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FEATURES OF PHASE SPECTRUM AND ITS CALCULATION IN SEISMIC DATA PROCESSING

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ABSTRACT. Using simple examples we show the main features of the phase spectra, which have to be taken into account in seismic data processing. In this case the peculiarities of phase spectra processing in terms of probabilistic and statistical characteristics are considered. Based on previous theoretical results and using the results of the analysis of the spectra calculated for the real seismic signals, there are formulated simple criteria to ensure high performance and stability of the statistical procedures for the phase spectra processing. It allowed us to construct a simple procedure for continuous extension of the phase spectra of signals observed on real traces, and thereby ensure the uniqueness of their determination. The uniqueness is important for the joint processing and analysis of large sets of phase spectra, calculated from the observed seismic signals. Such obtained spectra can be effectively used in various practical problems. Thus, based on the analysis of synthetic and real data, we show that using the phase spectrum can reduce the zone of uncertainty in determining the pinch-out points of a horizon.

Keywords: phase spectrum, seismic data, thin-layer reservoir, pinch-out zone.

RESUMO. Usando exemplos simples, mostramos as principais características dos espectros de fase, que devem ser consideradas no processamento de dados sísmicos. Neste caso, as peculiaridades do processamento de espectros de fase em termos de características probabilísticas e estatísticas são consideradas. Com base em resultados teóricos anteriores, e utilizando os resultados da análise dos espectros calculados para os sinais sísmicos reais, são formulados critérios simples para garantir alto desempenho e estabilidade dos procedimentos estatísticos para o processamento de espectros de fase. O que permitiu-nos construir um procedimento simples para a expansão contínua do espectro de fase dos sinais observados em traços reais e, assim, garantir a unicidade de sua determinação. A unicidade é importante para o processamento e a análise em comum de grandes conjuntos de espectros de fase, calculados a partir dos sinais sísmicos observados. Tais espectros obtidos podem ser usados com eficácia para resolver vários problemas práticos. Assim, com base na análise dos dados sintéticos e reais, é mostrado que o uso do espectro de fase pode reduzir a zona de incerteza na determinação dos pontos de acunhamento de um horizonte.

Palavras-chave: espectro de fase, dados sísmicos, reservatório delgado, acunhamento.

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INTRODUCTION

The phase component of the spectrum or phase spectrum of short-time interval of seismic trace is an extremely important characteristic in the analysis of seismic signals related to the local objects. Examples of such signals are reflections of different types, in particular, PP and PS waves associated with thin-layer target horizons. Unfortunately, the phase spectrum is given little attention in practical seismic exploration, and the number of publications on the subject is very limited. At the same time, the phase spectrum contains even more information than the amplitude component, which is usually used in the dynamic analysis of real seismic data. In this case, as a rule, it is not considered that the amplitude spectrum contains, basically, information about the integral or energy characteristics of the signal, while the phase spectrum contains information about its differential characteristics. Orientation to analysis of only the amplitude spectrum of the seismic signal leads to the fact that during the processing of real data there is a loss of important information about the structure of the phase spectrum of target signal. As a result of its reduction to the minimum-phase and even to the zero-phase spectrum information about the phase spectrum of the observed signal are lost or significantly distorted.

The current interest to the phase spectra of seismic signals may be connected with several reasons: first, some traditionalism in focus of the real data processing on the arrival times of observed signals. This unification of the form of signals (in particular, by their reduction to the zero-phase spectrum) guarantees improving their selection and correlation on the original seismograms. Secondly, such reasons are justified if to consider the original seismic trace as a realization of a random process. Similar assumptions are often used in the development of certain procedures, for example, the optimal Wiener filtering. Third, the processing of large sets of source signals (taking into account changes of the phase components) is much more complex than processing, based on the above assumptions.

Interest in the phase spectrum can be renewed for dynamic analysis of complex wave fields in connection with the solution of inverse dynamic problems for thin-layer objects. We only have to understand the limitations of the above assumptions (zero-phase, minimum-phase), which allowed us to exclude the phase component of the spectrum in the traditional real data processing. Thus, it is well-known that even for a single thin-layer we cannot completely restore the elastic parameters using only the amplitude spectrum of the reflected signal, see Berzon (1965) and Khudzinskii (1966). Understandings about the importance of phase spectra in the study of many technical processes have manifested in actively developing modern methods of spectral analysis. In particular, the absence of capacity to analyze this component in the local spectrum with wavelet analysis of temporal processes favored to the creation of S-transformation. A key feature of this transformation is the possibility of combining a frequency dependent resolution in time-space domain to obtain the local phase information, see Stockwell et al. (1996).

A low noise stability of the estimates obtained by real seismic traces is one of the main difficulties in the processing and interpretation of the phase spectra. The traditional ways of increasing stability of the estimates through the application of accumulative procedures, as well as other procedures for joint analysis of the phase spectra, make a demand on the unique determination of this component (such a requirement is completely absent when considering the amplitude spectra).

A simple model experiment, whose results are presented in Figures 1 and 2, illustrates the importance of the accumulation procedures in the unique determination of the phase spectra. This experiment used a simple pulse in the form of decaying sinusoids, see Figure 1(a). Its spectral components (amplitude and phase) are presented in Figure 1(b). In this experiment the phase component was determined in the interval $[-\pi, \pi]$ that is sufficient to perform a single-valued inverse Fourier transform. This definition of the phase spectrum is also sufficient to carry out the averaging of the spectral characteristics and to obtain a stable estimate, as a form of the initial pulse and its two spectral components. It requires the invariability of arrival times for the considered signals, or the exact values of the corresponding times. In case of violation of the requirement (it is a typical phenomenon for a real seismic experiment) the situation changes dramatically. Thus, when the difference in arrival times of signals is 10 ms, which is less than 1/3 of the period of this signal, we obtain a significant distortion when trying to construct estimates of the waveform by averaging the spectral components in the phase determined on the interval $[-\pi, \pi]$. The corresponding result is illustrated in the subsequent parts of Figure 1, which presents several methods of estimating the original signal using only the two observations. At the same time, Figure 1(c) shows the result of summation of two signals without taking into account the changing the time of their arrival. We see that in this case there is some distortion of the signal with changing (decreasing) its apparent frequency. Such a distortion of the signal is a well-known fact, which demanded the development of correlation methods for the observed seismic signals and adjusting their arrival times at the implementation of accumulation procedures for different purposes (estimating the velocity spectrum, constructing the stacked seismic sections, for example). Implementation of the procedure and the correlation of accounting changes

