Revista Brasileira de Geofísica
RBGf
Brazilian Journal of Geophysics

Revista Brasileira de Geofísica (2012) 30(4): 431-444

ISSN 0102-261X
www.scielo.br/rbg

EXPLORATION OF THE RESIDUAL TOPOGRAPHY EFFECT ON SOME FUNCTIONALS OF THE GRAVITY FIELD

Henry Diverth Montecino Castro, Karoline Paes Jamur and Silvio Rogério Correia de Freitas

Recebido em 16 junho, 2011 / Aceito em 27 março, 2012
Received on June 16, 2011 / Accepted on March 27, 2012

ABSTRACT. The topographical masses, considering their distribution and variations in density, have a measurable influence on Functionals of the Gravity Field (FGF). In this paper, the effect modeled by the Residual Terrain Model (RTM) exerted on the height anomaly, on the gravity anomaly, and on the deflections of the vertical components was explored. The SRTM30_PLUS and DTM2006.0 Digital Elevation Models (DEM), based on a high-pass filter were used in the context of RTM technique. Chile is a natural laboratory for studies in geodynamics considering the characteristics of its topography and crustal structure. Several studies consider the analysis of FGF described in the geopotential space. However, investigations linking the residual effects of topography on these FGF have not yet been reported. Thus, this paper seeks to include this analysis considering two different regions of Chile: the 1st and 8th regions, which have very different crustal characteristics. The results reveal the importance of the contribution of RTM in modeling the high-frequencies of the Earth's gravity field. The values of height anomaly obtained were between 1 m and –1 m, while the gravity anomalies were between 100 mGal and –100 mGal, whereas the deflections of the vertical components reach values between –20 and 18 arcseconds in different topographic settings. The study shows the possibility of retrieving large part of the omission errors in Global Geopotential Models (GGM) by using the RTM technique. Furthermore, the stability of different FGF in relation to the variation of the radius of integration, in the context of the Stokes-Pizzetti formulation was explored.

Keywords: residual terrain model, digital elevation models, global geopotential model, functionals of the gravity field.

RESUMO. As massas topográficas, tendo em vista suas distribuições e variações de densidade, exercem influências nas grandezas vinculadas ao campo de gravidade. Neste trabalho, o efeito gerado pela Modelagem Residual do Terreno (Residual Terrain Model – RTM) sobre a anomalia de altura, anomalia da gravidade e as componentes do desvio da vertical foi explorado. Os modelos digitais de elevação SRTM30_PLUS e o DTM2006.0, com base em uma filtragem de passa alta, foram usados no contexto da técnica RTM. O Chile tem-se constituído um laboratório natural para estudos em geodinâmica em vista das características de sua topografia e de sua estrutura crustal. Diversos projetos são desenvolvidos com base na análise de grandezas descritas no espaço do geopotencial. No entanto, não são reportadas investigações que vinculem os efeitos residuais do terreno sobre estas grandezas. Desta forma, busca-se a inclusão desta análise considerando-se duas regiões distintas do Chile: as regiões I e VIII, as quais têm características crustais bastante diversas. Os resultados revelam a importância da contribuição do RTM na modelagem da alta frequência do campo de gravidade. Na anomalia de altura os valores alcançados situaram-se entre 1 e –1 m, ao passo que as anomalias de gravidade situam-se entre 100 e –100 mGal, enquanto que as componentes do desvio da vertical apresentaram valores entre 18 e –20 segundos de arco nas diferentes situações topográficas. O estudo mostra a possibilidade de recuperação de grande parte do erro de omissão dos Modelos Globais do Geopotencial por intermédio da técnica RTM. Além disso, a estabilidade das diferentes grandezas em relação à variação do raio de integração, no contexto da formulação de Stokes-Pizzetti, foi explorada.

Palavras-chave: modelo residual do terreno, modelo digital de elevação, modelo global do geopotencial, funcionais do campo de gravidade.

LARAS (Geodetic Reference Systems and Satellite Altimetry Laboratory), Department of Geomatics, Universidade Federal do Paraná, P.O. Box 19001, Centro Politécnico, Jardim das Américas, 81531-990 Curitiba, PR, Brazil. Phone/Fax: +55(41) 3361-3153 – E-mails: henrymontecino@gmail.com, kjamur@ufpr.br, sfreitas@ufpr.br

INTRODUCTION

Since the last century the structure of the gravity field has been modeled predominantly via the analysis of the orbital disturbances of artificial satellites associated to the gravitational effects of the Earth and terrestrial gravimetry. Its preferred representation are based on spherical harmonic expansions, whose maximum degree and order of development express the best spatial resolution of the model. Numerous models have been produced, usually classified into three groups (Featherstone, 2002): a) *Satellite-only*, where only orbital satellites data are used in the analysis (nowadays reaching degree and order of up to 250 solving half wavelength to about 83 km (ICGEM, 2011); b) *Combined*, where the data from the satellite are combined with land, air and sea gravimetry as well as topography data from Digital Elevation Models (DEMs). Usually such models are presented to the degree and order 360, solving half wavelength up to 55 km (Amos & Featherstone, 2003). However, there are models developed up to higher orders and degrees, reaching a resolution of about 9.2 km such as the EGM2008 developed to the 2190 degree and 2159 order; c) *Tailored*, where the model does not have a global scope and purpose but intends to describe in detail certain region, and in general are linked to local references. These are usually associated with local geoids, such as the MAPGEO2010 (IBGE, 2011). The modeling of local geoids has been a central focus of research, and is operated mainly by the Remove-Restore technique – RR (Sansó, 1994; Featherstone et al., 2004; Sjöberg, 2005) which are based on a Global Geopotential Model (GGM), the terrestrial/airborne gravity and topography information. However, in view of current needs, mainly associated with geodynamics (Perrot et al., 1997; Sun & Sjöberg, 2001) and the necessity to link Local Positioning Systems (LPS) to the global GNSS (Global Navigation Satellite System) (Awange et al., 2010), other geopotential functionals have been demanded, such as the vertical deflection, abnormal height, indirect effects of anomalous masses, residual gravity anomalies, among others.

With advances in satellite gravimetry, Rummel et al. (2002) have predicted the huge impact arising from missions CHAMP, GRACE and GOCE, on the consistent description of the gravitational potential. In fact, nowadays these missions present significant contribution to modeling the geoid/quasigeoid with a resolution on the order of centimeters and wavelengths at about 83 km, thus encompassing the long wavelengths (classically associated with the degree and order 70 of spherical harmonics expansion) and even part of the middle wavelengths (Hecimovic & Bašić, 2005; Flury & Rummel, 2005). Due to the truncation of the expansion in spherical harmonics series, which is also known as omission errors , the GGMs until today failed to recover the high-frequency signals or to resolve half-wavelengths shorter than about 9 km (Torge, 2001). As the topography of the Earth is the primary source of high-frequency signals (Forsberg, 1984; Flury & Rummel, 2005), it becomes increasingly important to reduce the omission errors of the GGMs by the application of high-frequency models based on Digital Elevation Models (DEMs), in order to complete the spectrum of the gravity field (Denker, 2005).

