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GEOTHERMAL ENERGY — THE LONG TERM GLOBAL PERSPECTIVE

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A model thermal history of the Terrestrial planets assumes an initial fully molten state and that the mantle grows by extracting a low melting point fraction from the remaining core. The conservation of energy is represented for the whole body and for the core. The mean surface heat flux is estimated. The energy flux is taken as made up from that of thermal convection of the mantle and magma convection in a zone of partial melting. The rate of recirculation of upper mantle material is also obtained. The model is calibrated with existing planetary data, notably the core radius, surface heat flux and duration of volcanism.
The model behaviour suggests that the dominant mode of heat transfer during the first 1 Ga of planetary time is magma convection.

É proposto um modelo da história térmica dos planetas terrestres que assume um estado inicial inteiramente fundido em que o manto cresce pela extração de uma fração de baixo ponto de fusão do núcleo restante. A conservação da energia é descrita para o corpo integral e para o núcleo. O fluxo de calor superficial médio é estimado. Considera-se o fluxo de energia composto da energia da convecção térmica do manto e da convecção de magma numa zona de fusão parcial. Obtém-se também a taxa de circulação do material do manto superior. O modelo é calibrado com dados existentes dos planetas, principalmente o raio do núcleo, o fluxo de calor superficial e a duração do vulcanismo.
O comportamento do modelo sugere que a forma dominante da transferência de calor, durante o primeiro bilhão de anos da vida dos planetas, é a conveção de magma.

(Traduzido pela Revista)

INTRODUCTION

The Earth is losing energy from it's interior. Our interest in this energy is twofold: as an available and exploitable resource either directly from mining hot water substance or indirectly from mining ores collected and deposited in hydrothermal fluids; as a muted record of the Earth's thermal history. Whatever our interest, a major concern is in the time scales of the particular phenomena, for example: the response of a natural system to local rainfall; the response of an active hydrothermal system to exploitation; the lifetime of a hydrothermal system; the recirculation of the upper mantle; or the duration of volcanism on a particular planet.

The global structures of the planets change. Apart from near surface processes, these changes occur through internal rearrangement. As well as rearranging the distribution of matter the system is rearranging it's energy distribution and can only progress from one state to the next in so far as the energy can be transported through the body. The amounts and rate of that energy transport determine the time scale of the rearrangement.

For the Jovian planets, from a global viewpoint: there is no chemical development; the structural form remains the same; the thermodynamic development is simply a consequence of the continuous contraction of the body. For the Terrestrial planets, major chemical, physical and thermodynamic rearrangements occur.

This paper considers the thermal history of the Terrestrial planets by means of a lumped parameter model. The major assumption of the model is that at the beginning of geological time these bodies were completely molten; that thereafter their mantles began to form by growing downwards using a low melting point extract from the remaining liquid cores.

The lecture, in this year of extreme interest in the return of Comet P/Halley 1986, was dedicated to Edmond Halley (1656-1742). The similarities and differences between a comet and an active hydrothermal system, both seen as vigorous objects depending for their operation on thermal energy and water substance, were briefly described.

The lecture presentation was divided between the temporal aspects of hydrothermal systems and the global aspects of geothermal energy. This paper presents only the latter. My approach to hydrothermal systems is well covered elsewhere (see for example Elder, 1976, 1981). This paper, which covers the second part of the talk, presents a new look at the long term time scales and is part of the presentation of a model of the chemical, structural, and thermodynamic development of the terrestrial planets (Elder, 1987).

MODEL SYSTEM

Consider a model terrestrial planet losing heat from its interior through the surface with an interior hotter than the surface and the surface temperature maintained by external processes. See the sketch of Fig. 1.

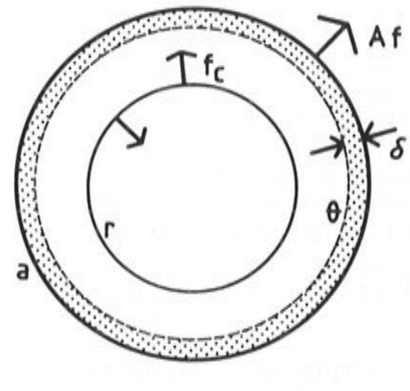

Figure 1 — Scheme for thermal development.

The body is assumed to be fully molten initially, and develops a mantle at a rate determinated by the available energy resources and heat transfer rate through the mantle.

Conservation of energy of the whole body requires

$$d(\varrho VE)/dt = -Af$$

where: A, V are the surface area, volume of the body; E is the mean total specific energy; ϱ is the mean density; and f is the net outward surface heat flux.

Conservation energy at the mantle-core interface requires

$$\varrho H \frac{dr}{dt} = -f_c,$$

where H is the energy released per unit mass in freezing at the core-mantle interface; and f_c is the net heat flux out of the core.

These two relationships provide a mere framework. The quantities in the relations need to be related; and the model calibrated. The key feature of the strategy of this study is to consider the energy budget by comparing the total energy at a particular moment with that at a reference moment — taken as t = 0, 5Ga ago.

TEMPERATURE DISTRIBUTION

The quantities E, H, f, f_c, are determined by the structure of the body; the properties of the matter; and the thermodynamic state. In this simple model the structure is given by the core radius; the properties of the matter are presumed known; and the thermodynamic state will be caracterized by a single parameter the "global representative" temperature, θ.

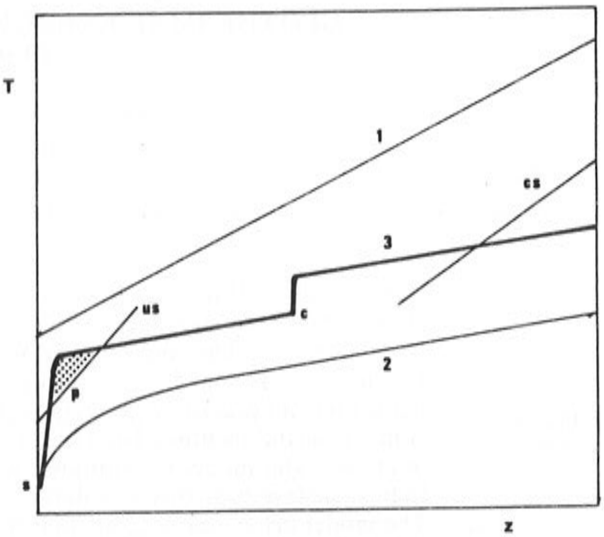

Figure 2 — Schematic temperature profiles used in the models. "3" is the intermediate temperature profile bounded by profile "1", the liquidus, and profile "2" the subsolidus of the undifferentiated material.

This representative temperature is obtained, in principle, from the mean temperature profile. Consider the mean temperature profile at a particular instant as sketched in Fig. 2. The profile — labelled "3" — is bounded by two lines — labelled "1", "2".

Profile "1". This temperature profile at geological time t = 0 is assumed to be that of the liquidus of the original undifferentiated material. The form of this curve will be that of the melting point for depth (MPD) relation — a column everywhere at its melting point, with gradients of order 1 K km^{-1}.

Profile "2". This profile is typical of a sub-solidus in a fully solid body. The form of this curve will be determined by the entire thermal history up to the moment all the material is frozen.