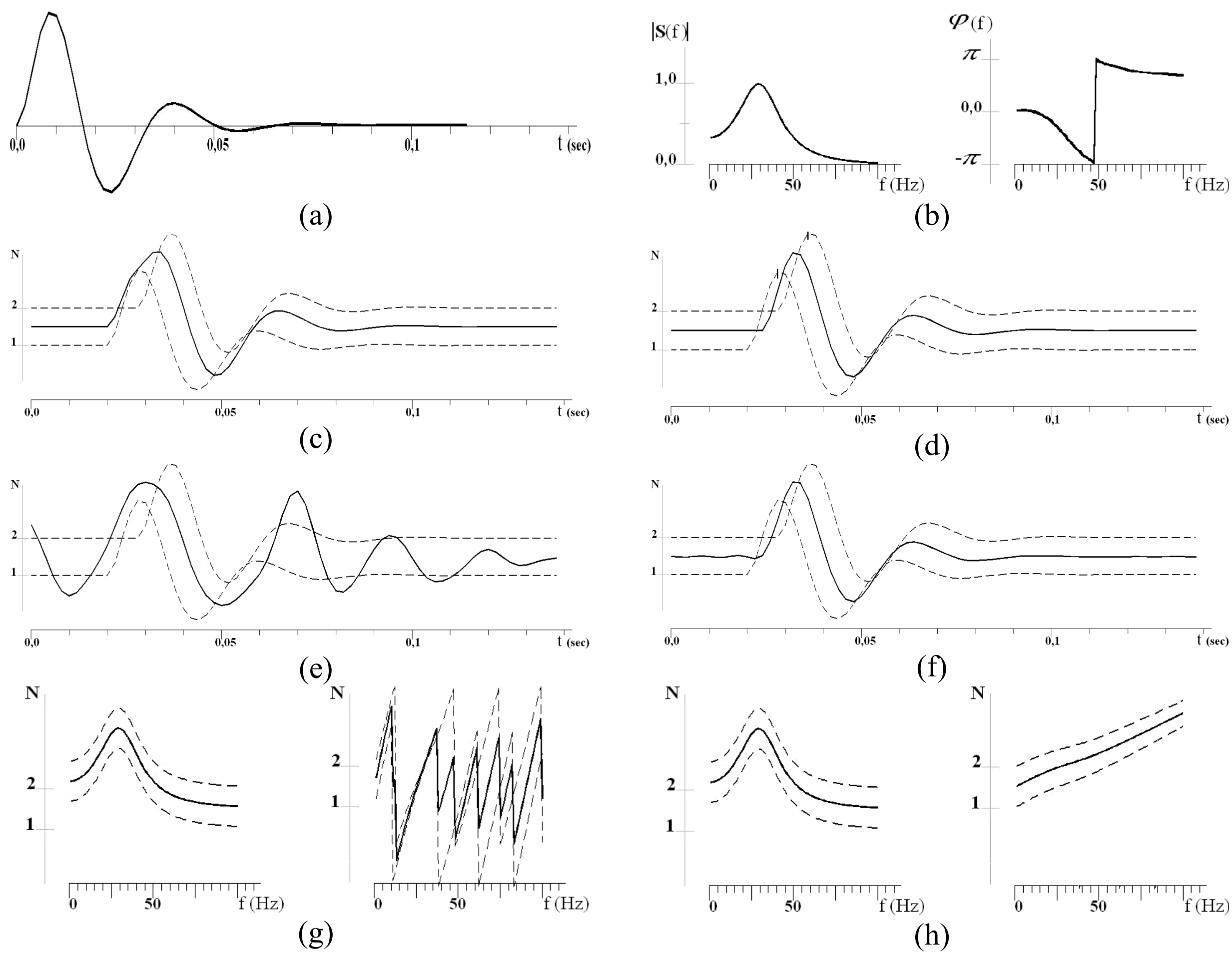

Figure 1 – Determination of the original signal form and its spectrum on the basis of two observed signals using different methods (explanation in text).

of arrival times of signals permit accurately to reconstruct the shape of the initial pulse in the summation of the observed pulses in the time domain and provide its location on the average time of arrival of the two considered signals (see Fig. 1(d)). However, the summation of the spectra of these signals in determining the phase component in the range provides a very significant distortion of the signal, which is much higher than what we had in the absence of correlation procedures (see Fig. 1(e)). At the same time using the continuation of the phase spectrum provides us a high accuracy in determining the shape of the initial pulse and its location in the time domain, see Figure 1(f).

The above situation and the problem remain unchanged with an increase in the number of accumulated signals and the possible reduction of the time shifts between neighboring signals, see Figure 2. Here we use the same type of signal, but for the accumulation there are taken 12 signals, which represent some part of the seismogram. The minimum delay between the arrival time of signals was 1 ms, the maximum time shift reached 20 ms. It is evident that if we ignore these shifts in the summation of signals in time domain, then we get above the distortion of the signal shape (see Fig. 2(a)), which is completely eliminated by correlation and accounting shifts of signals (see Fig. 2(b)). We also have a strong distortion of the signal in the summation of spectral components in the case of determining the phase component in the interval $[-\pi, \pi]$ (see Fig. 2(c)), and its continuation in the interval $(-\infty, \infty)$ permits us to calculate the shape of the signal with high accuracy. Obtained for these two cases, evaluation of amplitude and phase spectra are shown in red, see Figures 2(e) and 2(f). The latter figures show that the phase component of the spectrum can be measured not less stable than the amplitude component in the case of its uniqueness in the interval $(-\infty, \infty)$. It should be noted that ensuring the uniqueness of the determination of the phase component for each of the traces allows us, even in complicated models of wave field related to the multiplicative factor models, to obtain consistent estimates of unknown parameters, see Goldin (1976).