In spite of the importance of DEMs in many areas of knowledge and applications, there are still deficiencies in local DEMs with appropriate resolution to model high-frequency gravity fields in many regions. Great part of the countries generates its own DEMs from the digitization of topographic maps available in their databases. Generally, these bases have not an homogeneous coverage (Vergos et al., 2005). In recent years there has been considerable progress in the development of global DEMs, reaching horizontal resolutions of about 30 m (Kiamehr & Sjörberg, 2005). Among the different applications of the DEMs, they reached great interest in Geodetic Sciences in order to model the external gravity field. Nowadays the open access digital elevation models Shuttle Radar Topography Mission (SRTM30_PLUS) (Becker et al., 2009), provides an absolute horizontal accuracy of 20 m and 16 m vertical accuracy (Kiamehr & Sjörberg, 2005). In South America, several global DEMs have been evaluated (Blitzkow et al., 2005; Tocho et al., 2007). On the other hand, in the southern Andes there is still no regional DEM, and also difficulties to represent the topography with high resolution are faced. Besides the lack of local DEMs in Chile, there is a lack of gravimetry, especially in mountainous areas, which makes impossible the study of short-wavelength anomalous structures in the crust (Götze & Kirchner, 1997). However, there is a demand to explore alternatives for modeling the regional/local geoid or to provide gravimetric informations or functionals associated with anomalous gravimetric fields.

Chile is considered by the scientific community as a natural laboratory of geodynamics, by presenting relevant topographical, geological and geodynamic characteristics (Kendrick et al., 1999; Khazaradze & Klotz, 2003). In this context, geopotential information could improve the interpretation and permit other studies such as: relationships between seismic and gravity anomalies in the continental crust (Barton, 1986); viscosity and density of the 3D structure of the mantle (Richard & Wuming, 1991); using the vertical component deflection for the analysis of subsurface anomalies of density, among other applications dependent or correlated with the anomalous field, in geodetic, geophysical, geological and geodynamic applications (Watts & Daly, 1981; Chase, 1985; Zongjin & Xianglin, 1996; Kiamehr & Sjöberg, 2006; Vermeersen & Schotman, 2008). In the classical approach of Stoke's (Torge, 2001), information of global GGMs, local observations of gravity and digital elevation models are used in the context of RR technique for modeling local/regional gravity field.

Currently, some alternatives to improve the performance of GGMs have been addressed in the context of Residual Terrain Model (RTM). Applications of combined GGMs, such as EGM2008, on the functionals, height anomaly and vertical deflection, in regions of ermany have been tried (Hirt, 2009; Hirt et al., 2010a,b). GGMs employed in those works involved the local information and are thus contaminated by local references . Other regional studies related to the impact of horizontal and vertical resolution in DEMs in the modeling of the topographic effect have been made in recent years (Gučevic et al., 2010).

The purpose of this article is to explore the effect of topography on different functionals of the present residual gravity field (gravity anomaly, height anomaly and the deviation of the vertical components) in two regions of Chile. Regions with different topographic characteristics were tested to verify the possible advantages of combining satellite only data GGM with high-frequency models generated by the residual topography. Another explored subject is the effect caused by the variation of the integration radius in obtaining the potential of residual masses over different functionals.

DATABASE

Two regions of Chile were chosen as a study area. I – Tarapacá region, within the limits 20°S $< \varphi <$ 18°S and 70°W $< \lambda <$ 69°W, and VIII – Bio-Bio region located between 39°S $< \varphi <$ 36°S and 74°W $< \lambda <$ 70°W. These regions were chosen by their steep topographical character as shown in Figures 1(a) and (b).

Among the data used, was the global topography DTM2006.0 with harmonic development to degree and order 250. The high resolution DEM was SRTM30_PLUS, which incorporates the topography model SRTM30 and the ice topography model generated by ICE Sat (Becker et al., 2009). The SRTM30_PLUS is referred to the WGS84 reference system in the horizontal component and to the geoid model EGM96 in the vertical component (Denker, 2005). The model data are available on the site SRTM30_PLUS <http://topex.ucsd.edu/WWW_html/ srtm30_plus.html>, with a resolution of 900 m.

MODELING THE GRAVITY FIELD

The most common methodology used in the local/regional modeling gravity field is the Remove-Restore (RR). The RR technique consists in removing both the topography and the low degree harmonic development gravity signal before calculating the residual, restoring these effects after application of Stokes integral. In addition, the Stokes integral is truncated to a bounded region (Sjöberg, 2005). A brief description of the technique RR is presented below.

Considering that the observations of terrestrial gravity already contain the global and local effects corresponding to the long and short wavelengths respectively, the difference between the magnitudes ($Q_i = f(T)$) from global and local data, results in a residual component, as shown in Equation (1):

$$Q_r = Q - Q_{GGM} - Q_{DEM} \tag{1}$$

where Q_r is the residual component of any functional of the gravity field, Q the magnitude observed, Q_{GGM} a global component extracted from the GGM, and Q_{DEM} the component recovered from the effect of topography.

In the restoration stage, each part referring to the spectral division of the functional associated with the disturbing potential has to be transformed into a corresponding portion of the formula via Bruns, to finally restore the disturbing potential (Moritz, 1980):

$$T = T_l + T_m + T_c \tag{2}$$

where T_l, T_m and T_c represent information linked to long, medium and short wavelengths respectively.

An alternative technique used for high-frequencies modeling is the RTM (Hirt, 2010a). The technique was introduced by Forsberg & Tscherning (1981), where they describe some methods for calculating the terrain gravitational effects by a minimum squares approximation technique. The RTM technique consists in calculating the effects of small wavelengths generated by the topography/bathymetry on the different functionals of the gravity field. In this method a surface average height (closely linked with the reference GGM) is considered as a long wavelength filter, and later this effect is restored. The technique RTM, can be understood as a method for the spectral improvement of the GGM, reducing mainly the error of omission (Torge, 2001).

The formulas used to calculate the effect in terms of RTM prisms, over the main gravimetric functionals, are also shown in Forsberg & Tscherning (1981). Below are some of the formulas presented.