At the surface (s), for all $t \geqslant 0$, the temperature is determined by the solar flux, the planet's orbital distance and the state of the atmosphere — and taken here as independent of time.

The intermediate temperature profile "3" has two distinct parts, those of a solid mantle and liquid core, with a sharp rise in temperature at the mantle-core interface (c):

(i) There is an initial stage with a region in which mantle temperatures exceed the solidus temperatures (us) so that a zone of partial melting (p) exists.

(ii) The core will in general be hotter than the mantle. The bulk of this excess temperature presumably will be taken up in a thin boundary layer at the top of the core — a mush zone of thickness perhaps of order 10 km for the present Earth. It is this excess temperature (over that of a corresponding solid mantle extract) together with the latent heat of freezing which is modelled by the parameter H.

(iii) The inner core will be at temperatures below the core material-extract-solidus (cs). This zone is ignored here.

(iv) Otherside both the deep mantle and core temperatures distributions, on the assumption they are well stirred by convection, will be close to adiabatic ones (with small gradients of order 0.1 K km^{-1}).

The initial global mean temperature θ_o

Initial temperatures at depth, even for a homogeneous material, will exceed that of the surface melting point owing to the increase of melting point with pressure. This increment to the initial mean global temperature can be estimated roughly by considering the liquid sphere to be homogeneous and ignoring the role of compressibility. Thus the pressure $P = P_c (1 - r'^2)$ where $P_c = 2/3\pi G \varrho^2 a^2$, $r' = r/a$; the melting point temperature $T = T_o + \beta P$; and the mean melting point temperature $\bar{T} = \int T\,dm / \int dm = T_o + 2/5\beta P_c$, an increment of $2/5\beta P_c$. For example with $\beta = 10^{-3}$ K bar^{-1} for Mercury, Venus, Earth, Moon, Mars the increment (in K) is: 100, 550, 700, 20, 1000. There is insufficient information to specify these quantities reliably. In the illustrative models, unless stated otherwise, I have therefore chosen nominal values of 2400K with increments of 100K for Mercury, Mars and 600K for Venus, Earth to give θ_o: Mercury, 2500K; Venus, 3000K; Earth, 3000K; Moon, 2400K; Mars, 2500K.

Initial and boundary conditions

At the end of the proto-planetary stage a planet is assumed to be an entirely liquid body. Soon after a crust begins to form, this moment, taken as nominally 5 Ga ago, is time zero in the thermal history models presented here. Thus at $t = 0$, the temperature $\theta = \theta_o$ where θ_o is presumed known; the 'core' radius $r = a$, equals the radius of the planet; and $E = E_o$.

The surface temperature, θ_s is presumed to be constant in time. The values used are: Mercury, 445K; Venus, 750K; Earth and Moon, 300K; Mars, 225K. Possible variation of order 10^2K in these values have minor effects on the behaviour of the model.

ENERGY CONTRIBUTIONS AFTER t = 0

The major energy resources are: the internal thermal energy; with the extra contribution from the core arising from its latent heat and excess temperature treated separately; the gravitational energy. The contribution of radioactive material is presumed be confined to a thin zone near the surface and not to affect the internal budget.

Contribution from internal thermal energy

I choose the "global representative temperature" θ such that the total internal thermal energy for a planet of mass M, and nominal specific heat c (= 1 kJ kg^{-1} K^{-1}), is $Mc\theta$.

Contribution from the thermal energy of the core

The core material is of different material from that of the solid mantle, except at the core-mantle interface it will have a higher temperature than any solid extract, and in the liquid state carries the latent heat of melting. This extra thermal energy, over that of a mantle solid is modelled as the energy per unit mass H.

An order of magnitude estimated of H can be made.

$$H = L + (C_c - C_m)\,\theta_m + C_c\Delta\theta$$

where θ_m is the temperature at the core-mantle interface, $\Delta\theta$ is the temperature difference across a transition zone between the fully solid mantle and fully liquid core, and L is the latent heat of freezing. For example with $L = 0.5$ MJ kg^{-1}, $C_c = 1$ kJ kg^{-1} K, $C_m = 1$ kJ kg^{-1} K, $\Delta\theta = 500$K we obtain $H = 1$ MJ kg^{-1}.

The quantityt, H is a phenomenological constant for each planet. The corresponding contribution to the global specific energy is taken to be $H\,(r/a)^3$ — crudely proportional to the mass of the core.

Gravitational energy available from structural change

As a planetary body rearranges its mass distribution — with a growing mantle — its gravitational energy becomes smaller; the energy change providing a source of thermal energy. The gravitational work function has the form $W = 3/5\,\zeta\,GM^2/a$ where ζ is a dimensioless quantity close to unity, dependent on the mass distribution (for details, see Elder, 1987).

The quantity W/W^*, where W^* corresponds to that of a completely frozen body, for the terrestrial planets is shown in Fig. 3. The percentage change in W a fully molten to a fully solid body is: Mercury, 30; Venus, 18; Earth, 29; Moon, 0; Mars, 5. The change in W for Mars and the Moon is small and negligible for the past interval of geological time — the change in W can be ignored for these two bodies; the change in W is substantial for Mercury, Venus and Earth.

The initial gravitational energy per unit mass, $w = W/M$, for example for the Earth, is about 40 MJ kg^{-1}. this is very large compared to the other energy sources. The available fraction is 29% so that about 12 MJ kg^{-1} is available to run the global system during the solidification process.

Energy total

The total specific energy is

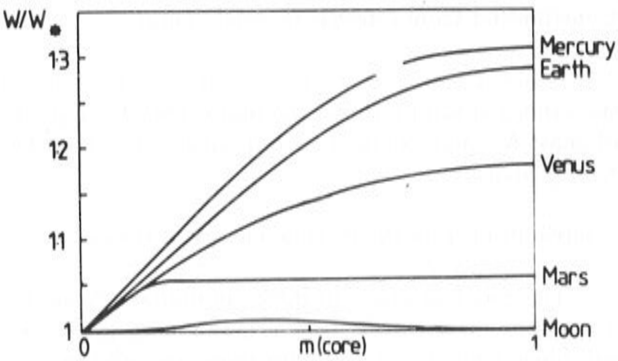

Figure 3 — Gravitational work function ratio, W/W_* as a function of core mass, m(core). Note: m(core) = 0, body completely frozen; m(core) = 1; completely molten.

$$E = c\theta + (\frac{r}{a})^3 H + w$$

comprising the contributions of thermal, core excess, and gravitational energy. The various terms, for example for Earth, have typical magnitudes in units of MJ kg^{-1}, respectively: 3; 5; 12. Of the three contributors only the thermal energy is a strong contributor for all the planets throughout geological time: the role of core excess heat being important early in geological time; the role of gravitational energy being important only for the large terrestrial planets Venus, Earth.

Note that, for a particular planet, the therm w is derived from a set of structural models which are compatible with the zero pressure densities of the phases present and their compressibilities. Otherwise the model treats a planet as a body of uniform density — equal to its mean density.