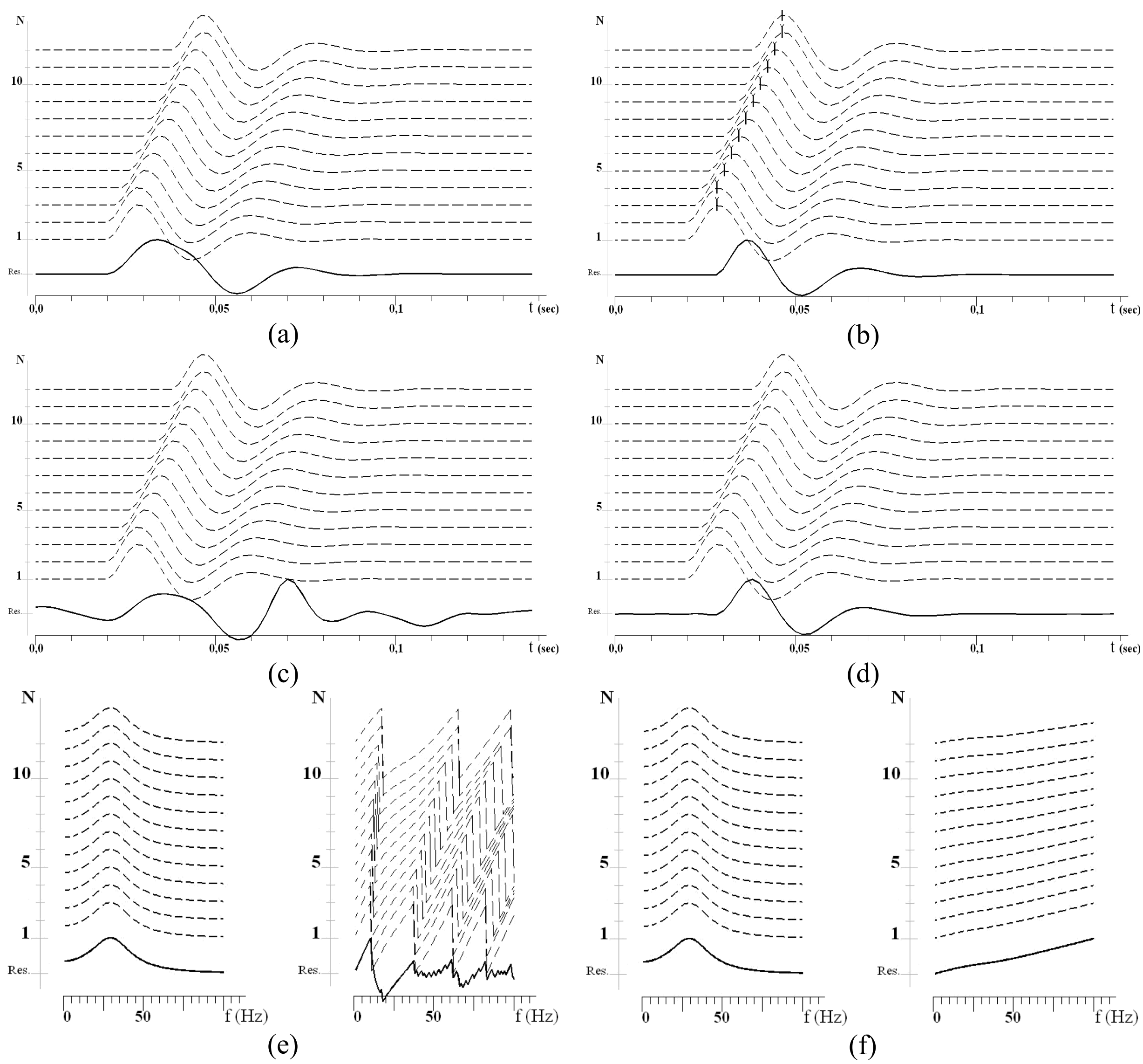

Figure 2 – Determination of the original signal form and its spectrum on the basis of twelve observed signals using different methods (explanation in text).

UNIQUENESS OF THE PHASE SPECTRA IDENTIFICATION USING THEIR STATISTICAL CHARACTERISTICS

Calculation of phase spectra of the real data

Before turning to the peculiarities of the phase spectra let us determine how they are calculated. Apparently, the calculation procedure of the phase spectrum using Fourier's transform presents no difficulty, see Titchmarsh (1948). But it is important to note that such studies were done for relatively short-time intervals of the traces. Often the time interval can be less than 100 milliseconds, and the number of samples does not exceed 100. Usually, in the selection of appropriate intervals ones use a rectangular window. However, it has been noticed that the estimate of the signal strongly depends on the width and shape of the applicable window, see Jenkins & Watts (1969). Using special windows can significantly improve the stability and the statistical significance of spectral estimates of the energy characteristics of random processes, see Hennan (1970). Selection of the seismic recording interval is similar as using some window in time domain. In frequency domain, it is equivalent to smoothing the signal and random noise spectra. Assume that we are able to use the best window in the selection of recording interval for subsequent spectral analysis. Then by the spectral analysis we shall understand the computation and analysis of individual spectra of discrete time intervals or parts of traces t_1^j, t_2^j, where j specifies the number of selected interval of the trace, and t_1^j, t_2^j determine, respectively, the initial and end points of the selected time interval. This approach allows us to separate a complex wave field on individual components, often differ in their spectral composition, and for each discrete given j-th time interval of the trace to write down the following model:

$$y(t) = s(t) + \zeta(t), \quad t \in \left[t_1^j, t_2^j\right] \tag{1}$$

Here $s(t)$ is a useful signal, whose spectrum is necessary to be calculated with highest accuracy, and $\zeta(t)$ is a part of the realization of a stationary random process with correlation function $2\sigma^2 R(\tau)$. Note that when considering the real seismic traces, the assumption of stationarity of the process with respect to $\zeta(t)$ can be satisfied only for relatively small time intervals. Under the useful signal, depending on the problem, there are understood separate seismic pulses or the whole set of such impulses.

