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REVERSE TIME MIGRATION BY INTERPOLATION AND PSEUDO-ANALYTICAL METHODS

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ABSTRACT. Within the seismic method, in order to obtain an accurate image, it is necessary to use some processing techniques, among them the seismic migration. The reverse time migration (RTM) uses the complete wave equation, which implicitly includes multiple arrivals, can image all dips and, therefore, makes it possible to image complex structures. However, its application on 3D pre-stack data is still restricted due to the enormous computational effort required. With recent technological advances and faster computers, 3D pre-stack RTM is being used to address the imaging challenges posed by sub-salt and other complex subsurface targets. Thus, in order to balance processing cost and with image's quality and confiability, different numeric methods are used to compute the migration. This work presents two different ways of performing the reverse time migration using the complete wave equation: RTM by interpolation and by the pseudo-analytical method. The first migrates the data with different constant velocities and interpolate the results, while the second uses modifications in the computation of the Laplacian operator in order to improve the finite difference scheme used to approximate the second-order time derivative, making it possible to propagate the wave field stably even using larger time steps. The method's applicability was tested by the migration of two-dimensional pre- and pos-stack synthetic datasets, the SEG/EAGE salt model and the Marmousi model. A real pre-stack data from the Gulf of Mexico was migrated successfully and is also presented. Through the numerical examples the applicability and robustness of these methods were proved and it was also showed that they can extrapolate wavefields with a much larger time step than commonly used.

Keywords: acoustic wave equation, seismic migration, reverse time migration, pseudo-spectral method, pseudo-analytical method, pseudo-Laplacian operator.

RESUMO. No método sísmico, a fim de se obter uma imagem precisa, faz-se necessário o uso de técnicas de processamento, entre elas a migração sísmica. A migração reversa no tempo (RTM) empregada aqui não é um conceito novo. Ela usa a equação completa da onda, implicitamente inclui múltiplas chegadas, consegue imagear todos os mergulhos e, assim, possibilita o imageamento de estruturas complexas. Porém, sua aplicação em problemas 3D pré-empilhamento continua sendo restrita por conta do grande esforço computacional requerido. Mas, recentemente, com o avanço tecnológico e computadores mais rápidos, a migração 3D pré-empilhamento tem sido aplicada, especialmente, em problemas de difícil imageamento, como o de estruturas complexas em regiões de pré-sal. Assim, com o intuito de equilibrar o custo de processamento com a qualidade e confiabilidade da imagem obtida, são utilizados diferentes métodos numéricos para computar a migração. Este trabalho apresenta duas diferentes maneiras de se realizar a migração reversa no tempo partindo da solução exata da equação completa da onda: RTM por interpolação e pelo método pseudo-analítico. No método de interpolação, a migração é aplicada utilizando-se várias velocidades constantes, seguido de um procedimento de interpolação para obter a imagem migrada através da composição das imagens computadas a partir dessas velocidades constantes. Já no método pseudo-analítico, introduz-se modificações no cálculo do operador Laplaciano visando melhorar a aproximação da derivada segunda no tempo, que são feitas por esquemas de diferenças finitas de segunda ordem, possibilitando assim propagar o campo de onda de forma estável usando-se passos maiores no tempo. A aplicabilidade das metodologias foi testada por meio da migração de dados bidimensionais sintéticos pré- e pós-empilhamento, o modelo de domo de sal da SEG/EAGE e o modelo Marmousi. Um dado real bidimensional, adquirido no Golfo do México não empilhado, também, foi usado e migrado com sucesso. Assim, através desses exemplos numéricos, mostra-se a aplicabilidade e a robustez desses novos métodos de migracão reversa no tempo no imageamento de estruturas complexas com os campos de ondas propagados com passos maiores no tempo do que os usados comumente.

Palavras-chave: equação da onda, migração sísmica, migração reversa no tempo, método pseudo-espectral, método pseudo-analítico, operador pseudo-Laplaciano.

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INTRODUCTION

Since the beginning of the seismic digital era, several migration methods have been developed in order to produce the most accurate subsurface image with less computational cost. The reverse time migration (RTM) proposed by Whitmore (1983), McMechan (1983) and Baysal et al. (1983) is considered the most accurate migration method. Using the complete wave equation, RTM implicitly includes all the arrivals and has no limitations on reflectors dip, allowing the imaging of complex structures. Historically it had its use restricted, mainly due to its high computational cost. But recently, with technological advances and faster computers, 3D pre-stack migration has been more commonly applied, especially in difficult imaging problems such as imaging of sub-salt complex structures.

For pos-tack data, RTM is performed propagating the recorded data in reverse time within the subsurface model, using half the velocity of the medium. After the reverse propagation of the recorded wave field, it is captured at $t = 0$, the image condition, to build the migrated image. For pre-stack data, the RTM algorithm requires a different image condition. When migrating unstacked data, typically, a shot is propagated in the direct way, the recorded data is similarly propagated but in the reverse direction or back propagated. In this case, the image is formed by cross-correlation of the source and receiver fields at each propagation step in time to determine when a reflection event occurs. RTM can also be considered as a reverse process of modeling and the same code used for modeling can also be used for performing the RTM.

From the wave equation or any of its solutions, one can use different strategies to solve its discrete equivalent by numerical methods. The proposition of such methodologies is the core of this study, in which are presented two ways for resolving the wave equation, RTM by interpolation and pseudo-analytical methods.

Normally, the wave equation is approximated using finite difference second-order schemes to approximate the time derivative, and fourth-order schemes for the spatial derivatives. Approximation of the time derivative can introduce numerical errors such as pulse shape distortion and numerical dispersion. To avoid these numerical problems, small time intervals have to be used, reducing the computational efficiency of this method. Consequently, methods of finite differences become slow when used for modeling large-scale, large offsets and high frequency acoustic data due to the need to use several points per wavelength and small time steps to minimize numerical undesirable artifacts.

The pseudo-spectral methods, where the spatial derivatives are evaluated in the wave number domain, can help reduce this numerical problem because they use accurate optimum operators for a given spatial sampling of the grid. Using the Fast Fourier Transform (FFT) to implement the pseudo-spectral method, one can achieve computational efficiency and a high degree of accuracy (Kosloff & Baysal, 1982; Reshef et al., 1988).

However, there is no single and simple solution to improve the computation of the time derivatives. This problem does not have an optimal solution and recently several alternatives have been proposed. In this paper, we propose two new methods to improve the time derivative computation.

The pseudo-analytical method proposed here, using the pseudo-spectral method to solve the spacial derivatives, allows the obtaining of more accurate and stable results with lower computational time than the conventional finite differences method (second-order on time) if applied to a problem that would require the same degree of precision.