CORE HEAT FLUX AND RADIUS

The rate of development of the model system is determined by the two fluxes f, fc. The flux f is determined by heat transfer in the mantle and can be obtained, in principle, from measured properties of the upper mantle. The flux f_c could be determined similarly but there is no information on the relevant parameters. Let us therefore make the strong assumption that the net loss from the body as a whole and the core arise in proportion to their masses. Then the core heat flux

$$f_c = (r/a)\, f.$$

Combining this with the conservation relations, provided H is a constant, gives

$$\log (r/a) = -\frac{1}{3H} (E_0 - E).$$

It is of interest to note that this result provides a method of obtaining the mean surface heat flux f over geological time. The net loss of energy, ΔE (per unit mass) from the interior in a time interval Δt requires

$$\bar{f} = 1/3\, \varrho a\, \Delta E/\Delta t = -a\, H \log(r/a)/\Delta t.$$

With Δt = 5 Ga and the measured or estimated values of r and H the mean fluxs for Mercury, Venus, Earth, Moon, Mars respectively, in mWm^{-2} are: 90, 450, 860, 20,70. (We regard each planet as a large calorimeter with the energy content read from the core radius.)

Furthermore, if the initial heat flux is f_0 we have the time scale of the core radius $\tau_c = \varrho\, a\, H/f_0$, where initially $dr/r = -dt/\tau_c$.

SOLID MANTLE HEAT FLUX

Three processes are envisaged as the main contributors:

(i) transfer by thermal conduction of the heat released by radioactive decay of radiogenic materials presumed to be concentrated in the crust;
(ii) free convection of the "solid" mantle;
(iii) free convection of magma in a zone of partial melting, aided in the case of the Earth by the circulation of water substance in the near surface zone.

Contribution from radioactivity

There is strong evidence that radiogenic materials are concentrated in the crust and outer parts of the mantle — assumed to have occured early in geological time. Therefore the radiogenic contribution is appropriately modelled by means of a modification to the surface boundary conditions rather than by incorporation in the global energy equation.

(1) There is an additional surface heat flux $\Delta f = 1/3\, \varrho\, a\, P$ where P is the global mean radiogenic specific power.

(2) There is an increment to the interior temperature $\Delta\theta = \Delta f\, D/K$ where D is the mean depth of the radiogenic layer (~10 km), and K the thermal conductivity.

Of the major contributors to internal heating by radioactive decay — namely the elements K, U, Th — K is dominant. Thus for a single component the specific radiogenic power (per unit mass) $P = P_0\, e^{-\lambda t}$ where $\lambda \approx$ 0.54 Ga^{-1} is the decay constant of K. P_0 is the initial power. (In order to have some idea of orders of magnitude note that a uniform value of $P_0 = 6 \times 10^{-11}$ W kg^{-1} in the Earth, provided the energy could reach the surface, would supply the present heat flux of 50 mWm^{-2} after 5 Ga.)

In this work I (somewhat arbitrarily) assume that no more than 10% of present day net power output from the Earth's interior is radiogenic. This requires $P_0 = 6 \times 10^{-12}$ W kg^{-1}. I use the same value for all the terrestrial planets. Thus the radiogenic contribution to the surface heat flux is $1/3\, \varrho\, a\, P_0\, e^{-\lambda t}$. Note: this flux is taken from the superficial crustal layer and does not affect the interior energy budget; and the temperature increment is negligible. Thus the net surface heat flux, f(surface) = f(interior) + 1/3 ϱ a P.

"Solid" convective heat transfer

The identification of the convective heat transfer process can be made by considering the net outward mantle heat flux of the present Earth. The mean global value is about f = 50 mWm^{-2}. To put this value in perspective it is useful to relate it to the corresponding passive state of a model body in which heat is transferred solely by thermal conduction — with a surface heat flux of scale K ΔT/a, where ΔT is the temperature difference across the (upper) mantle. The ratio of the actual to this conductive scale value defines the global Nusselt number N = Fa/KΔT such that if N $\approx$ 1 the dominant process is thermal conduction and if N >> 1 some other active process is operating.

An estimate of the present Earth's Nusselt number can be made using: K = 3 W m^{-1} K^{-1}; ΔT = 1500 K (a guess at this stage). This gives N = 70. Clearly conduction alone is inadequate and some vigorous heat transfer process is operating.

The key question is then what process or processes produce this high heat transfer rate. Elsewhere (Elder, 1981) I give an estimate of the energy budget of the present Earth's upper mantle (global power in units 10^{12} W): total recharge from the lower mantle, 57; total discharge to the lower mantle of recirculated and cooled upper mantle, 32; loss through the surface, 25 — of which the contribution from mass discharge at the surface, largely as surface volcanism, is 1.7. Thus the loss produced by volcanism is at present about 7% of the total loss. The bulk of the loss arises from the effects of mantle convection.

Laboratory studies of vigorous free-convection in a layer of fluid cooling from above show that the rate of heat transfer is increased, over what it would be without convection, by the ratio N, the Nusselt number, which for materials of large Prandtl number, ($\nu/\varkappa$), is determined principally by the Rayleigh number, A = $\gamma g \Delta T\, h^3/\varkappa\nu$ where: γ is the coefficient of cubical expansion; g is the acceleration of gravity; ΔT is the temperature drop across the convecting layer; h is the depth of the convection layer, $\varkappa$, ν are the thermal diffusivity and kinematic viscosity of the convecting fluid. For A > 10^6 we find experimentally that the fluid is turbulent and N = $(A/A_C)^{1/3}$ where $A_C \approx$ 700-750 and the power transferred is independent of the layer depth. Furthermore, the mean temperature profile across the layer, is such that the bulk of the material is at a uniform temperature (in the laboratory), of mean temperature excess ΔT and that the mean temperature differs from that at the surface only in a thin layer, the sublayer, of thicknes $\delta << h$. It is found that $\gamma g \Delta T\, \delta^3/\varkappa\nu$ = constant = A_C — namely that the sublayer is in a marginally stable state with a constant sublayer Rayleigh number of A_C. The heat flux and sublayer thickness are:

$$f = K\Delta T/\delta,\ \delta = A_C^{1/3}\,(\varkappa\nu/\gamma g\Delta T)^{1/3}.$$

A major difficulty arises when we attempt to apply the results from simple laboratory systems to planetary bodies mainly because of lack of knowledge of the properties of the medium. The material properties $\varkappa$, γ are reasonably well known, but that is not the case for ν. In spite of a vast amount of detailed work on measuring post-glacial uplift and related quantities, estimates of the kinematic viscosity range from 10^{16} — 10^{18} m^2 s^{-1} with typically 3.10^{17} m^2 s^{-1} for models with a uniform mantle; and 10^{16} m^2 s^{-1} for models with a relatively low viscosity "channel" of vertical extent of about 100 km (e.g. Morner, 1980). There is no obvious method for distinguishing between these values.

The kinematic viscosity presents its greatest problem through its variation with temperature. I use the form

$$\nu = \nu_O \exp b\,(\frac{1}{\theta} - \frac{1}{\theta_*}).$$

After considerable numerical experimentation the quantities ν_O, b, θ_* for the selected models have been chosen to give values of ν which straddle 10^{17} m^2 s^{-1} and do not have extreme ranges in a particular model. I choose ν_O = 10^{17} m^2 s^{-1}, b = 10^4 K, θ_* = 2300 K.