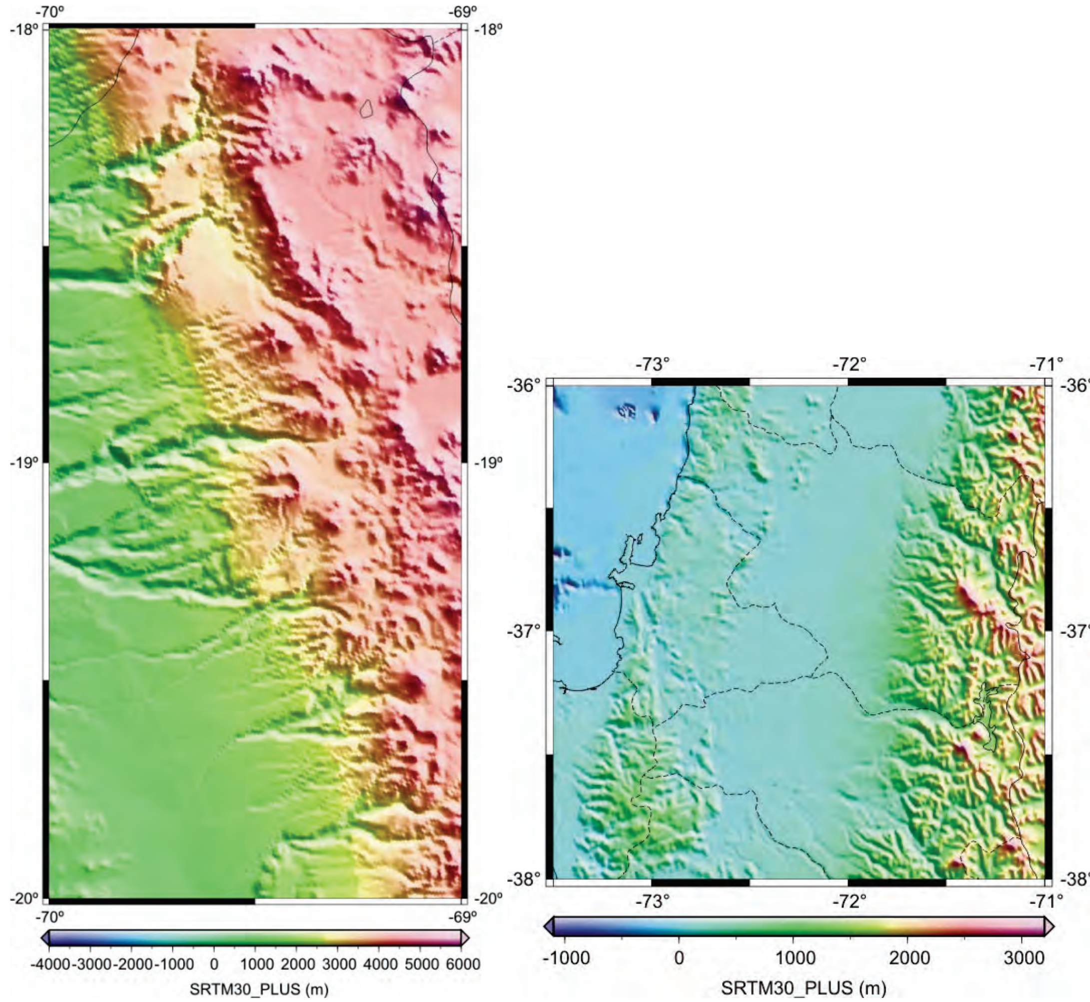

Figure 1 – Test area for Regions I a) and VIII b) in Chile. The scale bar indicates the altitude of the terrain from the SRTM30_PLUS.

It should be noted that the residual potential V_{RTM}, has several annotations in the literature, among these: V, T_m, V_{RTM}, $u(P)$.

$$V_{RTM} = \int_v \frac{dx \cdot dy \cdot dz}{r} = \int_{x_1}^{x_2} \int_{y_1}^{y_2} \int_{z_1}^{z_2} \frac{dx \cdot dy \cdot dz}{r} \tag{3}$$

Or:

$$V_{RTM} = G_\rho \left| \left| \left| xy \ln(z+r) + yz \ln(x+r) + zx \ln(y+r) - \frac{x^2}{2} \tan^{-1} \frac{yz}{xr} - \frac{y^2}{2} \tan^{-1} \frac{zx}{yr} - \frac{z^2}{2} \tan^{-1} \frac{xy}{zr} \right|_{x_1}^{x^2} \right|_{y_1}^{y_2} \right|_{z_1}^{z^2} \tag{4}$$

Being $r(x, y, z) = \sqrt{x^2 + y^2 + z^2}$, the radius of integration between the integration point and station, G is the gravitational constant, ρ the average density of the continental crust (2.67 g/cm^3).

Once the potential generated by the effect RTM is calculated, Bruns equation can be applied to recover the height anomaly (geoidal height) (Heiskanen & Moritz, 1967):

$$\zeta^{prisma} = \frac{V_{RTM}}{\gamma_Q} \tag{5}$$

$$\zeta^{RTM} = \sum_{i=1}^{k} \zeta_i^{prisma} \tag{6}$$

Due to the complexity and time consumption for this calculation, an approximate formula for the potential V_{RTM} was proposed by Forsberg (1984), to calculate the geoidal height based on the condensation of the mass of the prisms as a layer of mass located on a xy plan through the center of the prism.

$$V_{RTM} \cong G\rho(z_2 - z_1) \int_x y \int \frac{1}{r} dx dy \tag{7}$$

$$V_{RTM} = G\rho(z_2 - z_1) \Bigg| \Bigg| x \ln(y + r) + y \ln(x + r) - z_m \tan^{-1} \frac{xy}{z_m r} \Bigg|_{x_1}^{x^2} \Bigg|_{y_1}^{y^2} \tag{8}$$

where

$$z_m = \frac{(z_2 + z_1)}{2}, \qquad r = \sqrt{x^2 + y^2 + z_m^2}.$$

The contribution of the RTM in terms of the vertical deflection is obtained by the sum of the horizontal derivatives of the gravitational potential of all prisms from the RTM grid, divided by the normal gravity (Nagy et al., 2000).

$$\zeta^{RTM} = -\frac{1}{\gamma} \sum V_X \tag{9}$$

$$\eta^{RTM} = -\frac{1}{\gamma} \sum V_Y \tag{10}$$

Where:

$$V_X = G\rho \Bigg| \Bigg| \Bigg| y \ln(z + r) + z \ln(y + r) - x \tan^{-1} \frac{yz}{xr} \Bigg|_{x_1}^{x_2} \Bigg|_{y_1}^{y^2} \Bigg|_{z_1}^{y^z} \tag{11}$$

$$V_Y = G\rho \Bigg| \Bigg| \Bigg| z \ln(x + r) + x \ln(z + r) - y \tan^{-1} \frac{xz}{yr} \Bigg|_{x_1}^{x_2} \Bigg|_{y_1}^{y^2} \Bigg|_{z_1}^{y^z} \tag{12}$$

V_x, V_y: Horizontal derivatives of the gravitational potential.

METHODOLOGY

With the objective to get the residual terrain provided by the high frequency of MDE SRTM30_PLUS in relation to DTM2006.0 one high-pass filter was applied. The harmonic development model degree used for reference DTM2006.0 was 250.

The development in the harmonic expansion for DTM2006.0 in mathematical terms is shown in the following equation (EGM2008-Team, 2008):

$$H^{DTM\,2006.0}(\theta, \lambda) = \sum_{n=0}^{n_{\max}^{DTM}} \sum_{m=0}^{n} \left(\overline{HC}_{nm} \cos(m\lambda) + \overline{HS}_{nm} \sin(m\lambda) \right) \overline{P}_{nm} \cos\theta \tag{13}$$

where $\overline{HC}_{nm}$ and $\overline{HS}_{nm}$ are completely normalized altitude coefficients, $n_{\max}^{DTM}$ is the maximum degree of evaluation (250), (θ, λ) are the geocentric co-latitude and geodetic longitude $\bar{P}_{nm} \cos\theta$ the associated Legendre function completely normalized, and $H^{DTM\,2006.0}(\theta, \lambda)$ is the altitude at the point of coordinates (θ, λ).

The choice of the development degree of DTM2006.0 must have a close relationship with the development degree of GGMs used, when the objective is to minimize the omission error. In other words the GGM (data from the GOCE mission) used a development to the degree and order 250, however, the effects of the terrain below this resolution are already contained in GGM.

The grid spacing of the DEMs used for the generation of residual topography were 80 km to DTM2006.0 and 90 m for the SRTM30_PLUS. Statistics of Residual Topography for the two regions are shown in Table 1.

Table 1 – Statistics of residual topography in Regions VIII and I.

Statistics	Residual topography Region I [m]	Residual topography Region VIII [m]
Minimum	–2382	–5381
Maximum	2500	5035
Average	15	–430
Standard deviation	470	2283

After calculating the residual topographies for the two regions, the residual terrain effect was calculated for the following functionals: *height anomaly* (ζ); *gravity anomaly* (Δg); and the *deflexion of the vertical components* (η, ξ).