The RTM by the pseudo-analytical method is performed with a modification of the Laplacian operator, i.e., it uses a pseudo-analytical operator which is comprised of second-order spatial derivatives in order to improve the approximation by finite differences of the second-order time derivatives. The regular scheme to approximate the time derivatives using finite difference operators of second-order is widely used but requires small time sampling in order to avoid numerical errors, even if accurate spatial operators are used. Modifications on the Laplacian operator leads to a pseudo-differential operator that can be further simplified, assuming a constant reference velocity. Thus, it can be easily calculated in the wave number domain. It also shows that the pseudo-analytical Laplacian operator is reduced to the Laplacian operator when the time sampling interval used in the extrapolation is very small, i.e., analytically when the propagation time step becomes very small.

In RTM by interpolation, the wavefield evolution in time is based on the wave equation solution for constant velocity case. The wavefield propagation in time, i.e., its time evolution, is made considering some number of constant velocities, here implemented with the procedure proposed by Bagaini et al. (1995). The wavefield time evolution is taken in the Fourier domain, using the exact solution of the wave equation for constant velocity. Then, these fields are transformed back to the time-space domain and a process of linear interpolation is applied. The contribution of each wavefield in the migrated image is determined by a weight function which is calculated according to the velocity model.

The validation of the proposed methods is demonstrated here through post and pre-stack reverse time migration applied on the SEG/EAGE and Marmousi synthetic data, plus a real data from the

Gulf of Mexico. The numerical results show the proposed methods have the ability to image steep reflectors and complex structures using a larger time step than the commonly used by pseudo-spectral methods, while preserving the entire range of frequencies present in the data.

EXACT WAVE EQUATION SOLUTION

Given the complete wave equation:

$$\frac{\partial^2 P}{\partial x^2} + \frac{\partial^2 P}{\partial y^2} + \frac{\partial^2 P}{\partial z^2} - \frac{1}{v^2}\frac{\partial^2 P}{\partial t^2} = 0 \tag{1}$$

where $P = P(x, y, z, t)$ is the pressure field and $v = v(x, y, z)$ is the velocity, to evaluate it analytically as a function of time, it can be rewritten:

$$\frac{\partial^2 P(\mathsf{x}, t)}{\partial t^2} = v^2(\mathsf{x})\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}\right)P(\mathsf{x}, t) \tag{2}$$

where the vector position is defined by $(\mathsf{x}) = (x, y, z)$, the wave propagation velocity model is given by $v(\mathsf{x})$ and the pressure field expressed as $P(\mathsf{x}, t)$.

Following the deduction presented by Pestana & Stoffa (2010), an operator $-L^2$ can be defined:

$$v^2(\mathsf{x})\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}\right) = -L^2 \tag{3}$$

and the following differential equation is obtained:

$$\frac{\partial^2 P(\mathsf{x}, t)}{\partial t^2} = -L^2 P(\mathsf{x}, t) \tag{4}$$

Given the initial conditions $P(\mathsf{x}, t = 0) = P_0$ and $\frac{\partial P(\mathsf{x},t)}{\partial t}|_{t=0} = \dot{P}_0$, the equation solution is:

$$P(t) = \cos(Lt)\, P_0 + \frac{\sin(Lt)}{L}\, \dot{P}_0 \tag{5}$$

Using the solution presented at Eq. (5) it is possible to evaluate the fields $P(t + \Delta t)$ and $P(t - \Delta t)$. Summing then and using some trigonometric relations, it follows that:

$$P(\mathsf{x}, t + \Delta t) + P(\mathsf{x}, t - \Delta t) = 2\cos(L\Delta t)P(\mathsf{x}, t) \tag{6}$$

This equation can be used to perform the wavefield time evolution, both in the forward or reserve direction.

For the reserve time migration, the wave field $P(\mathsf{x}, t - \Delta t)$ can be computed using Eq. (6), knowing the fields $P(\mathsf{x}, t+\Delta t)$ and computing the operator $\cos(L\Delta t)$ applied to $P(\mathsf{x}, t)$ at each time step evolution.

RTM BY INTERPOLATION

The central idea of the reverse time migration by interpolation is to use the complete wave equation solution with constant velocities, propagate the wavefield using a certain number of different velocities followed by a process of interpolation in the time-space domain. The contribution from each wavefield is calculated based on the velocity model.

After a spatial Fourier transform, i.e., with the data at the time-wave number domain, the time evolution is made considering different constant velocities. Then, an inverse Fourier transform is applied, followed by a process of interpolation of the extrapolated sections on the time-space domain. This way, the method permits the propagation on the Fourier domain, using the complete wave equation and involving the most representative velocities of the wavefield. Conceptually, the idea is similar to the one used with the PSPI (phase-shift plus interpolation) migration, commonly applied to extrapolate the wavefield on depth but using the unidirectional wave equation in the frequency-wave number domain.

Starting with Eq. (4) and following the procedures at Raymond (1991), an pseudo-differential operator is defined to the j-th derivative of P with respect to t, which is given by:

$$\partial_t^j P \approx \frac{1}{(2\pi)^3}\int_{-\infty}^{\infty}\left[iL(\mathsf{x}, \mathsf{k})\right]^j \varphi(\mathsf{k}) e^{i(\mathsf{k}\cdot\mathsf{x})} d\mathsf{k} \tag{7}$$

where the $\mathsf{k} = (k_x, k_y, k_z)$ are the wave numbers corresponding to x and $\varphi(\mathsf{k})$ is defined by the equation:

$$\varphi(\mathsf{k}) = \int_{-\infty}^{\infty} P(\mathsf{x}) e^{-i(\mathsf{k}\cdot\mathsf{x})} d\mathsf{x} \tag{8}$$

It is known that the expansion by Taylor series of the wavefield $P(\mathsf{x}, t + \Delta t)$ from a known field $P(\mathsf{x}, t)$ is given by:

$$P(\mathsf{x}, t + \Delta t) = P(\mathsf{x}, t) + \sum_{j=1}^{\infty}\frac{\partial_t^j P(\mathsf{x}, t)}{j!}(\Delta t)^j \tag{9}$$

where ∂_t^j denotes the j-th derivative with respect to time.