In applying these laboratory studies to a planetary body it is convenient to choose the length scale h = a, the radius. Then writing A = $\gamma g\, a^3\, \Delta T/K\nu$ with initial value $A_o = \gamma g\, a^3\, \Delta T_o/K\nu$, we have

$$\delta = \delta_o(\Delta T/\Delta T_o)^{-1/3},\ \delta_o = a(A_c/A_o)^{1/3}.$$

Let us also estimate the corresponding global Rayleigh number A for the present Earth using: r = 10^{-5} K^{-1}; ΔT = 1500K; K = 3 W m^{-1} K^{-1}, ϱ = 5520 kg m^{-3}, c = 1 KJ kg^{-1} K^{1}, so that μ = 5.43 x 10^{-7} m^2 s^{-1}; ν = 3 x 10^{17} m^2 s^1. This gives A = 2.4 x 10^8 with convective N = 70, close to the value estimated (above) from measurement indicating that solid convection is the major contributor to the Earth's surface heat flux today.

The temperature across the sublayer, ΔT is related to the global representative temperature, θ by taking (θ — θ_S)/ΔT $\equiv \xi$, a constant. Here I use ξ = 1.25, the same for all the terrestrial planets (for details see Elder, 1981).

MAGMA CONVECTION

We find "solid" convection alone can provide an adequate transport mechanism for the smaller terrestrial planets. This is not the case for the larger bodies Venus and Earth for which the initial energy stocks are large owing to the large contribution from gravitational energy. For example, the Earth has an excess of about

10 MJ kg^{-1} which cannot be removed by "solid" convection.

Measurements of modern active hydrothermal systems, in which the working fluid — water substance — circulates freely in the crust, reveal heat transfer rates typically 10^2 to 10^3 those of simple conduction (see for example, Elder 1981). By analogy, the free circulation of magma in zones of partial melting could produce similarly enhanced heat transfer of rates of typically 10 W m^{-2}. Rates of this order would be sufficient to transfer the excess available energy.

The role of this magma convection process will not however be confined to the larger bodies — it will presumably operate for all the terrestrial planets while a zone of partial melting exists.

An attempt at a detailed study of the net effect of free convection in the partial melt zone would be not only beyond present knowledge but not be calibratable. For the moment it is sufficient to note that a key factor in the operation of this type of convection is the permeability of the medium. The permeability is a strongly varying function of the porosity, in this case the volumetric melt fraction e, varying at least as strongly as e^3. Thus early in the geological history when e is largest the process will be vigorous; and fall rapidly in vigor as e falls.

Thus I am forced to represent the effect of the process empirically by presuming its net effect is to produce an enhanced surface heat flux, $f_p = f_p$ (t). I choose $f_p = f_o e^{(-t/\tau p)}$ where f_o τp are phenomenological constants. The net effect of this process, after $t >> \tau p$, is the removal of a total amount of internal energy per uniot planetary mass of 3 f_o $\tau p/\varrho a$. In the selected models described below this quantity ranges from 0.7 MJ kg^{-1} for Moon to 10.8 MJ kg^{-1} for Earth. In the numerical models I have selected a common value for all the terrestrial planets, τp = 0.4 Ga, leaving f_o as a single free parameter. Note that after a time of about 3 τp, for which $f_p/f_o \approx 0.05$, the bulk of the effect of this process is past with a net effect thereafter for a particular planet dependent on f_o τp.

The total flux

The total flux out of the mantle is taken as

$$f\,(\text{interior}) = f_o\, e^{-t/\tau p} + K\Delta T/\delta$$

the sum of the contributions from magma convection and solid mantle convection.

UPPER MANTLE RECIRCULATION

A simple and useful picture of the mechanism of the sublayer is as follows (originally proposed by L Howard, see Elder, 1981, pp 43-51). Suppose (at time t = 0) at a particular moment in the vicinity of a localized patch of the surface that deep hot material has penetrated up to the surface so that the temperature locally is everywhere the same as at depth except at the cold surface and the local material is stationary. Heat will be lost from a zone near the surface; the heat will be transferred solely by thermal conduction; a zone of depth of order $(\varkappa t)^{1/2}$ will be affected. For an interval of time $0 < t < \tau_s$, say, the cooled layer, although statically unstable owing to its greater density, will be dynamically stable owing to the combined stabilizing role of thermal diffusion and viscosity. If, however, the Rayleigh number for the system as a whole is sufficiently large, instabilities within the cooled layer grow to finite amplitude, until the cooled mass falls into the interior and hot deep fluid is recirculated back towards the surface. The period of gestation in which the cooled layer grows by conduction is followed by a short interval in which the cooled material is ejected out of the cooled region to be replaced by deep hot material, thereby more or less restoring the conditions near the surface. As seen in the laboratory this process is observed to occur more or less at random over the surface zone.

In order to quantify the gross effects over geological time of the convective recirculation of mantle material into and cut of the upper mantle, consider n = n(t) the number of times the material of the upper mantle has been recycled where

$$dn/dt = 1/\tau_s,$$

$$\tau_s = \alpha\delta^2/\varkappa.$$

In the simple Howard model the constant $\alpha = 4/\pi$. In that model no allowance is made for the variation of viscosity with temperature. Also $\varkappa$ is the thermal diffusivity of upper mantle material rather than the global thermal diffusivity based on the global mean density. Rather, let us choose a value of α such that for the present Earth a single recycling occurs in a nominal time of 0.3 Ga — an interval during which substancial crustal rearrangement has occurred. We find $\alpha \approx 5$. This implies a single crustal rearrangement after 2.4, 2.5, 2.3, 3.9, 3.3 recyclings for the models of Mercury, Venus, Earth, Moon, Mars respectively.

PARTIAL MELTING

Partial melting of the upper mantle does not continue indefinitely. Volcanism ceased long ago on the Moon and probably also on Mercury and Mars. The cessation of volcanism provides a characteristic time marker with which we calibrate our model.

The global gives the global temperature $\theta(t)$. From θ (for details see Elder, 1981) we obtain the temperature profile $\Theta(z)$ in the upper mantle, presumed to be similar to that of a layer of fluid losing heat from its upper surface and in a vigorous state of thermal convection. If however $\Theta > T_m$, the local melting temperature, a portion of the material will be molten. Let the volumetric fraction in the molten state be $e = e(z)$ and the actual temperature be T_m; whereas Θ is the temperature that would have arisen if melting were not permitted. Conservation of energy then requires

$$\varrho_* c_* \Theta = (1 - e) \varrho_* c_* T_m + e\varrho (L + cT_m)$$

where ϱ_*, c_* refer to the solid and ϱ, c, L to the liquid and L is the latent heat of melting. It is convenient to write $\varrho = (1 - \zeta) \varrho_*$ and make the approximation $c = c_*$ whence

$$e = (\Theta - T_m)/\Theta_*$$

with $$\Theta_* = (1 - \zeta)L/c_* - \zeta T_m$$

(For example with $L = 3$ MJ kg^{-1}, $c_* = 1$ KJ kg^{-1} K^{-1}, $T_m = 10^3$ K.$\zeta = 0.1$, the quantity Θ_*, a property of the rock substance alone, is 2600 K).

The MPD (melting point for depth) is taken as $T_m = \alpha + \beta P$. Since our major interest, for the purpose of model identification, is in whether or not partial melting occurs at all I choose $(\alpha, \beta) = (1400$ K, 6×10^{-3} K bar$^{-1})$ values appropriate to a typical basaltic magma (other magmas will exist but if the system cannot produce basalt it is volcanically dead — or very close to it).