Using arbitrary types of windows in the selection of the interval for the spectral analysis, we can consider the input signal to the discrete Fourier transform in the following form:

$$\hat{s}(t) = g(t)\cdot y(t) = g(t)\cdot[s(t)+\zeta(t)], \quad t \in [0, T], \tag{2}$$

where $g(t)$ is the window function, which is different from zero at $t \in \left[t_1^j, t_2^j\right]$ and identically equal to 0 for all other values of time variable t. It follows from Equation (2) that the selected window should minimally distort the shape of the signal component and suppress as much as possible the disturbance. It is easy to write the discrete analog for this presentation, which is used in spectral analysis, see Mitrofanov (1979) for details.

Calculating the discrete spectrum of discrete signal given by Equation (2) allows us to construct estimates of the spectrum signal component at a fixed set of frequencies $\omega_l = l\Delta\omega$, $l = 1, \ldots, L$, where $\Delta\omega = 2\pi\Delta f$ is a step in frequency, which is defined in hertz (Hz) and can be chosen either randomly or in accordance with the duration of a given signal. The corresponding estimate of the spectrum component (or spectrum of the observed seismic signal) is a complex quantity. In that case the spectrum of the signal at a fixed frequency ω_l will be denoted by $\hat{S}_l$ and, by definition, this value can be regarded as the value of the Fourier transform constructed for the discrete analogue of Equation (2). In accordance with this representation, the imaginary and the real components of the calculated spectrum will always have the sum of the smoothed spectrum of the signal and noise. So, $\hat{S}_l, l = 1, 2, \ldots, L$, are random in nature and can be represented by using probabilistic characteristics, and the corresponding probabilistic characteristics will be different when considering different spectral components. It is important to always keep in mind such characteristics during the spectral analysis of real data.

Due to limited time-window function $g(t)$, we obtain a rather important property, which applies to all spectra obtained in accordance with Equation (2). They will be analytic in the corresponding frequency band, see Titchmarsh (1948). In particular, this applies to the values of the spectrum as well as to the values of the smoothed spectrum of the true signal $\tilde{S}_l$, and to the values of spectrum of the noise $\tilde{\zeta}_l$.

Based on the calculated values of $\hat{S}_l$ we can always define the value of the phase spectrum of the seismic signal at a fixed frequency ω_l, as

$$\begin{aligned} \hat{\phi}_l &= \operatorname{arctg}\left(-\frac{\operatorname{Im}\hat{S}_l}{\operatorname{Re}\hat{S}_l}\right) \\ &= \operatorname{arctg}\left(-\frac{\operatorname{Im}\hat{S}_l + \operatorname{Im}\tilde{\zeta}_l}{\operatorname{Re}\hat{S}_l + \operatorname{Re}\tilde{\zeta}_l}\right), \end{aligned} \tag{3}$$

which allows us to determine the value $\hat{\phi}_l$ in the interval $[-\pi/2, \pi/2]$. Here $\operatorname{Re}\tilde{S}_l$, $\operatorname{Im}\tilde{S}_l$ are the real and imaginary components of the true signal spectrum, respectively, and $\operatorname{Re}\tilde{\zeta}_l$, $\operatorname{Im}\tilde{\zeta}_l$ are the corresponding components of the spectrum of the noise. Since $\hat{\phi}_l$ is the angle of the radius-vector of point $\hat{S}_l$ with coordinates $\operatorname{Re}\tilde{S}_l$, $\operatorname{Im}\tilde{S}_l$, then analyzing the signs of coordinates, we can simply extend the definition of value $\hat{\phi}_l$ in the interval $[-\pi, \pi]$. This extended value of $\hat{\phi}_l$ will exactly be regarded as a calculated value of the phase spectrum of the real data. In this case, for the values of the phase spectrum related to the smoothed signal component there a notation $\tilde{\phi}_l$ will be used, but for brevity, in some cases the notation ϕ_l will be used for general representation of the phase spectrum component at a fixed frequency.

Discontinuity of the phase spectrum component

It was shown in the Introduction that a solution of the problem of unique determination of the phase spectrum component in the interval $(-\infty, \infty)$ is very important for joint analysis of the sets of phase spectra when the arrival times are not precisely known. In order to solve this problem we have to consider an important property of the phase spectrum component as piecewise continuity of the given function, see Mitrofanov (1986). At the same time two types of discontinuities may exist. *The first type of discontinuity* introduces a change in the phase spectrum on the value $\pm 2\pi$, and *the second type of discontinuity* leads to a change in the phase spectrum on the value $\pm\pi$. The nature of these types of discontinuities is quite different. For example, if the first type of discontinuities is associated with the transition to a new sheet of the Riemann surface for the values of the complex spectrum, the second type of discontinuity characterizes the behavior of the spectrum at a point where its value is zero. It is well-known that the information about the behavior of analytic functions at points of zero is greatly important. Therefore, the fact that this information is contained in the phase spectrum, rather than in amplitude, once again emphasizes on the importance of using the phase characteristics in the spectral analysis of seismic signals.

For a better understanding of the phase spectra of real seismic signals, the origin of these types of discontinuities will be analyzed in major details. Following Mitrofanov (1986) and using the above notations, we consider the behavior of the difference $\tilde{\phi}_{l+1} - \tilde{\phi}_l$, calculated for a given discrete signal $\tilde{s}(t)$ and obtained by "cutting" it out of the observed trace using the window $g(t)$. We have already noted that the spectrum $\tilde{S}_l$, calculated for this signal, is an analytic function. It is easy to specify the maximum width of windows, for which the amplitude spectrum $|\tilde{S}_l|$ is a continuous function of the frequency or the parameter $\omega_l = l\Delta\omega,\ \Delta\omega = \omega_{l+1} - \omega_l$, in the processed frequency band, see Khurgin & Yakovlev (1971). It is also easy to show that $\tilde{\phi}_{l+1} - \tilde{\phi}_l$ has the following properties:

$$\lim_{\Delta\omega\to 0}\left(\tilde{\varphi}_{l+1} - \tilde{\varphi}_l\right) = \begin{cases} \pm 2\pi \\ 0 \\ \pm\pi \end{cases} \tag{4}$$

The first two equalities in Equation (4) are obvious. They follow the analyticity of $\tilde{S}(\omega)$ in the corresponding frequency band, which gives continuity of $\tilde{\phi}(\omega)$ under the condition $|\tilde{S}(\omega)| \neq 0$ for any $\omega \in (\omega_{l+1}, \omega_l)$, and the definition of $\tilde{\phi}(\omega)$ in the interval $[-\pi, \pi]$, but not in the interval $(-\infty, \infty)$.