A number of approaches can be used to calculate the effect on the different RTM functionals. Among these we have the tesseroids, prisms and mass points (Heck & Seitz, 2007). However, in this study the prisms approach was applied. The calculations were made using the TC application (Forsberg, 1984). In TC three grids must be inserted. These are: a detailed grid, a grid of lower resolution than the detailed one, and a reference grid. The first is used to calculate the effect in the region nearest to the point of calculation, the second to estimate the effect of the most remote regions of the station and the grid of reference acts as a kind of filter. The constant continental crust density value applied was 2670 kg.m^{-3} since there is no density model available for this study.

RESULTS AND DISCUSSION

The residual effect of topography was calculated in different magnitudes for the two regions, considering an integration radius of 220 km. The results are shown in Tables 2 and 3.

From Figure 1 we infer the strong topographic contrast in this Region I, where more than 50% of the area has elevations between 4000 and 6000 m. On the other hand, the Region VIII has a less rugged topography, with the majority of the area between 0 and 2000 m.

A first analysis was made regarding the behavior of the different functionals in the two regions also considering a radius of integration of 220 km.

Regarding the functional *anomaly height*, higher values were found in Region I with an average and standard deviation of 32 cm and 53 cm, respectively (Fig. 2a). However, in Region VIII an average of –9 cm and a standard deviation of 34 cm were obtained (Fig. 2b).

With regard to the *gravity anomalies* the effect was less expressive in comparison with the *anomaly height*. While Region I (Fig. 3a) shows a more pronounced topography, the Region VIII (Fig. 3b) presented higher values of maximum gravity anomaly than Region I. The average and standard deviation values obtained for Region I were 0.10 and 34 mGal, respectively, and values for Region VIII were –2.93 mGal for the average, and 26.46 mGal for the standard deviation.

In the statistics of the deflexion of the vertical components, the residual effect of the terrain resulted in approximately $\pm 0.1''$ and $5''$ for the average and standard deviation, respectively. The average and standard deviation, values of the deflexion of the vertical components show the same tendency shown in Tables 2 and 3.

The second analysis was performed to evaluate the incident effect on the variation of the the radius of integration over the functionals *height anomaly*, *gravity anomaly* and the deflexion of the vertical components. The radius of integration used were 100, 220 and 300 km.

The behavior of the functional height anomaly for Region I was strongly distinguished by varying the radius of integration. For the radius 100, 220 and 300 km, the values were 18, 32 and 30 cm respectively. The largest change was for the radius from 100 to 220 km, showing almost twice the effect. The residual effect calculated with different radius of integration on the gravity anomalies can be seen in Table 6.

Regarding the behavior of the residual effect on the deflexion of the vertical components, differences smaller than one second were noticed. However, different tendencies were noticed in the different radius of integration. The average values obtained for the integration radius 100, 220 and 300 km were –0.085, 0.080 and 0.09 respectively (Table 4).

The results, once changing the radius of integration in functional *height anomaly* for Region VIII, showed a similar behavior in magnitude (proportional), as well as the same tendency as in Region I.

The residual terrain effect over the *gravity anomalies* and the deflexion of the vertical components to Region VIII after varying the radius of integration were slightly noticed, with differences in values smaller than 1 mGal and a second of arc respectively (Table 5).

Table 2 – Statistics on the RTM functionals in Region I.

Statistics	$\zeta_{RTM}[m]$	$\Delta g_{RTM}[mGal]$	$\eta_{RTM}['']$	$\zeta_{RTM}['']$
Minimum	–1.02	–137.70	–22.40	–21.48
Maximum	1.72	120.66	17.11	17.40
Average	0.32	0.10	–0.07	0.09
Standard deviation	0.53	34.65	4.69	5.93

Table 3 – Statistics on the RTM functionals in Region VIII.

Statistics	$\zeta_{RTM}[m]$	$\Delta g_{RTM}[mGal]$	$\eta_{RTM}['']$	$\zeta_{RTM}['']$
Minimum	–1.04	–105.59	–17.85	–17.90
Maximum	1.27	149.59	18.89	17.70
Average	–0.09	–2.93	–0.06	–0.17
Standard deviation	0.34	26.46	3.40	4.45

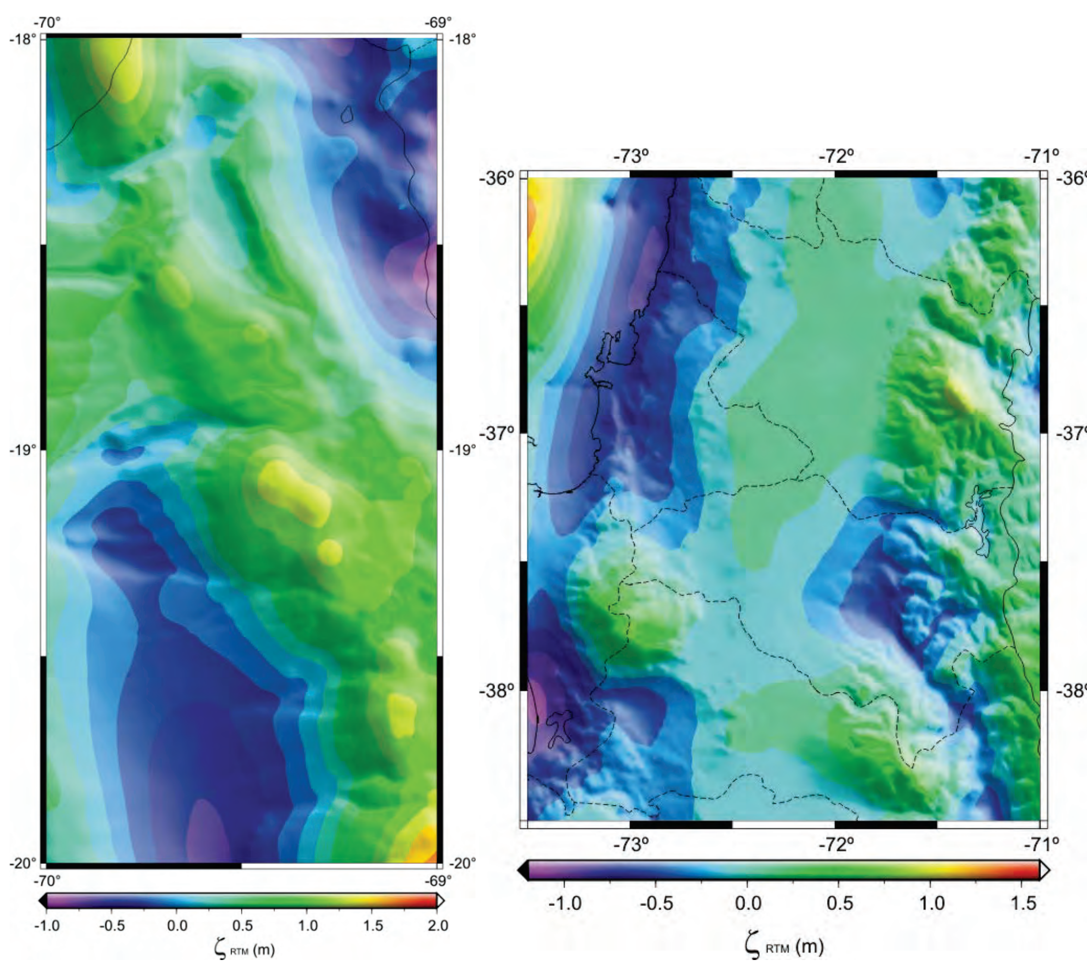

Figure 2 – Altitude anomalies at Regions I a) and VIII b).