Model results for the Earth are shown in Fig. 4. The system starts with a completely molten mantle and ends with a completely solid one. For the Earth, over an interval of about 7 Ga, there is a zone of partial melting of restricted vertical extent located in the upper portion of the mantle, and the extent of this zone diminishes with time.

Role of temperature fluctuations

Superimposed on the mean temperature Θ are temperature fluctuations which vary in space as well as time and have amplitude Θ' which from laboratory measurements of thermal turbulence are of order $0.1\,\Theta$. The depth to the top of the partially melted zone for the Earth at t = 5 Ga with model values is shown in Fig. 5 together with the corresponding value of e_{max}, the maximum value of e(z). For $\Theta' < -110$ K no melting is possible; otherwise the melting depth falls rapidly near $\Theta' = 0$; but for $\Theta' > 400$ K the change in melting depth is small. This suggests that for the present Earth partial melting of the upper mantle: is confined to depths greater than about 70 km; can be as deep as about 175 km; and for expected fluctuations of 200-300 K will be predominantly at depths of about 80-90 km with e = 0.12 — 0.16 to be compared with the global value at Θ' =) of e = 0.04. These depth ranges are similar to those suggested by the study of the petrogenesis of the parent magmas of basalts and of kimberlites.

Thus in general, partial melting will occur sporadically and with varying intensity — not necessarily being present everywhere — within a shell of restricted radial extent; a shell whose extent diminishes with time.

SIMPLIFIED MODEL

To demonstrate the behaviour of the model system consider a simplified version in which: the kinematic viscosity does not vary with temperature; the excess heat

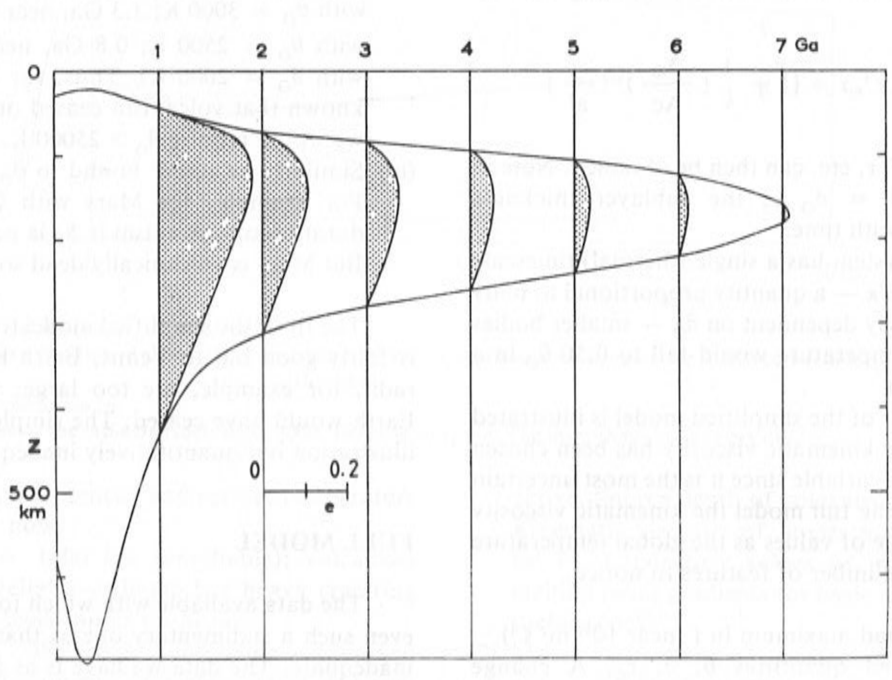

Figure 4 — Partial melt development for the Earth. Profiles of volumetric melt fraction, e as a function of depth, z; at times, t = 1(1)7 Ga, with time zero 5 Ga ago.

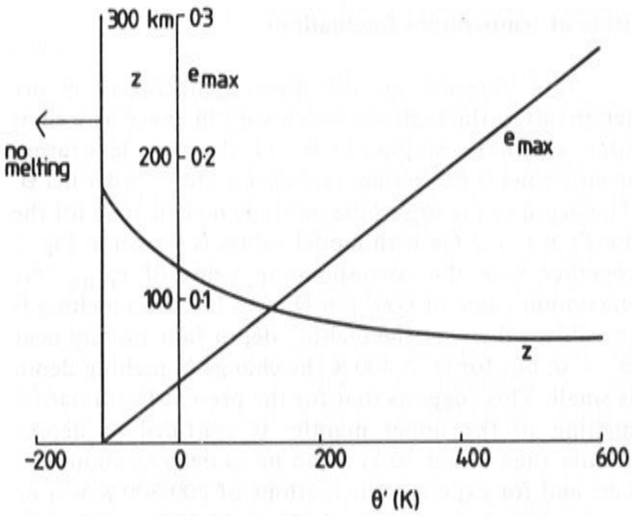

Figure 5 — Depth, z (km) to the top of the partially melted zone as a function of the temperature fluctuation (K). Also shown: the maximum over the distribution with depth of the volumetric partial melt fraction, e_{max}.

contribution is negligible, H = 0; the contribution of gravitational energy is negligible; the contribution of radiogenic power to the surface flux is negligible, P = 0; and magma convection does not occur, $f_O = 0$.

The relations for E and r are now decoupled; $E = c\theta$; the energy equation involves only the variables θ, t. Thus writing $K = \varrho c \varkappa$, so that $\varkappa$ is a thermal diffusivity based on the thermal conductivity of the upper mantle and the mean density of the whole body; the energy equation becomes

$$\xi \, d\Delta T/dt = -3\varkappa \Delta T / a\delta$$

which with the convective relation $\delta(\Delta T)$ integrates to

$$\Delta T = \Delta T_O \gamma^{-3}, \gamma = [1 + \frac{1}{\xi} (\frac{A_O}{A_c})^{1/3} \frac{\varkappa t}{a^2}]$$

The quantities δ, f, r, etc. can then be obtained. Note in particular that $\delta = \delta_O \gamma$; the sublayer thickness increases linearly with time.

This simple system has a single (thermal) timescale $\tau_\theta = (A_c/A_O)^{1/3} a^2/\varkappa$ — a quantity proportional to body radius a, and weakly dependent on θ_O — smaller bodies cool faster (the temperature would fall to 0.50 θ_O in a time about 0.2 τ_θ.).

The behaviour of the simplified model is illustrated in Fig. 6 where the kinematic viscosity has been chosen as the independent variable since it is the most uncertain parameter and in the full model the kinematic viscosity sweeps over a range of values as the global temperature falls. There are a number of features to notice.

(i) There is a broad maximum in f (near 10^{17} m² s⁻¹).

(ii) The associated quantities θ, δ, r_c, A change monotonically with ν; all except A increasing with ν.

(iii) The duration of volcanism has a maximum strongly dependent on θ_O (of 7.1 Ga, near $\nu = 10^{19}$ m² s⁻¹, with θ_O = 3000 K; 3.3 Ga, near $\nu = 2 \times 10^{18}$ m²s⁻¹, with θ_O = 2500 K; 0.8 Ga, near $\nu = 10^{17}$ m² s⁻¹, with θ_O = 2000 K). Thus, for example if it were known that volcanism ceased on Mars after 2 Ga, we would require $\theta_O > 25000$ K.