To prove the validity of the last equalities in Equation (4) in the interval (ω_{l+1}, ω_l) let us represent the imaginary and real components of the spectrum in the form of polynomials of a finite order: $\mathrm{Re}\,\tilde{S}(\omega) = P_n(\omega)$, $\mathrm{Im}\,\tilde{S}(\omega) = Q_m(\omega)$. Assume that at a point $\omega^0 \in (\omega_{l+1}, \omega_l)$ the polynomials $P_n(\omega)$, $Q_m(\omega)$ have the following representations

$$P_n(\omega) = (\omega - \omega^0)^{n_0} \cdot P^*(\omega), \quad n_0 \geq 1$$
$$Q_n(\omega) = (\omega - \omega^0)^{n_0} \cdot Q^*(\omega), \quad m_0 \geq 1$$

with $P^*(\omega^0) \neq 0,\ Q^*(\omega^0) \neq 0$. Due to this reason there is a valid equality which is given below:

$$\lim_{\omega\to\omega^0\pm 0} \operatorname{arctg}\left(-\frac{\mathrm{Im}\,\tilde{S}(\omega)}{\mathrm{Re}\,\tilde{S}(\omega)}\right)$$
$$= \begin{cases} \operatorname{arctg}(\pm\infty) = \pm\pi/2 + k\pi, & n_0 > m_0 \\ \operatorname{arctg}(-\alpha) + k\pi & n_0 = m_0 \\ \operatorname{arctg}(\pm 0) = 0 + k\pi, & n_0 < m_0 \end{cases}$$

where k is an integer and α is a real number; it proofs equality (4).

Thus, the function $\tilde{\phi}(\omega)$ can only have two types of discontinuity, which correspond to the first and third equalities in Equation (4). If the first type is associated with the definition of $\tilde{\phi}(\omega)$ in the interval $[-\pi, \pi]$, the second type characterizes the behavior of the radius-vector $\tilde{S}(\omega)$ in the vicinity of the discontinuity point, i.e., the second type has an useful information about the structure of the seismic signal, which can be used in the sequel in the dynamic interpretation. Discontinuities of the first kind can be eliminated by supplementing the definition of $\tilde{\phi}_l$ in the interval $(-\infty, \infty)$, or by using explicit expressions for the derivatives of $\tilde{\phi}(\omega)$ at corresponding points. Discontinuities of the second type are unremovable, which entails uncertainty of the infinite type in calculating derivatives of $\tilde{\phi}(\omega)$ at corresponding points.

It is followed from the above proof of (4) that the continuity of the phase component of the phase spectrum under of the seismic signal depends not only on the kind of used window, determining the band of analyticity of $\tilde{S}(\omega)$, but also on the ratio of orders of zeros of its real and imaginary components.

To understand the nature of discontinuities of the second type, as well as for the subsequent development of algorithms, the following statement is important, firstly formulated in Mitrofanov (1986): *if* $\min\{n^0, m^0\}$ *is an odd number, then the function* $\tilde{\phi}(\omega)$ *has the second type of discontinuity at the point* ω^0.

To prove the sufficiency of this statement, let us consider a pair $\{S_\omega^-, S_\omega^+\}$, where

$$S_\omega^\pm = \tilde{S}(\omega_0 \pm \omega), \quad \omega > 0.$$

In consideration of the orthogonality of the real and imaginary components of the spectrum, we can write the cosine of the angle between vectors S_ω^-, S_ω^+, see Gelfand (1989):

$$\cos\left(S_\omega^- \wedge S_\omega^+\right) = \frac{\mathrm{Re}\,\tilde{S}(\omega^0 - \omega) \cdot \mathrm{Re}\,\tilde{S}(\omega^0 + \omega) + \mathrm{Im}\,\tilde{S}(\omega^0 - \omega) \cdot \mathrm{Im}\,\tilde{S}(\omega^0 + \omega)}{|\tilde{S}(\omega^0 - \omega)| \cdot |\tilde{S}(\omega^0 + \omega)|} \tag{5}$$

In the conditions formulated above and analyticity of $\tilde{S}$, we have a monotonic convergence $|\tilde{S}(\omega^0 - \omega)|$ to $|\mathrm{Re}\,\tilde{S}(\omega^0)|$ or $|\mathrm{Im}\,\tilde{S}(\omega^0)|$ when $\omega \to 0$, depending on which of the orders of zeros is smaller. A similar convergence holds for $|\tilde{S}(\omega^0 + \omega)|$. Hence considering the oddness of the order of zero, we find that $\cos(S_\omega^- \wedge S_\omega^+) \to -1$ when $\omega \to 0$. Consequently, the angle between S_ω^-, S_ω^+ tends to π, and hence, the phase component has a jump when passing through the point ω^0.

Under the same orders of zeros in $\operatorname{Re}\tilde{S}(\omega)$, $\operatorname{Im}\tilde{S}(\omega)$, it is enough to transform Equation (5) into

$$\cos(S_\omega^- \wedge S_\omega^+) = \frac{\operatorname{Re}\tilde{S}(\omega^0-\omega)\cdot\operatorname{Re}\tilde{S}(\omega^0+\omega)}{|\operatorname{Re}\tilde{S}(\omega^0-\omega)|\cdot|\operatorname{Re}\tilde{S}(\omega^0+\omega)|}$$
$$\times\left(1+\frac{\operatorname{Im}\tilde{S}(\omega^0-\omega)\cdot\operatorname{Im}\tilde{S}(\omega^0+\omega)}{\operatorname{Re}\tilde{S}(\omega^0-\omega)\cdot\operatorname{Re}\tilde{S}(\omega^0+\omega)}\right)$$
$$\times\left(\left(1+\left(\frac{\operatorname{Im}\tilde{S}(\omega^0-\omega)}{\operatorname{Re}\tilde{S}(\omega^0-\omega)}\right)^2\right)\right.$$
$$\left.\times\left(1+\left(\frac{\operatorname{Im}\tilde{S}(\omega^0+\omega)}{\operatorname{Re}\tilde{S}(\omega^0+\omega)}\right)^2\right)\right)^{-1/2}$$

and, by taking into account the continuity of $\operatorname{Re}\tilde{S}(\omega)$, $\operatorname{Im}\tilde{S}(\omega)$ which follows from analyticity of $\tilde{S}(\omega)$, it is also easier to show that the second factor tends to 1. Accordingly, we find that $\cos(S_\omega^- \wedge S_\omega^+) \to -1$ when $\omega \to 0$ for odd-order zeros. It allows us to prove the necessity of the formulated statement about the existence of the second type of discontinuities in the function $\tilde{\phi}(\omega)$, since its violation leads to the fact $\cos(S_\omega^- \wedge S_\omega^+) \to 1$ when $\omega \to 0$.