Rewriting Eq. (9) and inserting Eq. (7), the next equation is obtained:

$$P(\mathsf{x}, t + \Delta t) = P(\mathsf{x}, t) + \sum_{j=1}^{\infty}\frac{\Delta t^j}{j!} \times \left[\frac{1}{(2\pi)^3}\int_{-\infty}^{\infty}\left[iL(\mathsf{x}, \mathsf{k})\right]^j \varphi(\mathsf{k}) e^{i(\mathsf{k}\cdot\mathsf{x})} d\mathsf{k}\right] \tag{10}$$

It is known that the Taylor series expansion of the exponential function e^x is given by $e^x = \sum_{j=0}^{\infty}\frac{x^j}{j!}$. Using this relation and expanding the summation on Eq. (10), it can be observed that the field $P(\mathsf{x}, t+\Delta t)$ can be written as:

$$P(\mathsf{x}, t+\Delta t) = \frac{1}{(2\pi)^3}\int_{-\infty}^{\infty} e^{[iL(\mathsf{x},\mathsf{k})\Delta t]}\varphi(\mathsf{k})e^{i(\mathsf{k}\cdot\mathsf{x})}d\mathsf{k} \quad (11)$$

Evaluating and summing the fields $P(\mathsf{x}, t+\Delta t)$ and $P(\mathsf{x}, t-\Delta t)$ according to the solution on Eq. (11):

$$\begin{aligned} &P(\mathsf{x}, t+\Delta t) + P(\mathsf{x}, t-\Delta t) \\ &= \frac{1}{(2\pi)^3}\int_{-\infty}^{\infty} \varphi(\mathsf{k}, t)e^{i(\mathsf{k}\cdot\mathsf{x})}\left[e^{iL(\mathsf{x},\mathsf{k})\Delta t} + e^{-iL(\mathsf{x},\mathsf{k})\Delta t}\right]d\mathsf{k} \end{aligned} \quad (12)$$

or:

$$\begin{aligned} &P(\mathsf{x}, t+\Delta t) + P(\mathsf{x}, t-\Delta t) \\ &= \frac{1}{(2\pi)^3}\int_{-\infty}^{\infty} \varphi(\mathsf{k}, t)\cdot 2\cos\left[L(\mathsf{x},\mathsf{k})\Delta t\right]e^{i(\mathsf{k}\cdot\mathsf{x})}d\mathsf{k} \end{aligned} \quad (13)$$

It can be noticed that for the constant velocity case, Eqs. (13) and (6) are exactly the same solutions for the acoustic wave equation.

It is worth to notice that the pseudo-differential operator in the space domain $L(\mathsf{x}) = v(\mathsf{x})\sqrt{-\nabla^2}$, when represented in the space-wave number domain, has the following form:

$$L(\mathsf{x}, \mathsf{k}) = v(\mathsf{x})\sqrt{k_x^2 + k_y^2 + k_z^2} \quad (14)$$

which is exactly the same pseudo-differential operator derived at Zhang & Zhang (2009).

Moreover, it can be noticed that the time extrapolation is done with the multiplication of the spatial Fourier transform of the field by a cosine function whose argument depends on the wave number and the velocity. This way, the procedure can be interpreted as a phase shift of the wave field applied on the Fourier domain.

The cosine function, in general, can be approximated by a separate two-term series:

$$2\cos[L(\mathsf{x}, \mathsf{k})\Delta t] \approx \sum_{j=0}^{n} a_j(\mathsf{x})b_j(\mathsf{k}) \quad (15)$$

where n is the number of terms, and $a_j(\mathsf{x})$ e $b_j(\mathsf{k})$ are separate real functions depending on x e k, respectively.

This way, Eq. (13) can be rewritten:

$$\begin{aligned} &P(\mathsf{x}, t+\Delta t) + P(\mathsf{x}, t-\Delta t) \\ &\approx \sum_{j=0}^{n} a_j(\mathsf{x})\frac{1}{(2\pi)^3}\int_{-\infty}^{\infty} \varphi(\mathsf{k}, t)b_j(\mathsf{k})e^{i(\mathsf{k}\cdot\mathsf{x})}d\mathsf{k} \end{aligned} \quad (16)$$

In the solution given by Eq. (16), $a_j(\mathsf{x})$, considering the two-dimensional case, represents a matrix that contains the weights used in the linear interpolation. They are calculated based on the velocity model for each reference velocity $(a_j(v(\mathsf{x})))$, which are computed using optimum reference velocities. In order to find these velocities, it was used the procedure proposed by Bagaini et al. (1995) in which the reference velocities $(v_1, v_2, \ldots, v_n)$ are computed using the entropy of the velocity distribution criteria in the $[v_m, v_M]$ interval, where v_m is the minimal velocity of the whole field and v_M the maximum one. The terms b_j are given by $b_j(\mathsf{k}) = \cos(v_j\sqrt{k_x^2 + k_y^2 + k_z^2}\Delta t)$, depending on each reference velocity and on the time marching step.

Finally, the reverse propagation on time can be written as:

$$\begin{aligned} &P(\mathsf{x}, t-\Delta t) = P(\mathsf{x}, t+\Delta t) \\ &+ \sum_{j=0}^{n} a_j(\mathsf{x})\, FT^{-1}\{b_j(\mathsf{k})\, FT\, P(\mathsf{x}, t)\} \end{aligned} \quad (17)$$

From Eq. (17), for each marching step on time, the method requires a fast Fourier transform (FT) and n inverse Fourier transform (FT^{-1}). This way, the computational cost to obtain $P(\mathsf{x}, t-\Delta t)$ is proportional to the number of terms (n) or the amount of reference velocities (v_j's) and to the cost of each Fast Fourier Transform.

Regarding the method stability, a study was done based on time and spatial sampling and on the reference constant velocity. Applying the eigenvectors methodology used in Pestana et al. (2011), the presented method is stable for the 2D case if:

$$v\sqrt{k_x^2 + k_z^2}\Delta t \leq \pi \quad (18)$$

Considering the highest spatial frequencies $k_x = \pi/\Delta x$ and $k_z = \pi/\Delta z$, and the maximum velocity present on the model:

$$\frac{v_{\max}\Delta t}{\Delta x\Delta z}\sqrt{(\Delta x)^2 + (\Delta z)^2} \leq 1 \quad (19)$$

If $\Delta z = \Delta x$:

$$\frac{v_{\max}\Delta t}{\Delta x} \leq \frac{\sqrt{2}}{2} \approx 0.71 \quad (20)$$

The same analysis can be done for the tri-dimensional case, in which the stability condition is given by the following inequation:

$$\frac{v_{\max}\Delta t}{\Delta x} \leq \frac{\sqrt{3}}{3} \approx 0.58 \quad (21)$$

Considering that the regular pseudo-spectral method, using a second-order approximation to the time derivative and computing the spatial derivatives by the Fourier method, according to Kosloff & Baysal (1982), for the 2D case, $\frac{v_{\max}\Delta t}{\Delta x} \leq 0.2$. This way, it can be seen that the presented method can perform the time evolution of the wave field in a stable way and without numerical dispersion using larger time steps than the regular pseudo-spectral method.