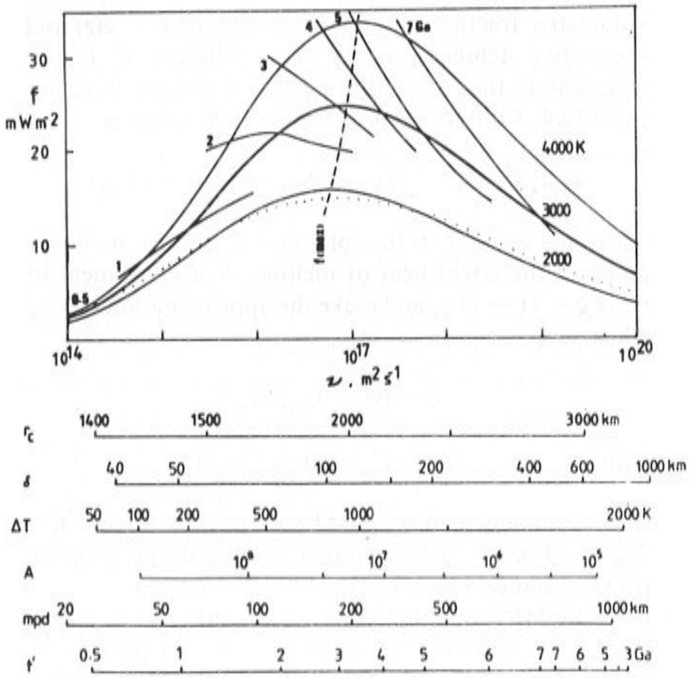

Figure 6 — Simplified model behaviour — properties after 5 Ga for Mars. Heat flux f, (mWm⁻²) as a function of kinematic viscosity ν, (m² s⁻¹) for initial global representative temperatures 2000, 3000, 4000 K together with the line of heat flux maxima. The duration of volcanism, for (0.5, 1, 2, 3, 4, 5, 7) Ga, is also indicated, for (α, β) = (1400 K, 2 x 10^{-2} K bar⁻¹); θ' = 100 K.
The attached scales refer to the θ_O = 3000 K data, t = 5 Ga (now) — except for the duration of volcanism scale, t' : r_c, core radius; δ, sublayer thickness; ΔT, temperature drop across the sublayer; A, Rayleigh number: mpd, depth to top of zone of partial melting.

(iv) Similarly an upper bound to θ_O can be estimated. For example, for Mars with θ_O = 4000 K the duration of volcanism is 5 Ga near $\nu = 10^{17}$ m² s⁻¹. But Mars is volcanically dead so that $\theta_O < 4000$ K.

The fit of the simplified model to the smaller bodies is fairly good but to Venus, Earth is poor — the core radii, for example, are too large; and volcanism on Earth would have ceased. The simple model is a useful illustration but quantitatively inadequate.

FULL MODEL

The data available with which to calibrate a model, even such a rudimentary one as that described here, is inadequate. The data we have is as follows.

(1) Properties at t = 0 (θ_O etc) — no data.

(2) Age of system, 5 Ga.

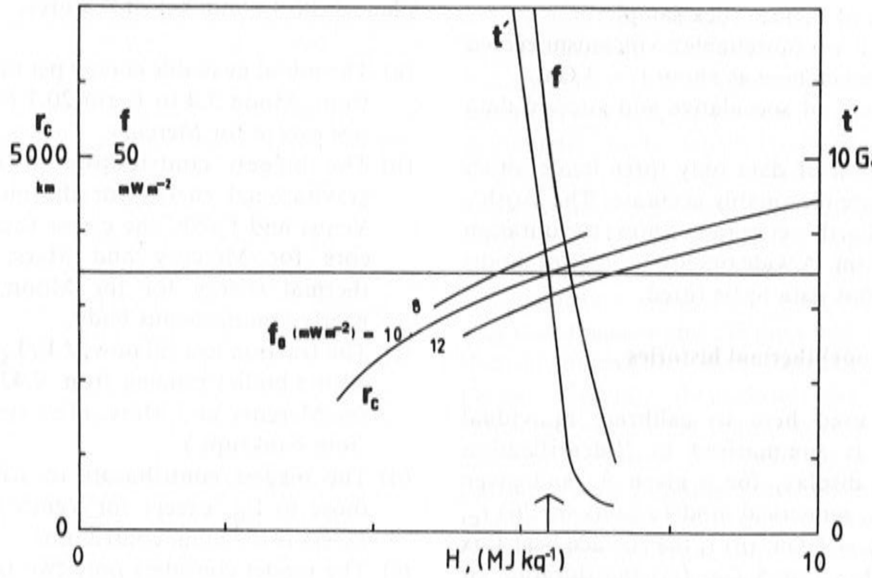

Figure 7 —Earth: identification diagram.
Properties as a function of H, (MJ kg⁻¹) at t = 5 Ga: labelled curves of core radius r_c (km) for flux f_O = 8, 10, 12 Wm⁻²; heat flux f, (mWm⁻²) for value of H at which r_c = 3470 km, the core radius, indicated by a horizontal line; and the corresponding duration of volcanism t′, (Ga). Linear scales — the ordinate labels 5000, 50, 10 refer to r_c, f, t′.

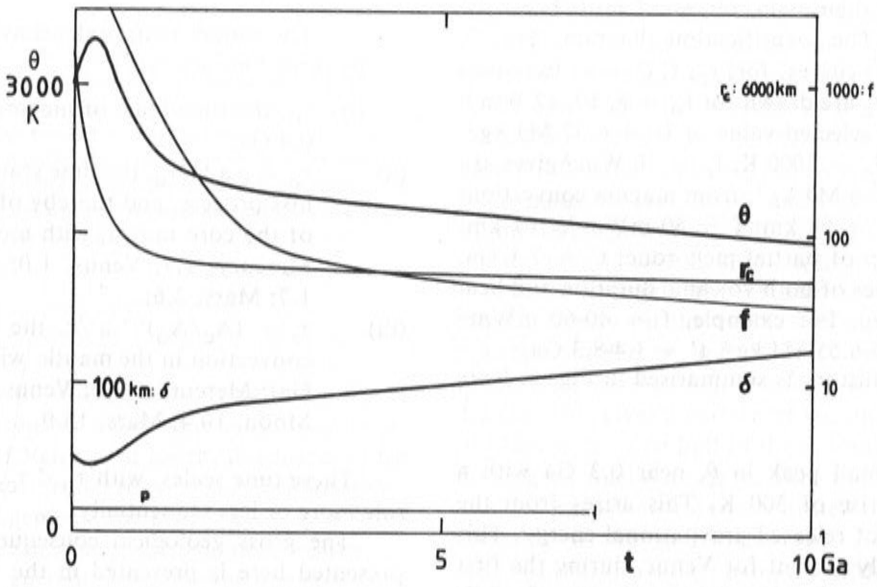

Figure 8 —Earth: thermal history.
Properties as a function of time t, (Ga): heat flux f, (mWm⁻²); representative temperature θ, (K); core radius r_c, (km); upper mantle (sublayer) thickness δ, (km); interval of volcanism, p.
Linear scales — the ordinate labels 6000, 3000, 100 refer to r_c θ δ. The logarithmic scale refers to f.