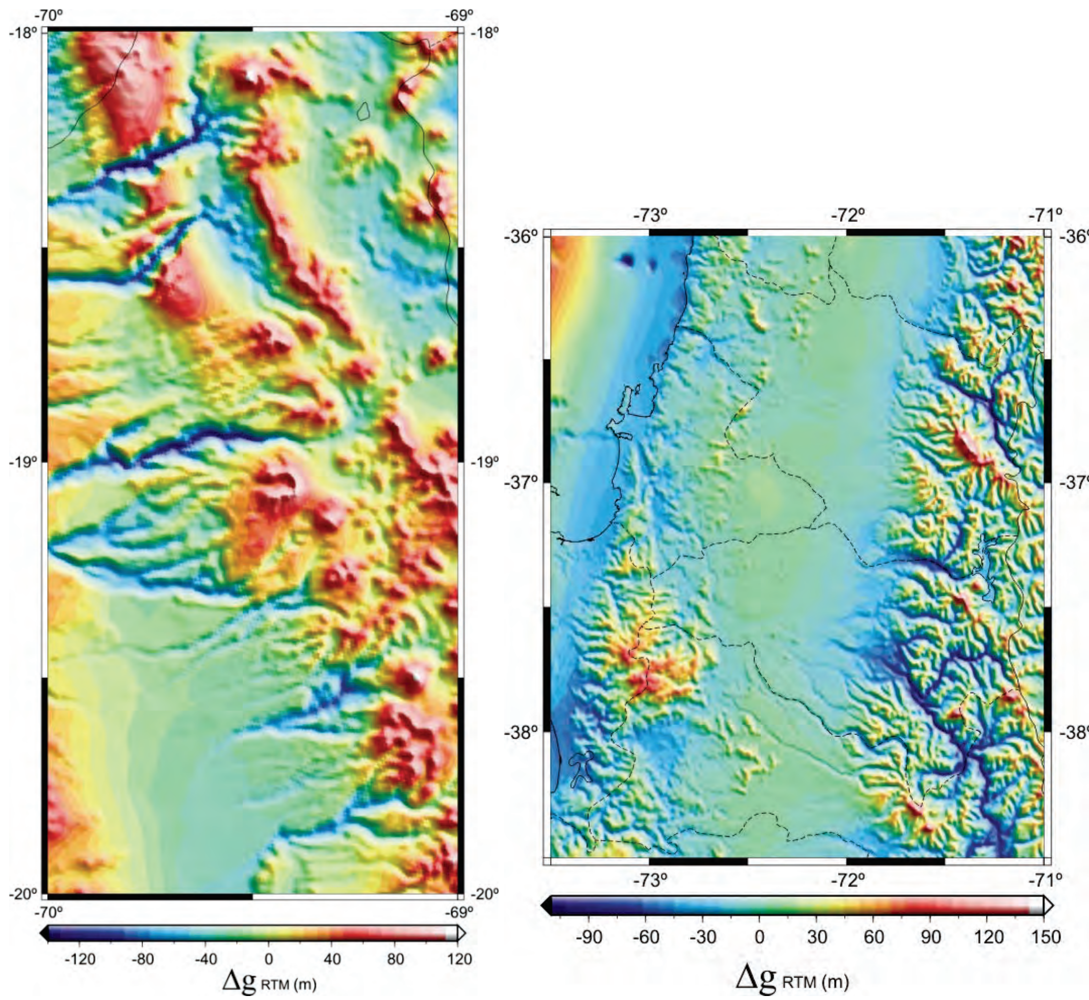

Figure 3 – Gravity anomalies at Regions I a) and VIII b).

Table 4 – Statistics on the RTM functional in Region I Radius 100 km.

Statistics	$\zeta_{RTM}[m]$	$\Delta g_{RTM}[mGal]$	$\eta_{RTM}['']$	$\zeta_{RTM}['']$
Radius 100 km				
Average	0.183	0.130	–0.085	–0.085
Standard deviation	0.392	34.660	4.690	4.690
Radius 220 km				
Average	0.32	0.10	–0.07	0.09
Standard deviation	0.53	34.65	4.69	5.93
Radius 300 km				
Average	0.300	0.090	–0.060	0.080
Standard deviation	0.370	34.650	4.680	5.910

Comparing the residual terrain effect for four functionals in the two study regions, opposite trends were found. For example: average values for the *height anomaly* presented a positive signal for the Region I, while for the Region VIII the values were negative. This opposite behavior was noticed in the four functionals (Tables 6 and 7).

