorithm which considers both linear and nonlinear constraints (Holmström et al., 2008). This algorithm is based on performing a space-filling experimental design and estimating the aimed value likelihood for a calculated model. Then, the model is tested for consistency and accuracy (Pintér, 1995; Schonlau, 1997; Jones et al., 1998).

The MCS (Multilevel Coordinate Search) is used for bound-constraint problems and is based on performing the partition of the search space into boxes with an evaluated base point (Huyer and Neumaier, 1999; Neumaier et al., 2005).

The last algorithm used in this work is the TOMLAB/LGO, which is based on providing access to several derivative-free optimization solvers (Jones, 2001; Holmström et al., 2008). The LGO (Local and Global Optimization) solver is used as a combination of global and local nonlinear solvers implemented as a combination of a Lipschitzian-based branch-and-bound algorithm with deterministic and stochastic local search (Pintér, 1995; Pintér et al., 2006).

In Table 2, it is possible to observe that the PS event takes a significant higher processing time to perform the inversion, varying from a time 3 to 13% higher depending on the optimization algorithm, but showing almost no difference concerning the time decrease between the L2- to L1-norm for this comparison. However, comparing the L2- and L1-norm for the same reflection event, the mean decrease is around 20% in the processing time, which is an important improvement in the time to perform the inversion. If the ASA optimization algorithm is considered as the reference concerning the processing time, due to the fact that it presented the lowest time to perform the inversion, it is possible to compare the increase of the processing time of the more robust optimization algorithms. Thus, the RSM optimization algorithm presented a mean processing time 31% higher, while the TOMLAB/EGO and the MCS presented a mean processing time 44% and 62% higher, respectively. The TOMLAB/LGO presented the highest mean processing time, 168% higher.

Table 2: Processing time (in seconds) to perform the inversion routine with each optimization algorithm for PP and PS reflection events with L2- and L1-norm.

| **Algorithms** | **PP event with L2-norm** | **PP event with L1-norm** | **PS event with L2-norm** | **PS event with L1-norm** |
|---|---|---|---|---|
| ASA | 131.5 | 109.6 | 142.0 | 118.3 |
| RSM | 177.2 | 147.7 | 181,6 | 151.3 |
| TOMLAB/EGO | 194.4 | 162.0 | 199.2 | 166.0 |
| MCS | 208.1 | 173.4 | 234.6 | 195.5 |
| TOMLAB/LGO | 344.2 | 286.8 | 388.2 | 323.5 |

Figure 3 shows that the ASA optimization algorithm presented the worst result for both reflection events, with a significantly higher error than the other algorithms. The RMS algorithm presented a good result for both reflection events, but the TOMLAB/EGO algorithm presented an even better result. However, the optimization algorithms which presented the best results concerning the accuracy were the MCS and the TOMLAB/LGO with a very similar set of results. Even with a more accurate result from the TOMLAB/LGO, the processing time must also be considered.

Comparing the L2- and L1-norm by the mean results of the five optimization algorithms, what can be observed, in Figure 3C, is that the L1-norm presented a sensible improvement concerning the accuracy, which is also related to the narrower global minimum region.

## CONCLUSIONS

Concerning the topology of the objective function, the tests performed with the approximation showed a very sensitive behavior of the additional parameter, which is essential to perform a more accurate recovering of the RMS velocity information. The use of the L1-norm rather than the L2-norm showed a narrower global minimum region for the PP and the PS events, which is essential to reach an even better RMS velocity characterization. Therefore, the use of the L1-norm presented a little more accurate result, and an important decrease in the processing time. However, it is important to understand that not every optimization algorithm works for L1-norm, which always demands the use of a derivative free optimization algorithm.

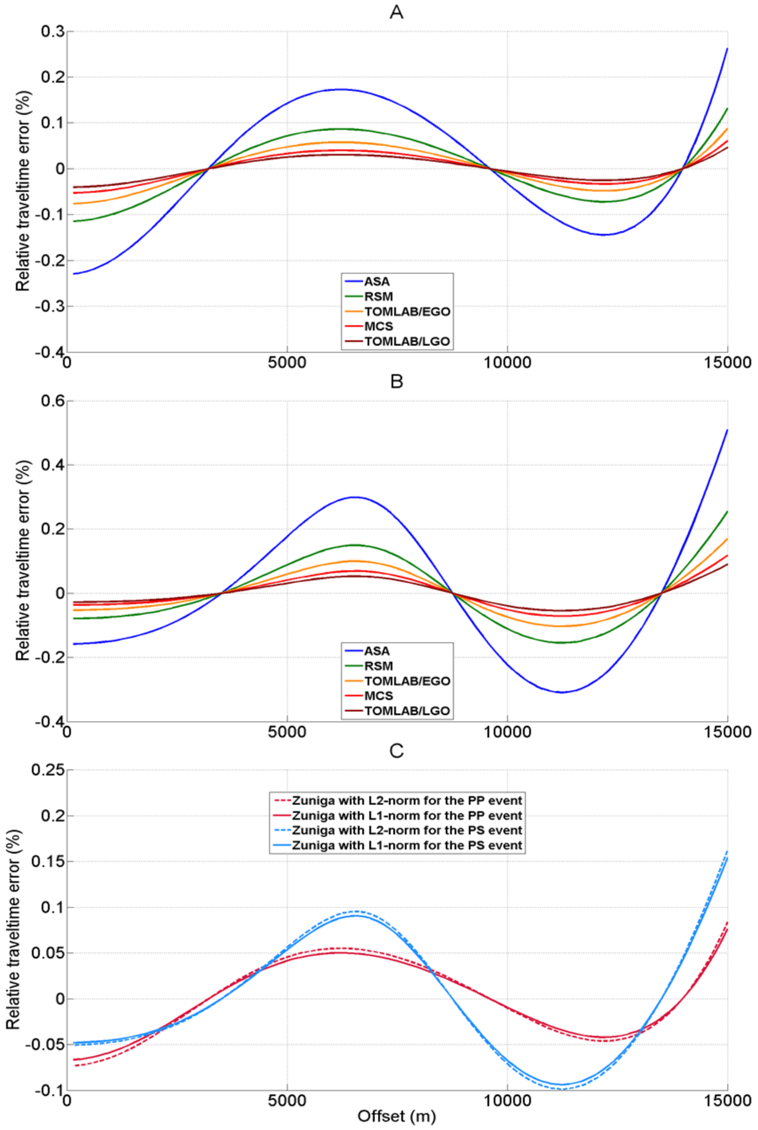

**Figure 3:** Relative errors in traveltime between the observed curve and the calculated one of the Zuniga (2021) approximation for (A) PP reflection event with each optimization algorithm, (B) PS reflection event with each optimization algorithm and the mean behavior of the inversion of the five optimization algorithms for L2- and L1-norm.

With the tests performed in this work, we found that the optimization algorithms showed an accurate set of results; however, the TOMLAB/LGO algorithm showed the best one, that is only a little more accurate than the set of results showed by the MCS algorithm, which presented the second most accurate results. Even though with the ASA algorithm presenting the lowest processing time, the combination of accuracy and efficiency of the MCS algorithm makes it the best general option to be used jointly with the tested approximation, as the TOMLAB/LGO takes too much processing time.

In a general way, the approximation proposed by Zuniga (2021) showed to be an important approach to perform the velocity analysis, working very well with all tested optimization algorithms and for both norms concerning the processing time and the accuracy. However, the most appropriate combination to use with this nonhyperbolic multiparametric approximation was found to be the L1-norm and the MCS optimization algorithm, which presented to be the most efficient combination for this kind of scenario.

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