(3) Mass, radius, mean density and surface temperature of each planet now.

(4) Mercury: r_c = 1806 km (unreliable); volcanism ceased — no reliable estimate but heavy cratering suggests early cessation as on Moon.

(5) Venus: r_c = 3930 km (unreliable); observed atmospheric phenomena indicate current volcanic activity.

(6) Earth: r_c = 3470 km; f = 50 mWm⁻²; volcanically active. Source depth of volcanism of order 100 km. Kinematic viscosity of upper mantle of order 10^{17} m² s⁻¹. Laboratory values of melting points and melting point gradients for basic and ultrabasic near surface rocks.

(7) Moon: r_c = 980 km (unreliable); volcanism ceased at t = 2.0 Ga (Dubious estimate, discounted here, of f = 16-22 mWm⁻² at Apollo 17, 15 sites with unreliable shallow probe method). An excellent

dated collection of surface rock samples.

(8) Mars: r_c = 2620 km (unreliable); volcanism ceased at t = 2-4 Ga, taken here as about t = 3 Ga.

(9) A very wide range of speculative and guessed data of no value.

Of this collection of data only three items, other than items (1)-(3), are reasonably accurate: The Earth's heat flux now; the Earth's core radius now; the duration of de Lunar volcanism. A wide range of choice of model parameters allows this data to be fitted.

Identification of model thermal histories

The method used here to calibrate individual thermal histories is summarised in "identification diagrams". These display, for a given θ_O and given material properties, numerical model results for: (i) r_c, the core radius at t = 5 Ga; (ii) f, the surface heat flux for the model with r_c, at 5 Ga; (iii) the duration of volcanism, t', namely the value of t at which the partial melt fraction e first reaches zero everywhere in the mantle.

The identification and behaviour of the thermal histories of the terrestrial planets is illustrated by that for Earth (for detailes, and models of other planets see Elder (1987). The diagrams for the terrestrial planets all have similar form; their main common feature being the sensitivity to H. The identification diagram, Fig. 7, shows three sets of curves, for: r_c, f, t' — as functions of H. Curves for r_c are drawn for f_O = 8, 10, 12 Wm^{-2}; the others for the selected value of H = 6.37 MJ kg^{-1}. This choice with θ_O = 3000 K, f_O = 10 Wm^{-2} gives at t = 5 Ga: ΔE = 10.8 MJ kg^{-1}, from magma convection; r_c = 3470 km; δ = 98 km; f = 50 mWm^{-2}; 107 km, mean depth to top of partial melt zone; t' = 7.1 Ga. Note that the curves of both volcanic duration and heat flux are very steep. For example, f = 40-60 mWm^{-2} requires H = 6.25-6.55 MJ kg^{-1}, t' = 3.4-8.3 Ga.

The thermal history is summarised in Fig. 8. Note the following.

(i) There is a small peak in θ, near 0.3 Ga with a temperature rise of 300 K. This arises from the early effects of released gravitational energy. This effect is barely present for Venus, during the first 0.2 Ga; for all the other planets θ falls monotonically.

(ii) The heat flux range is great, falling from 10 Wm^{-2} to 0.1 Wm^{-2} in the first 2 Ga.

(iii) The change in δ is small and slow, especially after the first 1 Ga —δ barely doubles in 10 Ga. The growth of δ is much faster for the smaller planets.

(iv) The initial growth of the mantle is rapid. The core radius today is little different from its size after 1 Ga. This feature is common to all the planets.

GEOLOGICAL SIGNATURE

Energy budget

The energy budgets for the (model) terrestrial planets show a number of features.

(a) The initial available energy per unit mass, E_O ranges from, Moon 3.4 to Earth 20.1 MJ kg^{-1}, in order of size except for Mercury.

(b) The biggest contributors to E_O are: available gravitational energy for the most massive bodies Venus and Earth; the excess thermal energy of the core for Mercury and Mars; the solid planet thermal energy for the Moon, the small, more nearly homogeneous body.

(c) The fraction lost till now, $\Delta E/E_O$, is close to 0.5 for all the bodies ranging from 0.42 for Moon to 0.58 for Mercury and Mars. (The system is a long way from bankrupt.)

(d) The biggest contributors to ΔE are the same as those to E_O, except for Venus for which the core excess is the main contributor.

(e) The model considers only two transport processes, "magma" and "solid" convection. Magma convection is the dominant process for all except the Moon — till now the fraction of energy lost through magma convection ranges from Earth, 93% to Moon, 42%.

The time scales

The model temporal behaviour is determined by three time scales:

(i) τ_p, the time scale of magma convection, set at 0.4 Ga;

(ii) $\tau_c = \varrho a H/f_O$, the time scale of the core energy loss process, and thereby of the rate of change of the core radius, with model values (in Ga): Mercury, 1.7; Venus, 1.0; Earth, 0.7; Moon, 1.7; Mars, 2.6;

(iii) $\tau_\theta = (A_c/A_O)^{1/3} a^2/\kappa$, the time scale of solid convection in the mantle with model values (in Ga): Mercury, 14.0; Venus, 18.8; Earth, 19.1; Moon, 10.4; Mars, 15.9.

These time scales, with $\tau_p < \tau_c < \tau_\theta$ thus play their role more or less sequentially.

The gross geological consequences of the model presented here is presented in the "signature" of the terrestrial planets shown in Fig. 9. For each planet two curves are drawn: (i) a measure of the vigour of volcanism, the maximum partial melt fraction e(max), (at a given time, the largest value of the radial profile of e); (ii) a measure of the vigour of convection in the mantle, the number of times the mantle has stirred itself, n.

Remarks on crustal rearrangement

A ubiquitous feature of geological activity is the continual recycling of the mantle. There is however a striking difference in the intensity of this process between the small planets Mercury, Moon, Mars where it is weak with only 6, 3, 8 recyclings and the larger

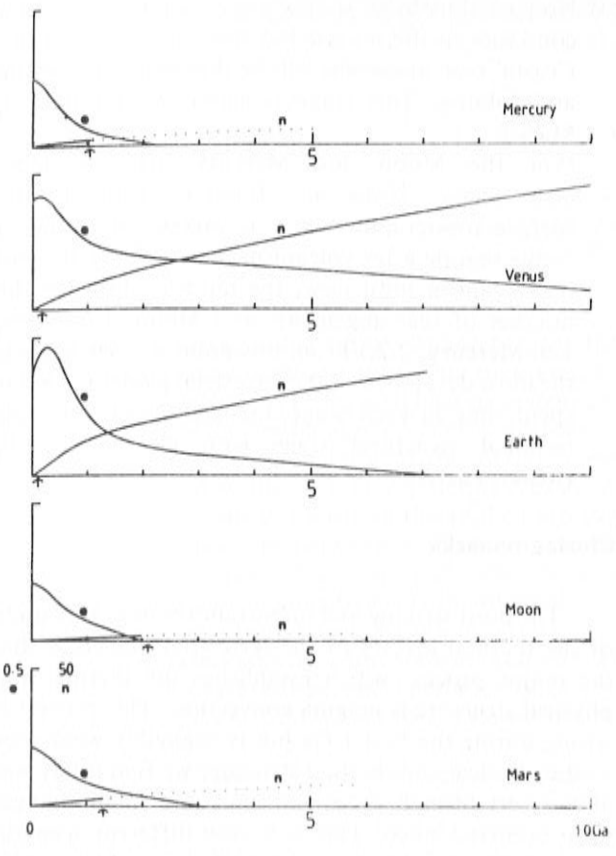

Figure 9 — Geological signature of the terrestrial planets. (i) The maximum partial melt fraction e(max) as a function of time, (Ga) for selected model. (ii) The number of times, n, the upper mantle and crust have been rearranged — calibrated to once for the present Earth in 0.3 Ga. The arrow indicates when partial melting no longer reaches to the base of the mantle.

planets Venus, Earth where it is strong with 28, 31 recyclings. Crustal rearrangement is a major feature of the physiognomy of Venus and Earth, much less so for the others. I leave it to reader to consider the plausibility of about 30 rearrangements for Earth, keeping in mind the contemporary acceptance of vigorous rearrangements during the current episode but the apparent reluctance to extrapolate this observation back into the past. The model presented here is unequivocal in its indication of the great vigour of our planet's geology throughout its geological life.