RTM BY PSEUDO-ANALYTICAL METHOD

As seen before, numerical methods that uses second-order finite differences schemes to approximate the time derivative, even the ones that use the Fourier method to compute the spatial derivatives, require a small time step in order to minimize such problems as numerical dispersion. There are several solutions to this problem and the pseudo-analytical method is one of them, proposed here to be applied with the reverse time migration.

The pseudo-analytical method, presented by Pestana et al. (2011), consists in a modification on the Laplacian operator and, this way, permitting a stable propagation of the wavefield using larger time steps than the ones possible with the conventional pseudo-spectral method. By the use of pseudo-analytical operators, a better approximation of the time derivative can be obtained. Modifications to the way the Laplacian operator is computed lead to a simplified differential pseudo-operator using a constant reference velocity which is easily calculated in the wave number domain.

From Eq. (6), summing $-2P(\mathsf{x},t)$ to both sides of the equation:

$$\begin{aligned}&P(\mathsf{x},t+\Delta t)-2P(\mathsf{x},t)+P(\mathsf{x},t-\Delta t)\\&\quad=2\big[\cos(L\Delta t)-1\big]\,P(\mathsf{x},t)\end{aligned}\tag{22}$$

Rewriting it in a convenient way, in a similar way to the second-order approximation of the time derivative by finite differences:

$$\begin{aligned}&\frac{P(\mathsf{x},t+\Delta t)-2P(\mathsf{x},t)+P(\mathsf{x},t-\Delta t)}{\Delta t^2}\\&\quad=v^2(\mathsf{x})\frac{2\left[\cos(L\Delta t)-1\right]P(\mathsf{x},t)}{v^2(\mathsf{x})\Delta t^2}\end{aligned}\tag{23}$$

Expanding the cosine function on the right side of the Eq. (23) and considering a constant velocity (v_0), the following pseudo-Laplacian operator can be defined:

$$\begin{aligned}\Gamma(L_0,\Delta t)&=\frac{2[\cos(L_0\Delta t)-1]}{v_0^2\Delta t^2}\\&\approx\frac{2}{v_0^2\Delta t^2}\left[-\frac{(L_0\Delta t)^2}{2}+\frac{(L_0\Delta t)^4}{24}-\ldots\right]\end{aligned}\tag{24}$$

where $L_0^2=-v_0^2\nabla^2$ and $-\nabla^2$ in the Fourier domain is equal to K^2, where $K=\sqrt{k_x^2+k_y^2+k_z^2}$ while k_x, k_y and k_z are the spatial wave numbers.

Now, the operator $\Gamma(L_0,\Delta t)$ can be rewritten according to the following expression:

$$\Gamma(L_0,\Delta t)\approx\nabla^2+\frac{v_0^2\Delta t^2}{12}\nabla^4-\ldots\tag{25}$$

In addition, it can be noticed that the operator $\Gamma(L_0,\Delta t)$ is exactly the same Laplacian operator (∇^2), when the time interval Δt is very small. This way, the constant velocity v_0, that appears on higher orders, is seen as a compensation velocity applied to each term of the pseudo-Laplacian operator.

The pseudo-analytical method introduced by Etgen & Brandsberg-Dahl (2009) is here called the zeroth order approximation and is given by the following equation:

$$\begin{aligned}&P(\mathsf{x},t+\Delta t)-2P(\mathsf{x},t)+P(\mathsf{x},t-\Delta t)\\&\quad=(v(\mathsf{x})\Delta t)^2\,\Gamma(L_0,\Delta t)\,P(\mathsf{x},t)\end{aligned}\tag{26}$$

where $\Gamma(L_0,\Delta t)$ in the Fourier domain is written as:

$$\Gamma(L_0,\Delta t)\Longleftrightarrow\frac{2[\cos(v_0^2\,K^2\Delta t^2)-1]}{v_0^2\Delta t^2}\tag{27}$$

As the $\Gamma(L_0,\Delta t)$ operator varies slightly with respect to v_0, more terms can be used to represent the pseudo-Laplacian operator, this way, improving the approximation of the second derivative in time.

Again applying the Taylor series expansion of the cosine function on Eq. (22), the wavefield propagation equation can be rewritten as:

$$\begin{aligned}&P(\mathsf{x},t+\Delta t)-2P(\mathsf{x},t)+P(\mathsf{x},t-\Delta t)\\&\quad=-(L\Delta t)^2P(\mathsf{x},t)+O(L,\Delta t)P(\mathsf{x},t)\end{aligned}\tag{28}$$

where $O(L,\Delta t)$ represents a higher-order time derivatives operator that, when applied to $P(\mathsf{x},t)$, tends to improve the approximation of the second derivative in time.

Replacing Eq. (6) on Eq. (28):

$$O(L,\Delta t)=2\left[\cos(L\Delta t)-1+\frac{(L\Delta t)^2}{2}\right]\tag{29}$$

Using the operator $O(L,\Delta t)$ obtained on Eq. (29), Eq. (28) is used to derive the expression for the second-order pseudo-analytical method, given by:

$$\begin{aligned}&P(\mathsf{x},t+\Delta t)-2P(\mathsf{x},t)+P(\mathsf{x},t-\Delta t)\\&\quad=-(L\Delta t)^2P(\mathsf{x},t)+(v(\mathsf{x})\Delta t)^4\,\Gamma_2(L_0,\Delta t)\,P(\mathsf{x},t)\end{aligned}\tag{30}$$

where the pseudo-Laplacian second-order operator $\Gamma_2(L_0,\Delta t)$ is given by:

$$\begin{aligned}\Gamma_2(L_0,\Delta t)&=\frac{2}{(v_0\Delta t)^4}\\&\times\left[\cos(L_0\Delta t)-1+\frac{(L_0\Delta t)^2}{2}\right]\end{aligned}\tag{31}$$

It can be noticed that the first term on the right side of Eq. (30) is exactly the same expression for the pseudo-spectral approximation of the second derivative in time. And the second term acts as a correction term.