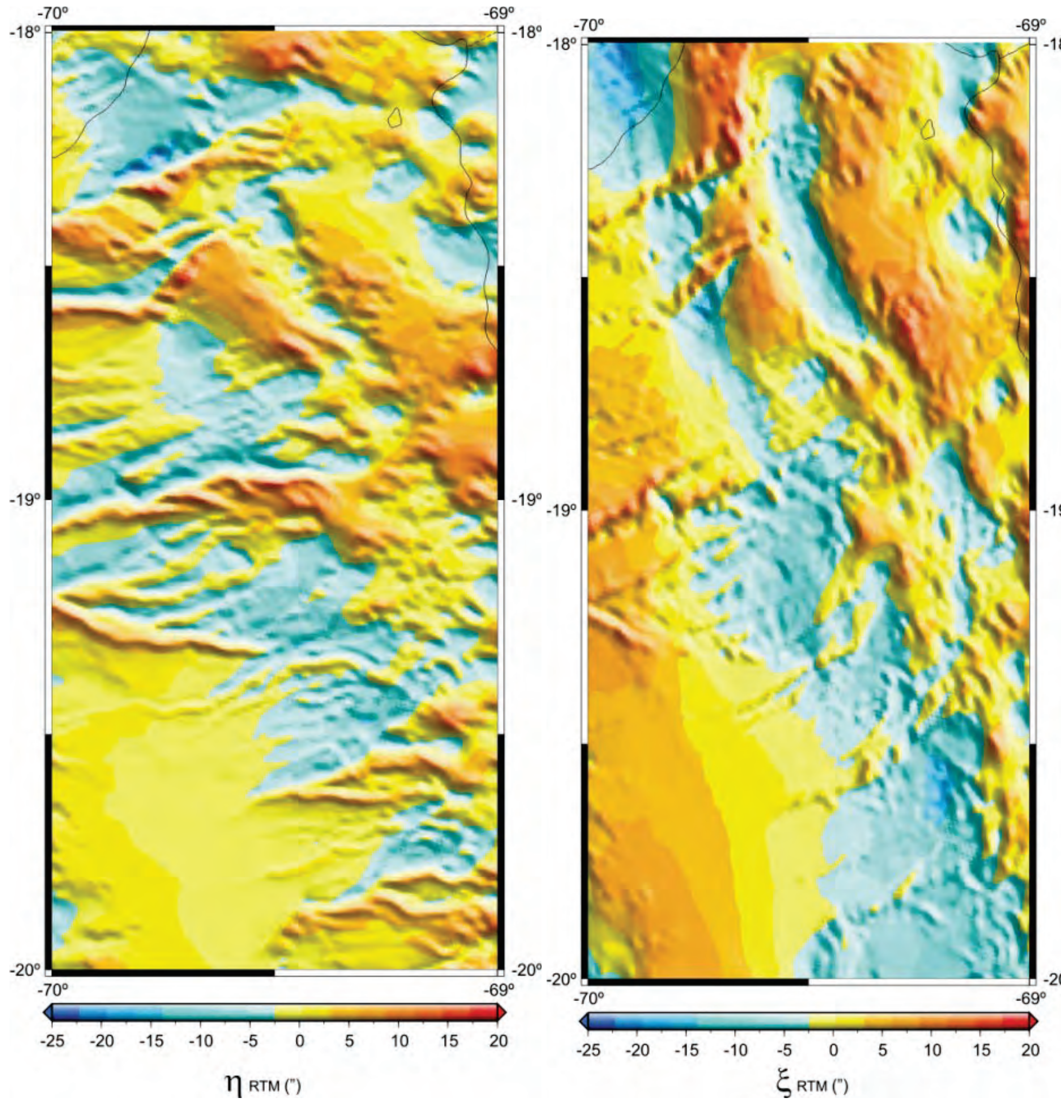

Figure 4 – Deflexion of the vertical components at Region I η a) and ξ b) [$''$].

From the analysis of the variables in the two regions, we can infer the different behavior of the anomalous field in regions with different topographic characteristics, and possibly different densities of the topographical masses.

After analyzing the four functionals in the two regions, it is inferred that the functional that reached higher values from the residual terrain effect is the *height anomaly*. The other functionals are slightly affected. Although the values obtained for the *deflexion of the vertical components* are small, these are significant, once the current methods for calculating the deviation of the vertical are able to measure the vertical deflection with a precision between 0.2" and 0.4" (Boyarsky et al., 2010).

A study by Jekeli (1999) based on the analysis of the *deflexion of the vertical components* derived from a GGM, presents comparisons in relation to those obtained in astronomy, indicating a M.S.E. around 4$''$, which raises the possibility of completing this information with the RTM technique.

On the other hand, the integration of GNSS (*Global Navigation Satellite System*) with a geoid/quasigeoid model is crucial in a series of applications where one seeks to recover the orthometric height/standard height e.g. upgrading the systems of altitudes), RTM effect offers a possibility to improve GGMs.

With the evolution of GGMs, the long and medium wavelengths are best resolved. This allows methods to recover the

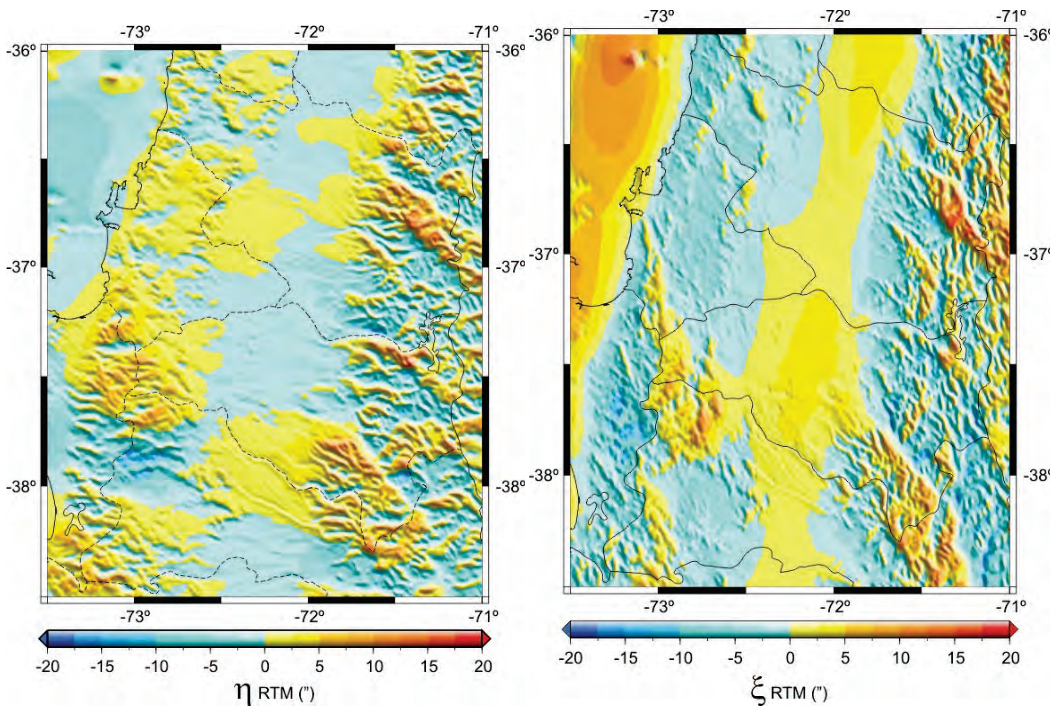

Figure 5 – Deflexion of the vertical components at Region VIII η a) and ξ b) $['']$.

Table 5 – Statistics on the RTM functional in Region VIII radius 100 km.

Statistics	$\zeta_{RTM}[m]$	$\Delta g_{RTM}[mGal]$	$\eta_{RTM}['']$	$\zeta_{RTM}['']$
Radius 100 km				
Average	–0.040	–2.890	–0.050	–0.080
Standard deviation	0.270	26.460	3.410	4.470
Radius 220 km				
Average	–0.09	–2.93	–0.06	–0.17
Standard deviation	0.34	26.46	3.40	4.45
Radius 300 km				
Average	–0.140	–2.930	–0.070	–0.180
Standard deviation	0.330	26.460	3.400	4.450

short wavelengths and minimize the error of omission of the GGMs with information from DEMs and the density models become more important.

Due to the deficiency in quantity and distribution of data from terrestrial/aerial gravimetry and GNSS data available in the Chilean territory, limitations arise for the modeling of a regional geoid. The classical approach (Stokes) and the current one (Molodenskii) require data from terrestrial/aerial gravimetry, what in rugged and inaccessible territories, as a part of the study regions, raise difficulties to the surveys, or even prevents its acquisition. Therefore the refinement of a GGM from the RTM effect can contribute with information to the gravity field and to get the functionals with the appropriate precision to the desired applications. This approach can be summarized by the following expressions for the different functionals studied:

$$\begin{aligned} \Delta g^{GGM/RTM} &= \Delta g_{GGM} + \Delta g_{RTM} \\ \zeta^{GGM/RTM} &= \zeta_{GGM} + \zeta_{RTM} \\ \eta^{GGM/RTM} &= \eta_{GGM} + \eta_{RTM} \\ \xi^{GGM/RTM} &= \xi_{GGM} + \xi_{RTM} \end{aligned} \tag{14}$$

Table 6 – Statistics estimation of functional with radius 100, 220 and 300 km in Region I.

Statistics	$\zeta_{RTM}[m]$	$\Delta g_{RTM}[mGal]$	$\eta_{RTM}['']$	$\zeta_{RTM}['']$
Difference Radius 100 km – R220 km				
Average	–0.138	0.035	–0.018	–0.171
Standard deviation	–0.134	0.015	0.001	–1.244
Difference Radius 220 km – R300 km				
Average	0.021	0.005	–0.007	0.006
Standard deviation	0.155	–0.005	0.009	0.024
Difference Radius 100 km – R300 km				
Average	–0.117	0.040	–0.025	–0.165
Standard deviation	0.022	0.010	0.010	–1.220

Table 7 – Statistics estimation of functional with radius 100, 220 and 300 km in Region VIII.