Mantle-wide partial melting

Shield volcanoes can be represented as manometric systems in lithostatic equilibrium embedded in the crust and upper mantle (Elder, 1981, 81-131). This viewpoint is adequate for Earth and Mars with source depths of order 100 km (current), 300 km (at the cessation of volcanism) and major volcanic systems of heights of order 9 km, 20 km.

It is plainly, however, not adequate for Mercury and Moon, both of which have had a vigorous volcanic history but with volcanoes of heights at most a few kilometers. There is no obvious modification of the manometer model which could account for the small heights — indeed at the cessation of volcanism, in the models, the depth to partial melting is 330 km, 610 km respectively. Some other effect must have operated.

Inspection of the Moon model, over a wide range of parameters compatible with the presumed known radius of the core and the duration of volcanism, shows that, in contrast to Venus, Earth and Mars, the zone of partial melting, throughout the volcanic history, extends down to the mantle-core boundary. This is a situation different from that of a localized zone of partial melting bounded both above and below by solid mantle. The essential feature of the manometer model of volcanism is the pressure difference between that in the surrounding solid mantle and that in the liquid column — it is this pressure difference, lithostatic minus "magmastatic" which drives the system and provides the head to elevate magma above the paleosurface. Where however the entire mantle is partially molten (and even if the distribution of melt is patchy) the ambient pressure will be closer to "magmastatic" than lithostatic. The available head to drive a volcanic system will be reduced. Shallow discharges will be readily possible — indeed would be the commonest eruptive mode — but high volcanoes would not. Small shield volcanoes would be possible in a region of marginal partial melt. Here then is the other effect — and the data on duration and core radius is sufficiently reliable for a plausible case to be made that volcanism on the Moon occurred with partial melting extending throughout the mantle for the entire volcanic stage.

There remains the enigma of Mercury. In the model, largely on the visible evidence that Mercury is cratered similar to and at least as much as Moon, parameters were chosen to give a volcanic duration of 2.2 Ga. This gives a pattern of volcanism similar to that of Mars — the first half of the volcanic stage has partial melting throughout the mantle but during the latter interval partial melting is confined to a localized zone bounded above and below by solid mantle. This suggests that the model chosen for Mercury needs to be modified, taking f_O = 1 Wm^{-2}, H = 3.8 MJ kg^{-1} for which volcanism is active for 0.9 Ga with mantle wide partial melting for 0.8 Ga. A repeat of the Apollo programme on Mercury (and Mars too, please!) will be necessary to resolve questions of this kind.

This early part of the volcanic stage during which partial melting extends to the base of the mantle will occur for all the terrestrial planets. The model results give the following values for the duration and mantle thickness at its close:

Mercury: 1.0 Ga, 520 km (alternatively, 0.8 Ga, 500 km);
Venus: 0.2 Ga, 730 km;
Earth: 0.1 Ga, 650 km;

Moon: 2.0 Ga, 600 km;
Mars: 1.3 Ga, 580 km;

During this interval, by analogy with the Moon, surface volcanism takes the form of shallow, extensive flows of hot, low viscosity lavas with minor central volcanism of small height. On Earth and Venus it is extremely unlikely that any relics of this early volcanism remain, following the obliteration produced by a continuing vigorous geology — whereas this process has formed the preserved lunar volcanic surface and is probably a major contributor to the preserved volcanic surface of Mercury and Mars.

Geological style

All the terrestrial planets start their geological lives with great vigour. The Earth itself would have been an awesome place with frequent volcanism and endemic intense hydrothermal and phreatic activity (such as seen today in small restricted geothermal areas on land and near ocean ridges — the term "Hadean" era, for an early interval of thermally intense geological activity, used by some authors, is certainly apt). There will however be little preserved from that time. The bulk of the possible evidence has been obliterated by cratering, flood volclanism or many cycles of crustal reworking.

During its geological life a terrestrial planet passes through three stages determined by the partial melt structure of the mantle.

(1) Mantle wide partial melting. This early stage occurs for all the terrestrial planets. It is characterized by vigorous sheet volcanism. Crustal rearrangement also occurs possibly with very small regions of coherent crustal displacement. The Moon and possibly Mercury spend their entire volcanic history in this stage. These are the most primative bodies with the least developed crustal systems. This stage is very brief for the large terrestrials Venus, Earth.

(2) Restricted vertical zone of partial melting. This next stage is characterized by both sheet and central volcanism together with crustal rearrangement, of the style found today on Earth and presumably on Venus — these two spend the bulk of their volcanic history in this stage. The vigour of the mantles of these two bodies leads to a highly evolved crustal system.

(3) No partial melting. In this stage "solid" convection continues in the mantle but there is no volcanism. Crustal rearrangement will be dominated by rifting and folding. This stage is clearly seen today on Mars.
(For the Moon and Mercury there is some postvolcanic rifting and faulting visible in the surface topography but it is muted, indicating a feeble mantle after volcanism ceased. Since the end of volcanism until now, the model values for the number of rearrangements are: Moon, 1.0; Mars, 1.9; Mercury, 2.2.).From this point of view Mars is the most developed of the terrestrial planets — it has spent time in each stage and has passed into this terminal structural stage with its mantle still (weakly) active.

Closing remarks

The most striking and important result of this study of the thermal history of the terrestrial planets is that the major process which establishes the thermal and physical structure is magma convection. This process is strong during the first 1 Ga but is negligibly weak now — the physical and thermal structure we find today was already established 4 Ga ago and only small changes have occurred since. This is a very different scenario from that previously envisaged in which solid convection was the leading candidate not only for global rearrangement but also as the main heat transport mechanism throughout geological time. Solid convection remains as the leading candidate after 1 Ga of geological time. But in that initial stage it was of minor significance — magma convection was dominant.

Little is known or even conjectured on the mechanics of global magma convection. What is needed is a number of laboratory and numerical experiments and corresponding analytical studies comparable to the large body of work done for ordinary convection — in order to set up and explore models of the first 1 Ga of the geological life of the terrestrial planets — in order to give the insight into what to look for and how to decode the fragments, as yet to be discovered, if any — of the beginnings of geology. Such studies will also be of fundamental importance in helping our understandings of volcanism and in particular the role of hydrothermal systems.

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