If the operator $\Gamma_2(L_0, \Delta t)$ is calculated in the Fourier domain, Eq. (30) is written as:

$$\begin{aligned} P(\mathsf{x}, t+\Delta t) - 2P(\mathsf{x}, t) + P(\mathsf{x}, t-\Delta t) \\ = -(v\Delta t)^2 \nabla^2 P(\mathsf{x}, t) \\ +(v(\mathsf{x})\Delta t)^4\, FT^{-1}\{\Gamma_2(K, \Delta t)\, FT\, P(\mathsf{x}, t)\} \end{aligned} \tag{32}$$

With the same technique used in the second-order approximation, higher orders approximations can be derived for the pseudo-analytical method. For the fourth order approximation, the solution for the wave equation has the form:

$$\begin{aligned} P(\mathsf{x}, t+\Delta t) - 2P(\mathsf{x}, t) + P(\mathsf{x}, t-\Delta t) \\ = -(L\Delta t)^2 P(\mathsf{x}, t) + \frac{1}{12}(L\Delta t)^4 P(\mathsf{x}, t) \\ + (v(\mathsf{x})\Delta t)^6\, \Gamma_4(L_0, \Delta t)\, P(\mathsf{x}, t) \end{aligned} \tag{33}$$

where $\Gamma_4(L_0, \Delta t)$ is the pseudo-Laplacian operator given by:

$$\begin{aligned} \Gamma_4(L_0, \Delta t) = \frac{2}{(v_0\Delta t)^6} \\ \times \left[\cos(L_0\Delta t) - 1 + \frac{(L_0\Delta t)^2}{2} - \frac{(L_0\Delta t)^4}{24}\right] \end{aligned} \tag{34}$$

Concerning the method's stability, applying again the eigenvalues method, the stability condition for the zeroth order approximation is given by:

$$\left[1 + \frac{1}{2}(v\Delta t)^2\Gamma(K)\right]^2 \leq 1 \tag{35}$$

If $\frac{1}{2}(v\Delta t)^2\Gamma(K) < 0$, the method is stable. This way:

$$\frac{1}{2}(v\Delta t)^2 F(K) = v^2\,\frac{\cos(v_0\Delta t K) - 1}{v_0^2} < 0 \tag{36}$$

Satisfying the equation (36) implies that $|\cos(\phi)| < 1$ where $\phi = v_0\Delta t\sqrt{k_x^2 + k_y^2 + k_z^2}$. Considering the maximum frequency present in the data:

$$\begin{aligned} \phi &= v_0\Delta t\sqrt{k_x^2 + k_y^2 + k_z^2} \\ &= 2\pi\Delta t f \leq 2\pi\Delta t f_{\max} \end{aligned} \tag{37}$$

This way, the largest sampling interval Δt that still ensures accuracy for the method is related to the maximum frequency presented. Thus, $f_{\max} = \frac{1}{2\Delta t}$, which corresponds to $\phi = \pi$.

For the two-dimensional case, using again the maximum spatial frequencies $k_z = \frac{\pi}{\Delta z}$ and $k_x = \frac{\pi}{\Delta x}$, and still considering $\Delta x = \Delta z$, from equation (37) follows that:

$$\frac{v_0\Delta t}{\Delta x} \leq \frac{\sqrt{2}}{2} \approx 0,71 \tag{38}$$

The method can still be extended to higher order approximations. For the second-order approximation:

$$\frac{1}{2}\big[-(v\Delta t K)^2 + (v\Delta t)^4\Gamma_2(K)\big] < 0 \tag{39}$$

Rewriting the operator Γ_2 as:

$$\Gamma_2(K) = \frac{\Gamma(K) + K^2}{(v_0\Delta t)^2} \tag{40}$$

and replacing it on equation (39):

$$\frac{(v\Delta t K)^2}{2}\left[\left(\frac{v^2}{v_0^2} - 1\right) + \frac{v^2\Gamma(K)}{v_0^2 K^2}\right] < 0 \tag{41}$$

Considering the approximation

$$\Gamma(K) = -K^2 + \frac{v_0^2\Delta t^2}{12}K^4,$$

and doing the same considerations about the spatial frequencies k_z and k_x and for $\Delta x = \Delta z$, the stability condition is achieved to the second-order approximation:

$$\frac{v_0\Delta t}{\Delta x} \leq \frac{\sqrt{6}}{\pi} \approx 0,78 \tag{42}$$

RESULTS

In order to test the applicability of the proposed methods, two-dimensional synthetic datasets were migrated, as well as a real non stacked data.

To perform the post-stack migration, it was used the SEG-EAGE salt dome model, whose velocity model is shown on Figure 1 and has velocities varying from 1524 m/s to 4481 m/s. The grid spacing in x and z directions are both 12.19 m. The migrated image, in this case, is obtained through the reverse propagation in time of the seismic data recorded along the surface, $z = 0$, from the final sample up to $t = 0$.

Figure 2 shows the result for the RTM by interpolation with 15 velocities and Figure 3 the migration by the pseudo-analytical method using the fourth order approximation for the pseudo-Laplacian operator with $v_0 = v_{\min}$. Both results were obtained using the sample interval of the seismic data, Δt =0,004 s.

The pre-stack migration was tested with the Marmousi model, the image condition used here is the cross correlation of the propagated wavefield from the sources and the receivers. The final

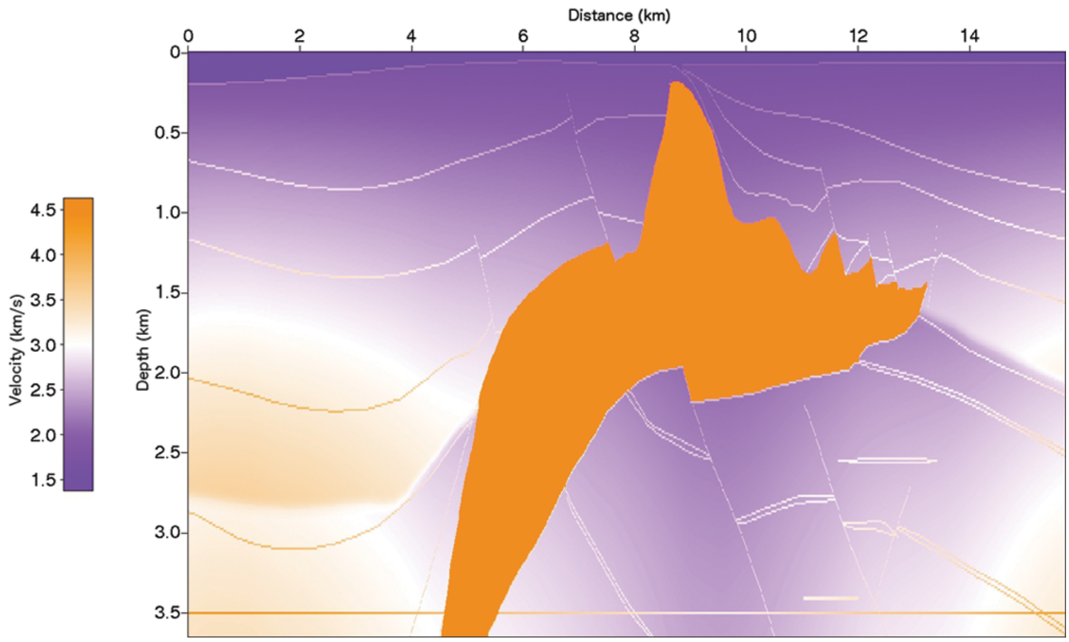

Figure 1 – SEG/EAGE salt model velocity field.