Statistics	$\zeta_{RTM}[m]$	$\Delta g_{RTM}[mGal]$	$\eta_{RTM}['']$	$\zeta_{RTM}['']$
Difference Radius 100 km – R220 km				
Average	0.047	0.040	0.009	0.087
Standard deviation	–0.065	0.000	0.009	0.015
Difference Radius 220 km – R300 km				
Average	0.053	0.000	0.011	0.013
Standard deviation	0.005	0.000	0.001	0.005
Difference Radius 100 km – R300 km				
Average	0.100	0.040	0.020	0.100
Standard deviation	–0.060	0.000	0.010	0.020

where the index GGM/RTM represents the fulfillment of the functional effect of the geopotential by RTM. Should be emphasized that the deflexion of the vertical components obtained from GGMs are in agreement with the Theory of Molodenskii, but for comparisons with the same obtained by astro-geodetic methods, corrections should be applied (Jekeli, 1999).

It should be emphasized that currently a series of evaluations of GGMs present M.S.E. values around 30 cm and 40 mGal in terms of geoidal height and gravity anomalies respectively (Huang & Véronneau, 2004; Sadiq & Ahmad, 2009; Hirt et al., 2010a), which raises the possibility to obtain satisfactory results through the integration of a GGM plus RTM effect.

CONCLUSION

The RTM effect was explored starting from four magnitudes of the gravity field, in two areas(Regions I and VIII) in Chile. The functionals studied were; *height anomaly*, *gravity anomaly* and the *deflexion of the vertical components* η and ξ.

The values obtained through the residual effect of the terrain show a greater contribution as a part of the functional *anomaly height*, however, in other functionals they manifest slightly. The effect of the RTM on the *deflexion of the vertical components* was small, but considerable from the point of view of current requirements. Furthermore, the opposite sign of the values obtained for the different functionals in both regions allow us to infer the local possible behavior of the anomalous field.

With respect to the incident variation of the radius of integration in the calculation of different functionals, it was found that the functional which has a greater dependence (in the lengths analyzed) with the length of the radius of integration is the *height anomaly*. However, the vertical deflection components show effects from hundredths of second to one second for the west-east, η, and south-north, ξ, components. The gravity anomalies values vary around hundredths of mGal.

The possible improvements in a GGM from the residual effect on *height anomaly*, *gravity anomaly* and the *deflexion of the* *vertical components* are considerable, due to the fact that in the regions analyzed this effect reached 2 m, 120 mGal and 25″, respectively, which is normally around the M.S.E. obtained from some evaluations of GGMs.

REFERENCES

AMOS MJ & FEATHERSTONE WE. 2003. Comparisons of Recent Global Geopotential Models with Terrestrial Gravity Field Observations Over New Zealand and Australia. Geomatics Research Australasia, 79: 1–20.

AWANGE JL, GRAFAREND EW, PALÁNCZ B & ZALETNYIK P. 2010. Algebraic Geodesy and Geoinformatics. 2nd ed., Springer-Verlag Berlin Heidelberg. 378 p.

BARTON PJ. 1986. The relationship between seismic velocity and density in the continental crust – a useful constraint. Geophysical Journal of the Royal Astronomical Society, 87: 195–208.

BECKER JJ, SANDWELL DT, SMITH WHF, BRAUD J, BINDER B, DEPNER J, FABRE D, FACTOR J, INGALLS S, KIM S-H, LADNER R, MARKS K, NELSON S, PHARAOH A, TRIMMER R, VON ROSENBERG J, WALLACE G & WEATHERALL P. 2009. Global Bathymetry and Elevation Data at 30 Arc Seconds Resolution: SRTM30_PLUS. Marine Geodesy, 32(4): 355–371.

BLITZKOW D, DE MATOS ACOC & CINTRA JP. 2005. SRTM evaluation in Brazil and Argentina with emphasis on the Amazon region. In: TREGONING P & RIZOS C (Eds.). Dynamic Planet: Monitoring and Understanding a Dynamic Planet with Geodetic and Oceanographic Tools. IAG Symposia, 128: 266–271. Springer.

BOYARSKY EA, AFANASYEVA VL, KONESHOV VN & ROZHKOV YE. 2010. On Calculation of the Vertical Deflection and Geoid Undulation from Gravity Anomalies. Physics of the Solid Earth, 46: 538–543.

CHASE CG. 1985. The geological significance of the geoid. Ann. Rev. Earth Planet. Sci., 13: 97–117.

DENKER H. 2005. Evaluation of SRTM3 and GTOPO30 terrain data in Germany. IAG Symposia, 129: 218–223. Springer Verlag.

EGM2008 – Team 2008. Description of files related to the EGM2008 Global Gravitational Model. U.S. National Geospatial-Intelligence Agency (NGA) EGM2008 Development Team.

FEATHERSTONE WE. 2002. Expected contributions of dedicated satellite gravity field missions to regional geoid determination with some examples from Australia. Journal of Geospatial Engineering, 4(1): 1–19.

FEATHERSTONE WE, HOLMES SA, KIRBYJF & KUHN M. 2004. Comparison of Remove-Compute-Restore and University of New Brunswick Techniques to Geoid Determination over Australia, and Inclusion of Wiener-Type Filters in Reference Field Contribution. Journal of Surveying Engineering, 130: 40–47.

FLURY J & RUMMEL R. 2005. Future Satellite Gravimetry for Geodesy. Earth, Moon, and Planets, 94: 13–29.

FORSBERG R. 1984. A study of terrain reductions, density anomalies and gravity field modelling. 129. Report n. 355. Department of Geodetic Science and Surveying, The Ohio State University. Columbus.

FORSBERG R & TSCHERNING CC. 1981. The use of Height Data in Gravity Field Approximation by Collocation. Journal of Geophysical Research, 86: 7843–7854.

GÖTZE H & KIRCHNER A. 1997. Interpretation of gravity and geoid in the Central Andes between 20° and 29°. Journal of South American Earth Sciences, 10: 179–188.

GUČEVIĆ J, OGRIZOVIĆ V, DELČEVĆ S & VASILIĆ V. 2010. Optimum Resolution and Size of DTM during Modelling Topographic Effect. Geodetski list, v.64 (87) No.3.

HEĆIMOVIĆ Ž & BAŠIĆ T. 2005. Terrain Effect on Gravity Field Parameters using Different Terrain Models. Newton's Bulletin, 3: 92–102.

HECK B & SEITZ K. 2007. A comparison of the tesseroid, prism and point-mass approaches for mass reductions in gravity field modelling. Journal of Geodesy, 81: 121–136.

HEISKANEN WA & MORITZ H. 1967. Physical Geodesy. W.H. Freeman, San Francisco. 364 p.

HIRT C. 2009. Prediction of vertical deflections from high-degree spherical harmonic synthesis and residual terrain model data. Journal of Geodesy, 84: 179–190.

HIRT C, FEATHERSTONE WE & MARTI U. 2010a. Combining EGM2008 and SRTM/DTM2006.0 residual terrain model data to improve quasigeoid computations in mountainous areas devoid of gravity data. Journal of Geodesy, 84: 557–567.