Thus, the discontinuities of the second type of function $\tilde{\phi}(\omega)$ are possible only at the points where $|\tilde{S}(\omega)| = 0$. The statement mentioned above allows us to better understand the process of formation of such discontinuities. The radius-vector, decreasing to zero, enters in a relevant discontinuity point, having one direction, and turns out in a different direction. In the paper Mitrofanov (1986) several examples are considered regarding appearance and the lack of discontinuities in $\tilde{\phi}(\omega)$ at the points where the modulus of the spectrum is equal to zero. Two such examples are shown in Figure 3. It is interesting that at the first look the discontinuous phase spectrum (of the second type) has more regular behavior of $\tilde{S}_l$ at the origin than the continuous one, see Figures 3(a) and 3(b).

Furthermore, the above proof scheme may serve as a basis for constructing algorithms for isolating such discontinuous points. These algorithms must provide singling out the points of the anomalous decreasing in $|\tilde{S}(\omega)|$, and then to analyze converging the right-hand part of Equation (5) to -1 in the vicinity of such points, or by analyzing the decay rate of $\operatorname{Re}\tilde{S}(\omega)$, $\operatorname{Im}\tilde{S}(\omega)$ to determine the point of sign alteration at most slowly changing component. Although these algorithms are very time consuming computationally, their implementation has allowed us to analyze and study of these types of discontinuities in the real seismic data processing.

A somewhat different approach for isolating points of discontinuities of the second type can be based on their significant differences (in magnitude) from the discontinuities of the first type. So, we can assume that there is a discontinuity of the second type in a point $\omega \in \Delta\omega$, if the following inequality is true

$$\pi - \delta < |\tilde{\phi}_{l+1} - \tilde{\phi}_l| < \pi + \delta \tag{6}$$

where $0 < \delta < \pi$. Obviously, such an algorithm, having an exceptional simplicity, will reliably single out points of such discontinuities for small $\Delta\omega$, see Sysoev & Evdokimov (1986) for details. But the question of choosing the appropriate value and finding the optimum value for this criterion remains unresolved. Some recommendations will be given below.

Another important point in the analysis of phase spectra and their processing at the discontinuity points of the second type is the question of choosing the direction of the discontinuity, i.e., sign of π when passing through the corresponding point. The main difficulty here is that the discontinuity of this type may lead to $\tilde{\varphi}(\omega) \notin [-\pi, \pi]$. This question is nontrivial, see Sword (1984), where it was done as an attempt to construct an algorithm for determining the direction of the discontinuity based on the methods of integer optimization. But these algorithms have high complexity. We propose below a different scheme of analysis of these situations, based on the statistical properties of the phase components of the spectra, calculated from the observed seismic signals.

Statistical properties of phase spectra and algorithms for their continuation

The above-described character of discontinuities of the phase spectra leads to the idea of their isolation and use in the unambiguous definition of this component for each of the processed signal. This can be done by analyzing the difference of the phase spectra, computed at two adjacent frequencies, i.e., for differences $\tilde{\phi}_{l+1} - \tilde{\phi}_l$. In this case, taking into account the presence of noise in real data, it is important to study their joint density. It allows us not only to construct the stable procedure to continue the phase component in the interval $(-\infty, \infty)$, but also to guarantee conformality of the spectral transformation, which will involve the phase spectra of the real seismic signals, such as procedures for decomposing a wave field using a multiplicative factor models, see Mitrofanov et al. (1993). This study was performed in papers Mitrofanov (1979, 1986). Corresponding expression was constructed for the joint distribution density of the two values of the phase spectrum ϕ_l, ϕ_m calculated at arbitrary frequencies ω_l, ω_m, and computational features arising in the construction

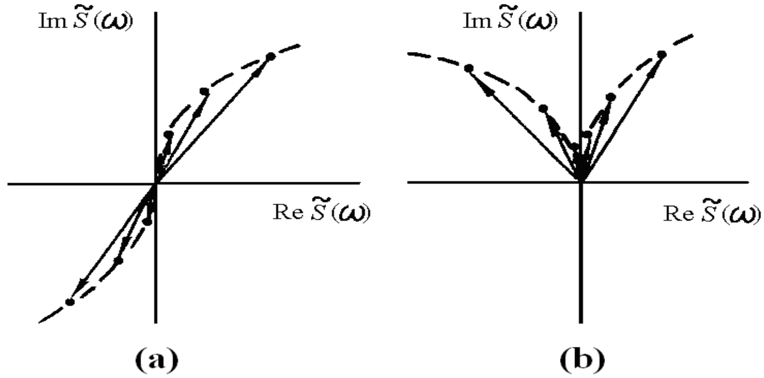

Figure 3 – Examples of discontinuous (a) and continuous (b) behavior of the phase spectrum.

of this function were studied. General view of the constructed function for the case of the phase spectrum of the noise $\tilde{\zeta}_l$ (after application of a smoothing window) is shown in Figure 4(a). Its projections for various correlation coefficients ρ_{lm}, which are possible between the values of the spectrum obtained at different frequencies, are shown in Figure 4(b). It should be noted that the resulting values, correlated with the real and imaginary part of the spectrum, are direct consequences of the windows used for the selection of signals in the spectral analysis.