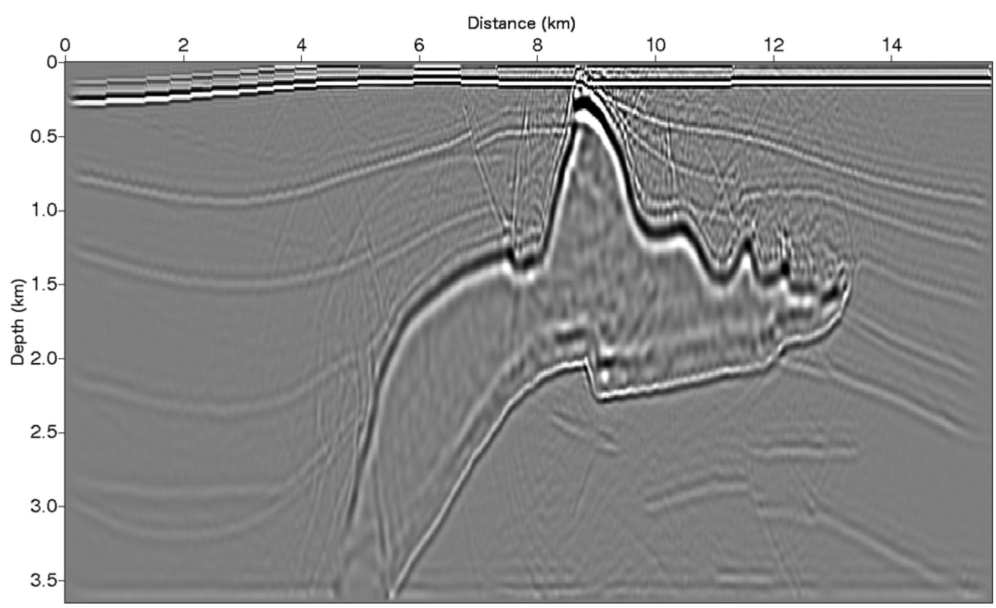

Figure 2 – Section from SEG/EAGE salt model migrated using the interpolation method with 15 velocities.

result is given by the sum of all migrated common source gathers. The velocity field with velocities varying from 1500 m/s to 5500 m/s is shown on Figure 4. The velocity field grid has 369 points in the horizontal direction (x) and 375 on the vertical one (z) and the spacings are $\Delta x = 25$ m and $\Delta z = 8$ m.

The migrated section using the interpolation method is shown on Figure 5, while the one using the pseudo-analytical method with a second-order approximation for the pseudo-Laplacian operator appears on Figure 6. These results were obtained migrating 240 common source sections spaced by 25 m, where each common source gather is formed by 96 channels also spaced by 25 m.

At last, a 2D real data was migrated. It was acquired in the central regions of the Gulf of Mexico, near the Mississippi canyon,

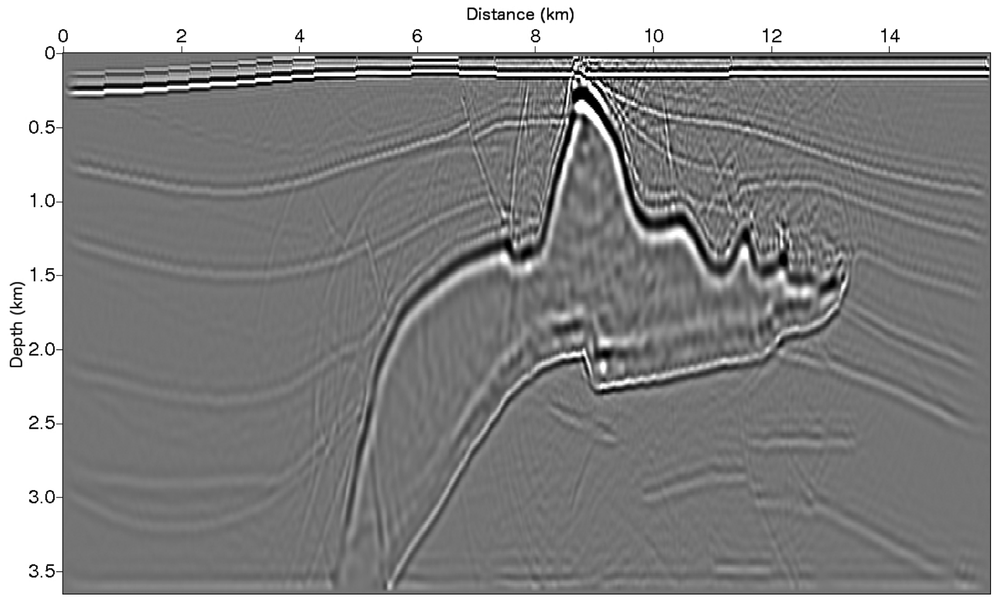

Figure 3 – Section from SEG/EAGE salt model migrated using the pseudo-analytical method with fourth order approximation for the pseudo-Laplacian operator.

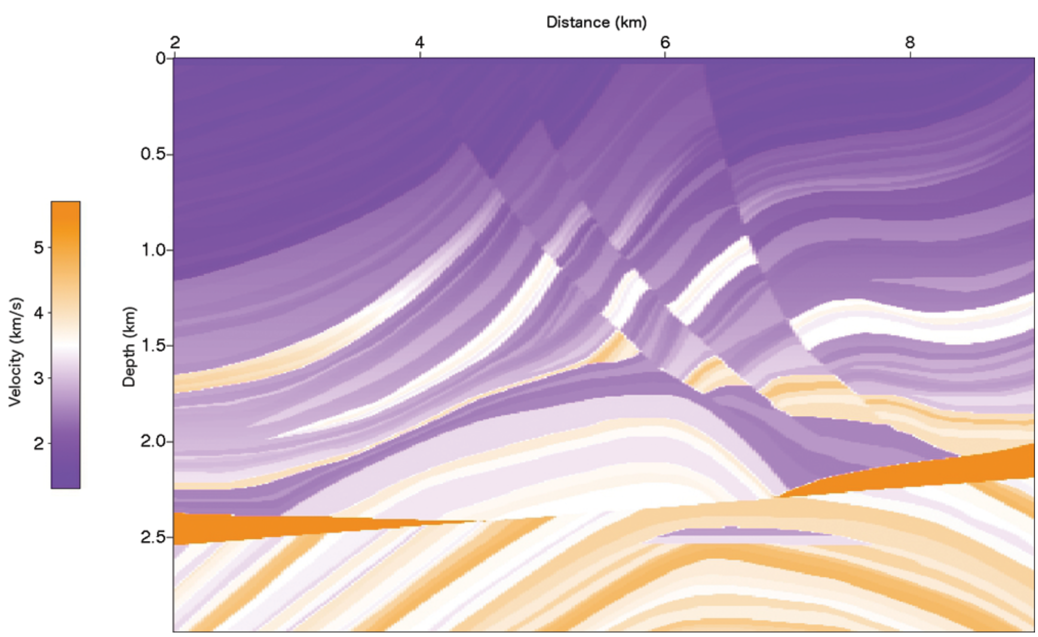

Figure 4 – Marmousi model velocity field.