HIRT C, MARTI U, BÜRKI B & FEATHERSTONE WE. 2010b. Assessment of EGM2008 in Europe using accurate astrogeodetic vertical deflections and omission error estimates from SRTM/DTM2006.0 residual terrain model data, Journal of Geophysical Research – Solid Earth, 115(B11): B10404.

HUANG J & VÉRONNEAU M. 2004. GPS-leveling and CHAMP&GRACE geoid models. Joint CHAMP/GRACE Science Meeting, GFZ, July 5-8, 2004. Available on: <http://www-app2.gfz-potsdam.de/pb1/JCG/>. Access on: May 26, 2011.

IBGE. Instituto Brasileiro de Geografia e Estatística. Available on: <http://www.ibge.gov.br/home/geociencias/geodesia/modelo_geoidal.shtm>. Access on: May 24, 2011.

ICGEM. International Centre for Global Earth Models. Available on: <http://icgem.gfz-potsdam.de/ICGEM/>. Access on: May 24, 2011.

JEKELI C. 1999. An analysis of vertical deflections derived from high-degree spherical harmonic models. Journal of Geodesy, 73: 10–22.

KHAZARADZE G & KLOTZ J. 2003. Short and long term effects of GPS measured crustal deformation rates along the South Central Andes. J. Geophys. Res., 108(B6), 2289, 2ng: 10.1029/2002 JB001879.

KENDRICK EC, BEVIS M, SMALLEY JR RF, CIFUENTES O & GALBAN F. 1999. Current rates of convergence across the Central Andes: Estimates from continuous GPS observations. Geophysical Research Letters, 26(5): 541–544.

KIAMEHR R & SJÖBERG LE. 2005. Effect of the SRTM global DEM on the determination of a high-resolution geoid model: a case study in Iran. Journal of Geodesy, 79: 540–551.

KIAMEHR R & SJÖBERG LE. 2006. Impact of a precise geoid model in studying tectonic structures – A case study in Iran. Journal of Geodynamics, 42: 1–11.

MORITZ H. 1980. Advanced physical geodesy. Karlsruhe: Herbert Wichmann Verlag. 500 p.

NAGY D, PAPP G & BENEDEK J. 2000. The gravitational potential and its derivates for the prism. Journal of Geodesy, 74: 552–560.

PAVLIS NK, HOLMES SA, KENYON SC & FACTOR JK. 2008. An Earth gravitational model to degree 2160: EGM2008 EGU General Assembly. 13-18 April 2008, Vienna.

PERROT K, TARITS P & GOSLIN J. 1997. Regionalized comparison of global Earth models of seismic velocity, geoid and topography: implications on the geodynamics of ridges and subductions. Physics of The Earth and Planetary Interiors, 99: 91–102.

RICHARD Y & WUMING B. 1991. Inferring the viscosity and 3D density structure of the mantle from geoid, topography and plate velocities. Geophysical Journal International, 105: 561–571.

RUMMEL R, BALMINO G, JOHANNESSEN J, VISSER P & WOODWORTH P. 2002. Dedicated gravity field missions – principles and aims. Journal of Geodynamics, 33: 3–20.

SADIQ M & AHMAD Z. 2009. On the selection of optimal global geopotential model for geoid modeling: A case study in Pakistan. Advances in Space Research, 44: 627–639.

SANSÓ F (Ed.). 1994. Lecture notes. International School for the Determination and Use of the Geoid. International Geoid Service, DIIAR – Politechnico di Milano.

SJÖBERG LE. 2005. A discussion on the approximations made in the practical implementation of the remove-compute-restore technique in regional geoid modelling. Journal of Geodesy, 78: 645–653.

SUN W & SJÖBERG LE. 2001. Permanent components of the crust, geoid and ocean depth tides. Journal of Geodynamics, 31: 323–339.

TOCHO C, VERGOS GS & SIDERIS MG. 2007. Evaluation of the SRTM 90 m DTM over Argentina and its implications to gravity field and geoid modelling. In: FORSBERG R & KILIÇOGLU A (Eds.). 1st International symposium of the international gravity field service "Gravity Field of the Earth", General Command of Mapping, Special Issue 18, pp. 324–329.

TORGE W. 2001. Geodesy. 3rd ed. De Gruyter. Berlin, 416 p.

VERGOS GS, GRIGORIADIS VN, KALAMPOUKAS G & TZIAVOS IN. 2005. Accuracy assessment of the SRTM 90 m DTM over Greece and its implications to geoid modelling. In: TREGONING P & RIZOS C (Eds.). Dynamic Planet: Monitoring and Understanding a Dynamic Planet with Geodetic and Oceanographic Tools. IAG Symposium, 130: 309–315. Cairns, Australia. Berlin: Springer-Verlag.

VERMEERSEN LLA & SCHOTMAN HHA. 2008. High-harmonic geoid signatures related to glacial isostatic adjustment and their detectability by GOCE. Journal of Geodynamics, 46(3-5): 174–181.

WATTS AB & DALY SF. 1981. Long wavelength gravity and topography anomalies. Ann. Rev. Earth Planet. Sci., 9: 415–448.

ZONGJIN M & XIANGLIN G. 1996. Three global-scale seismotectonic systems and the geoid. Journal of Southeast Asian Earth Science, 13: 337–340.

NOTES ABOUT THE AUTHORS

Henry Diverth Montecino Castro. Master Degree in Geodetic Sciences in the Department of Geomatics of Universidade Federal do Paraná, UFPR, Brazil supervised by Prof. Dr. Silvio Rogério Correia de Freitas em 2011. Scholarship from Coordenação de Aperfeiçoamento de Pessoal de Nível Superior (CAPES). Graduation in Engineering in Land Surveys (2006) at Universidad de Concepción, UDEC, Chile, under the supervision of Prof. Dr. Juan Carlos Baez Soto. Experience in Geosciences with an emphasis in Physical Geodesy with interest on the following subjects: Geodetic Reference Systems, gravimetry, satellite altimetry, geodetic surveys, geodetic instrumentation.

Karoline Paes Jamur. Scholarship of the Conselho Nacional de Desenvolvimento Científico e Tecnológico (CNPq). Ph.D. in progress in Geodetic Sciences of Universidade Federal do Paraná, UFPR, Brazil under the supervision of Prof. Dr. Silvio Rogério Correia de Freitas. Master's degree in Geodetic Sciences in 2007 at UFPR. Graduated in Cartographic Engineering (2004) at UFPR under the supervision of Prof. Dr. Luis Koenig Veiga and Prof. Dr. Silvio Correia de Freitas. Has experience in Geosciences with an emphasis in Physical Geodesy and Geodynamics, with interest on the following subjects: Geodetic Reference Systems, gravimetry, satellite altimetry, geodetic surveys, geodetic instrumentation.

Silvio Rogério Correia de Freitas. Professor, Department of Geomatics at Universidade Federal do Paraná. Graduated in Physics at Universidade Federal do Paraná (1975), Master's Degree in Geodesic Sciences Universidade Federal do Paraná (1980) and a Ph.D. in Geophysics at Universidade de São Paulo (1993). Member of the Board of Trustees of the Universidade Federal do Paraná, research fellowship from Conselho Nacional de Desenvolvimento Científico e Tecnológico (CNPq) level 1B, coordinator partnership – Deutsches Geodätisches Forschungsinstitut. Experience in the areas of Geodetic Reference Systems, gravimetry, satellite altimetry, geodetic surveys, geodetic instrumentation and applications of GPS/GNSS in Geodynamics.