Enough attention was given to the properties of the phase spectrum distribution fixed frequency in Levin (1957). Using these results, it is easy to construct an expression for the distribution density of $\tilde{\phi}(\omega)$ at a fixed frequency ω, by taking into account the signal and the random components of the spectrum. It has a fairly simple form:

$$g(\phi_l) = \frac{e^{-\mu_l^2/2}}{2\pi}\left(1 + e^{-\chi_l^2/2}\cdot\Phi(\chi_l)\cdot\chi_l\right),$$

where $\Phi(x)$ is the probability integral, see Gradshtein & Ryzhik (2000), $\chi_l = \mu_l\cos(\phi_l - \phi_l^{\tilde{S}})$, and $\mu_l = |\tilde{S}_l|/\sigma_l$ is the ratio of signal-to-noise ratio in the spectral domain at the frequency ω_l. View of this function, constructed for different values $\tilde{\phi}_l^{\tilde{S}}$ and signal-to-noise ratio is shown in Figure 5.

Summarizing the results of these works we can indicate the following basic statistical properties of the phase spectrum of the seismic signal:

First – A typical feature of the distribution function of the phase spectrum at a fixed frequency ω_l is that it has a maximum at $\tilde{\phi}_l = \tilde{\phi}_l^S$, whose value depends of the signal-to-noise ratio for a given frequency.

Second – Using windows to select the signals from the initial observations leads to the appearance of correlation in the phase spectrum of noise, which approximates the structure of this function to the distribution of the phase spectrum, containing the signal component.

Third – Transition to the difference of the phase spectra, in particular, to the difference $\tilde{\phi}_{l+1} - \tilde{\phi}_l$, substantially simplifies the structure of distribution, actually bringing it closer to the structure of the projections shown in Figure 4(c), where the location of the maximum is determined by Equation (4).

These properties, as well as Equation (4), allow us to construct a simple algorithm for extending the phase spectrum, calculated on real data, in the interval $(-\infty, \infty)$ on the basis of its continuous extension. At the same time the continuous extension of the phase spectrum will be understood as follows: on the basis of $\tilde{\phi}_l$, defined in the interval $[-\pi, \pi]$, it is required to find out the discontinuities of the first type and take them into account in calculating $\tilde{\phi}_l$. In order to determine the discontinuities of the first type the following criterion is introduced: *there is discontinuity of the giving type in the interval* $[\omega_l, \omega_{l+1}]$*, if the following inequality is valid:*

$$\left|\tilde{\phi}_{l+1} - \tilde{\phi}_l\right| > \delta^I \tag{7}$$

where δ^I *is a pre-specified threshold.* A removal of these discontinuities is carried out by adding the appropriate value $+2\pi$ or -2π providing a single-valued definition of the phase component in the interval $(-\infty, \infty)$.

The most serious moment in the implementation of criterion (7) is the determination of δ^I. This value should be selected by taking into account the possible changes in the smoothed signal spectrum $\tilde{S}_l$ and the existing distribution of the noise component $\tilde{\zeta}_l$. Obviously, it must satisfy the following inequalities:

$$\max_{\omega\in\Delta\omega}\left|\tilde{\phi}_{l+1}^{\tilde{S}} - \tilde{\phi}_l^{\tilde{S}}\right| < \delta^I < 2\pi - \max_{\omega\in\Delta\omega}\left|\tilde{\phi}_{l+1}^{\tilde{S}} - \tilde{\phi}_l^{\tilde{S}}\right|$$

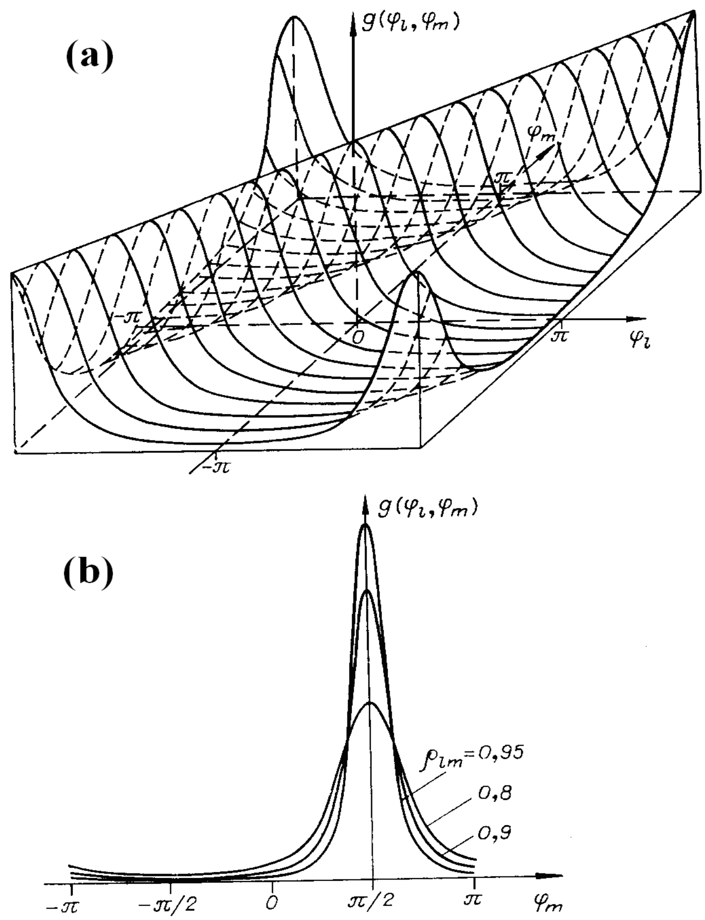

Figure 4 – Probability density function for couple values of phase spectrum (a) and its projection (b) when $\varphi_l = \pi/2$.