one of world's most productive regions for oil and gas and where, according to Chowdhury & Borton (2007), the hydrocarbon trapping is strongly related to the presence of salt.

Data acquisition was made using the end-on technique with the receiver line having 180 receivers spaced by 26.67 m. 1001 shots were registered, also spaced by 26.67 m. Each trace has 6.0 s with a time sampling of 4 ms, which gives a total of 1501 samples in each seismic trace. The velocity field grid, on Figure 7, has velocities varying from 1485 m/s to 4000 m/s, having 1000 points in the horizontal direction (x) and 1185 on the vertical one (z), with horizontal sampling interval of 26.67 m and vertical sampling interval of 13.21 m.

The result from the RTM by interpolation applied with 5 velocities can be seen at Figure 8 while Figure 9 shows the result of the pseudo-analytical method with a second-order approximation for the pseudo-Laplacian operator.

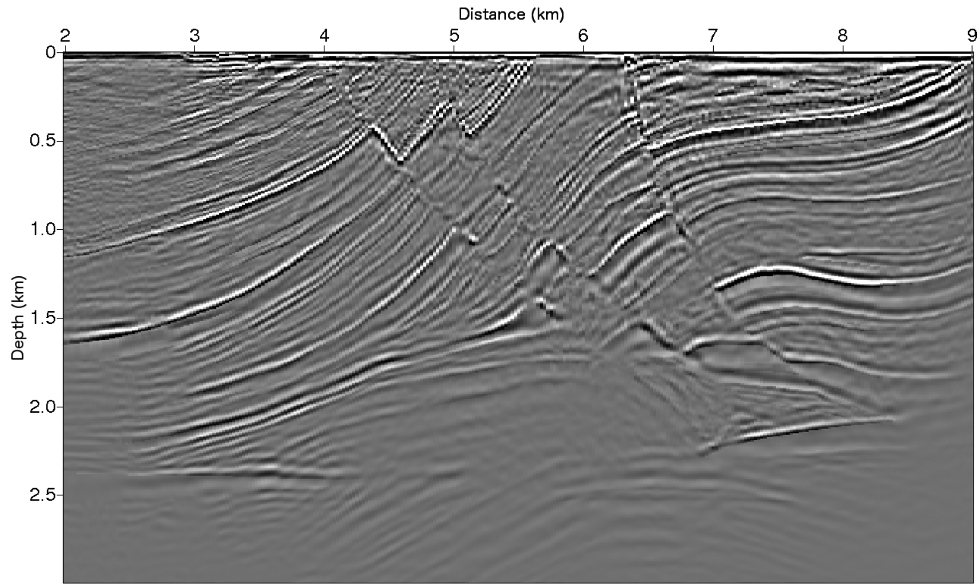

Figure 5 – Section from Marmousi model migrated using the interpolation method with 10 velocities.

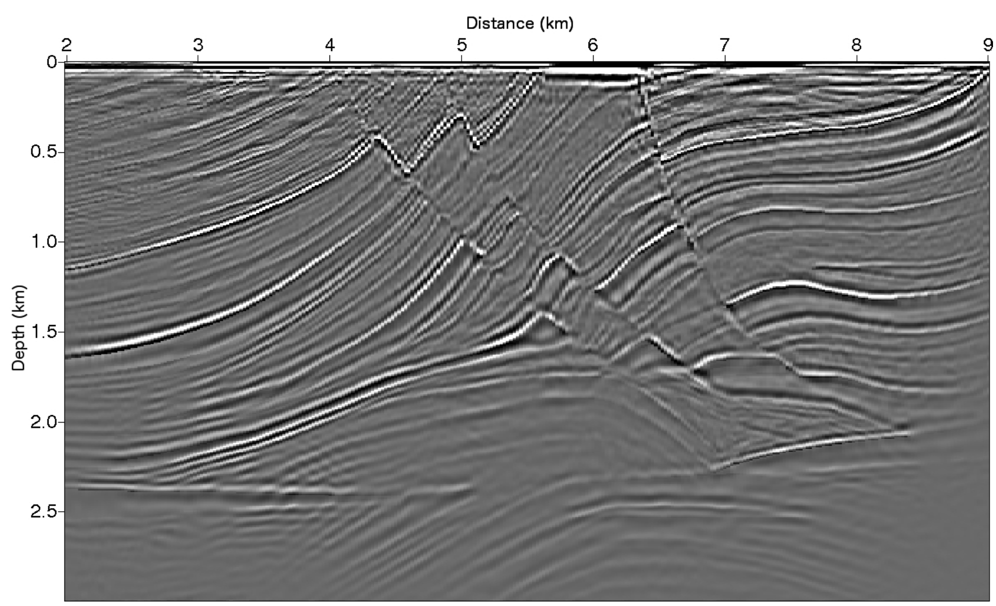

Figure 6 – Section from Marmousi model migrated using the pseudo-analytical method with second-order approximation for the pseudo-Laplacian operator.

To all the migrated images presented, a Laplacian filter has been applied in order to attenuate low frequency events that appear due to the cross correlation image condition.

CONCLUSIONS

The presented methods were both successful migrating the synthetic and real data. Application on synthetic data is very interesting because the geological model is known accurately and, consequently, that also applies to the velocity model. Applied on the SEG/EAGE salt model and the Marmousi data, both interpolation and pseudo-analytical methods were able to reproduce the synthetic model precisely.

The SEG/EAGE salt model is characterized by strong vertical and lateral velocity variations. For this reason, the migration by interpolation required a larger number of velocities to achieve a satisfactory result. This fact demanded a greater computational expense compromising the efficiency of the method. However, in the Marmousi model, characterized by geological com-

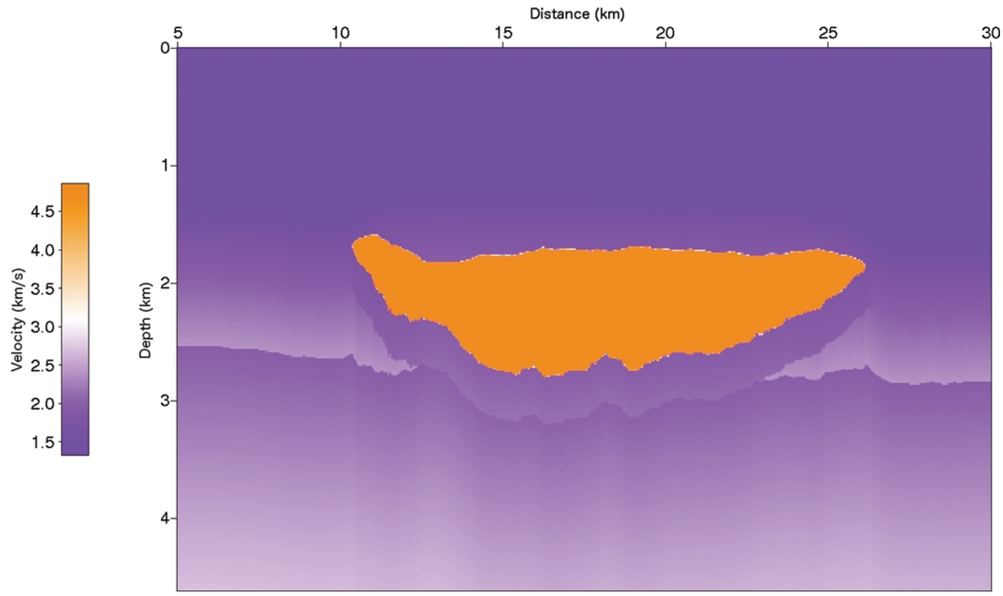

Figure 7 – Gulf of Mexico data velocity field.

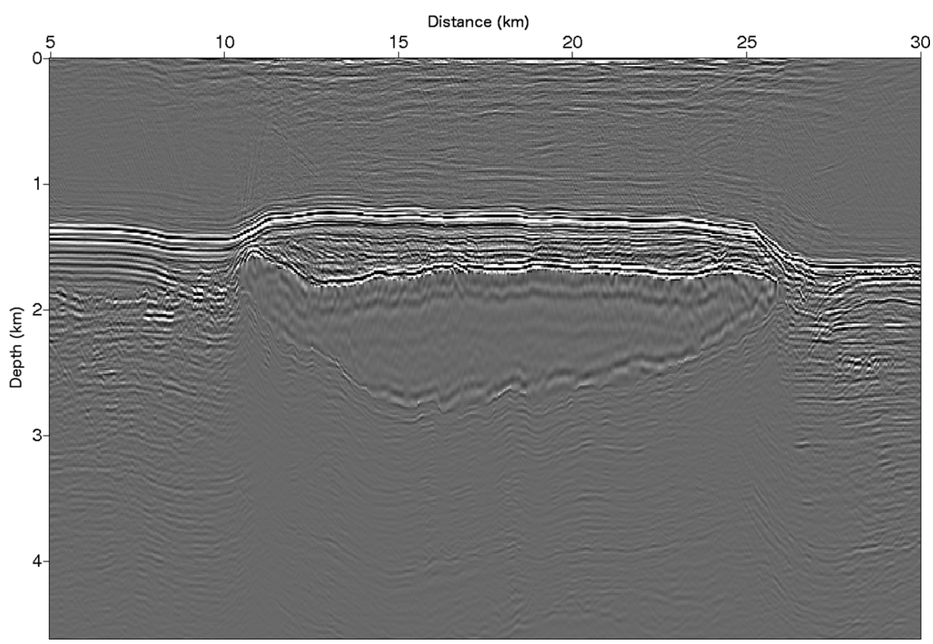

Figure 8 – Section from the Gulf of Mexico data migrated using the interpolation method with 5 velocities.

plexity, interpolation could be applied accurately using less velocities, which also demanded less computational effort. The pseudo-analytical method was also successful imaging both models and is highly recommended because it has a lower computational cost.

Alternatives to the present methods may include the use of Chebyshev polynomials instead of Taylor series for the approximations of the pseudo-Laplacian operator in the case of the pseudo-analytical method. Regarding the interpolation, different ways to distribute the velocities across the model are already present in the literature and can be tested.

Finally, the real Gulf of Mexico data migration proved the applicability of the present methods in two-dimensional data. Application on 3D data should still be implemented and tested.

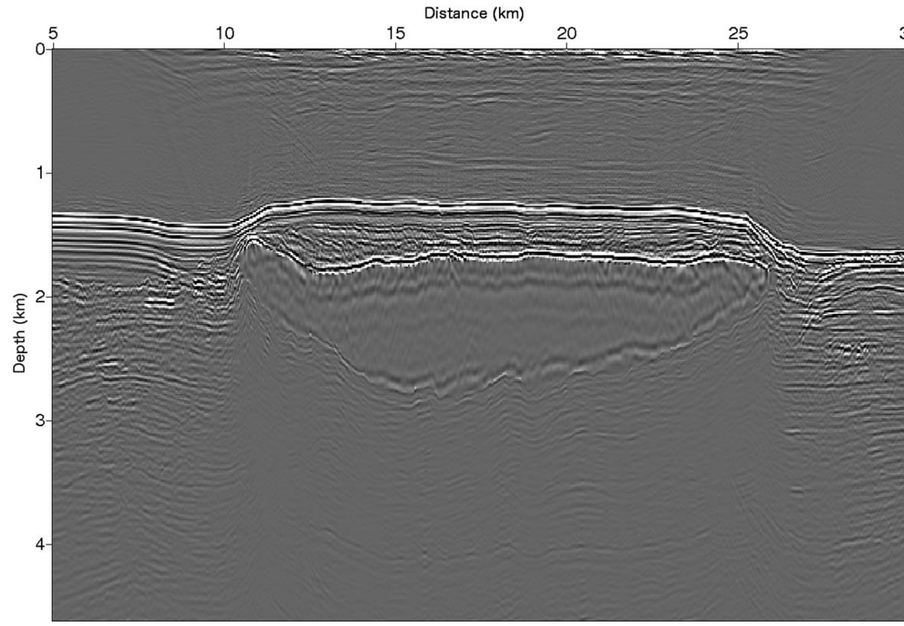

Figure 9 – Section from the Gulf of Mexico data migrated using the pseudo-analytical method with second-order approximation for the pseudo-Laplacian operator.

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