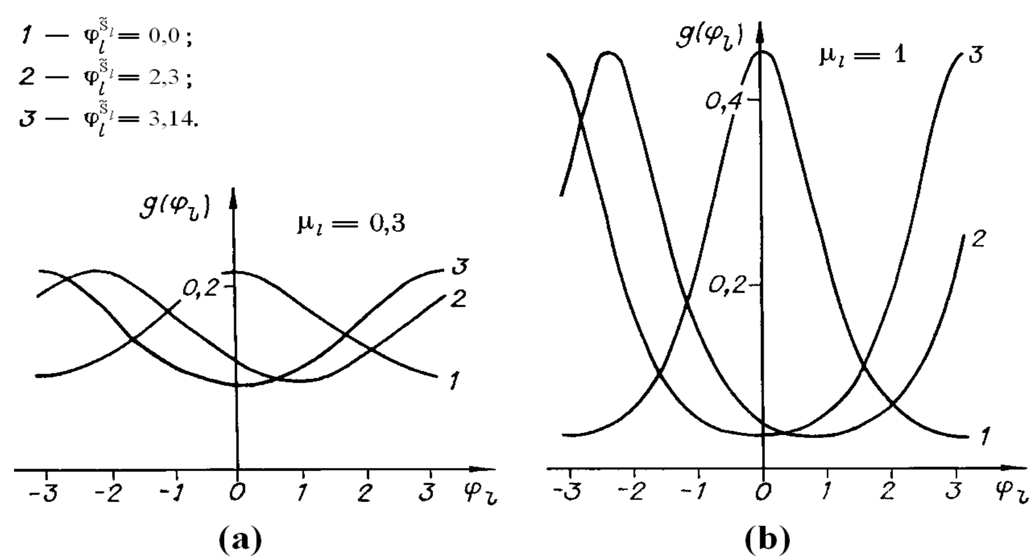

Figure 5 – Probability density function for different signal-to-noise ratio μ_l and values of $\tilde{\varphi}_l^{\tilde{S}}$.

But this estimate presented above is very inaccurate. In the paper Mitrofanov (1979) an attempt was made in order to build a more accurate estimate of this quantity based on the theory of testing hypotheses and using the constructed distribution function $g(\phi_l, \phi_m)$, which gives the possibility of constructing a density function of the difference between the values of the phase spectra, $g(\phi_{l+1} - \phi_l)$. At the same time possible errors were considered, which include an omission of true change in the magnitude of the phase spectrum on 2π or -2π, as well as false changes of the phase spectrum at these values. The appearance of such errors is associated to the fact that the subsequent value ϕ_{l+1} should be determined in the interval $[\phi_l - \pi, \phi_l + \pi]$, but not in $[-\pi, \pi]$. This is clearly seen in Figure 6(a). Not taking into account this fact leads to errors in the phase spectra processing. The number of possible discontinuities or the probability of discontinuity of the first type in the absence of discontinuities of the second type is equal to an integral over the interval $[-\pi, \phi_l - \pi]$ when $\phi_l > 0$, and in the interval $[\pi + \phi_l, \pi]$, if $\phi_l < 0$ (the corresponding region is shaded in Fig. 6(a)). When $\delta^I \neq \pi$, we can skip some of these discontinuities for $\delta^I = \delta_1 > \pi$ (region I in Fig. 6(a)) or indicate false discontinuities when $\delta^I = \delta_2 < \pi$ (region II in Fig. 6(a)). Due to the fact that the value ϕ_l can be arbitrary, the two-dimensional fields must be considered in the analysis of the errors, see Figure 6(c). Calculation of integrals in these areas allows us to determine the optimal δ^I.

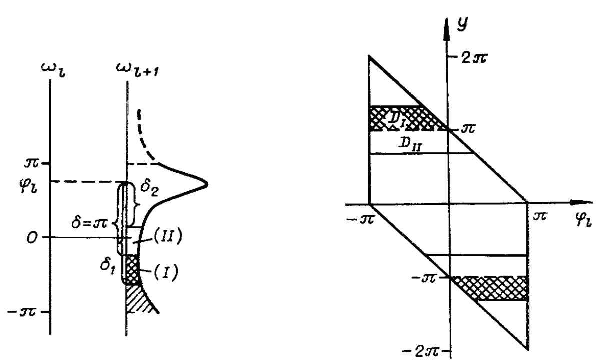

Figure 6 – Determination of probability of different kind breaks when φ_l is a fixed value (a) or changed evenly (b).

However, in this work we have not yet received the results of the second type of discontinuities, that follow from Equation (4). So, the frequencies, where $|\tilde{S}(\omega)| = 0$, had to be excluded from the analyzed intervals, that led to an increase in errors of false change of the phase spectrum in practice. The obtained results regarding to behavior of the phase spectrum in the points $|\tilde{S}(\omega)| = 0$ help us to correct the interval of possible changes δ^I to the interval (4.2, 4.4), which also provides optimal continuation of the phase spectrum in the real data processing.

The analysis of phase spectra of real and synthetic seismic signals confirms the obtained theoretical results very well. Figure 7(a) shows a histogram of real phase spectra, calculated on these frequencies. It is seen that at the fundamental frequency 12 Hz, related to the initial values of the phase spectrum, the histogram has the form close to the theoretical curve. For other frequency histograms have more complicated structure. The latter is connected not only with the change in the ratio signal/noise at other frequencies, but also with a significant dependence of the distribution of the phase spectrum of observed signals on the value of the phase spectrum of useful signal. This dependence, due to the changes in the arrival times of useful signals and the heterogeneity of the reflection properties of the real medium, can lead to a significant difference between the empirical densities of the theoretical functions type.

Despite the heterogeneous nature of the histograms of the original phase spectra, their structure begin to change when we turn to the differences $\Delta\phi_l = \phi_{l+1} - \phi_l$, defined by the difference $\Delta\omega$ (see Fig. 7(b)). As we can see, in this case, the histogram have a more uniform behavior with well-marked extremes, which, according to our research, should be close to one of three values 0, π or 2π. Grey dotted line in all figures shows the histograms corresponding to zero values of the amplitude spectra.

Such properties of the phase components are confirmed by other experiments as well. For example, Figure 7(c) presents the results that were obtained in the investigation of phase spectra of synthetic seismograms, and the remaining three figures show the summary results of other experiments, obtained using variable windows during the signal selection and decreasing in step frequency. It is clearly seen that the choice of the optimal window and step frequency not exceeding 1 Hz, allows us, using the above thresholds δ^I in the criteria (7), to extend the phase spectrum in a continuum way practically without errors, keeping in mind the information about the discontinuities of the second type.

EXAMPLES

Let us turn to the model experiments, which were carried out under the research project of the Petrobras, S.A., see Priimenko et al. (2005) and Mitrofanov et al. (2005). The main goal of the project was studying pinching-out zones of thin layers, as well as searching ways of analyzing and processing seismic data to enhance certainty in picking the pinching-out points. Accurate picking the pinching-out zone is one of the most actual and complex problems, because in such tasks the resolution of seismic methods is essentially important.

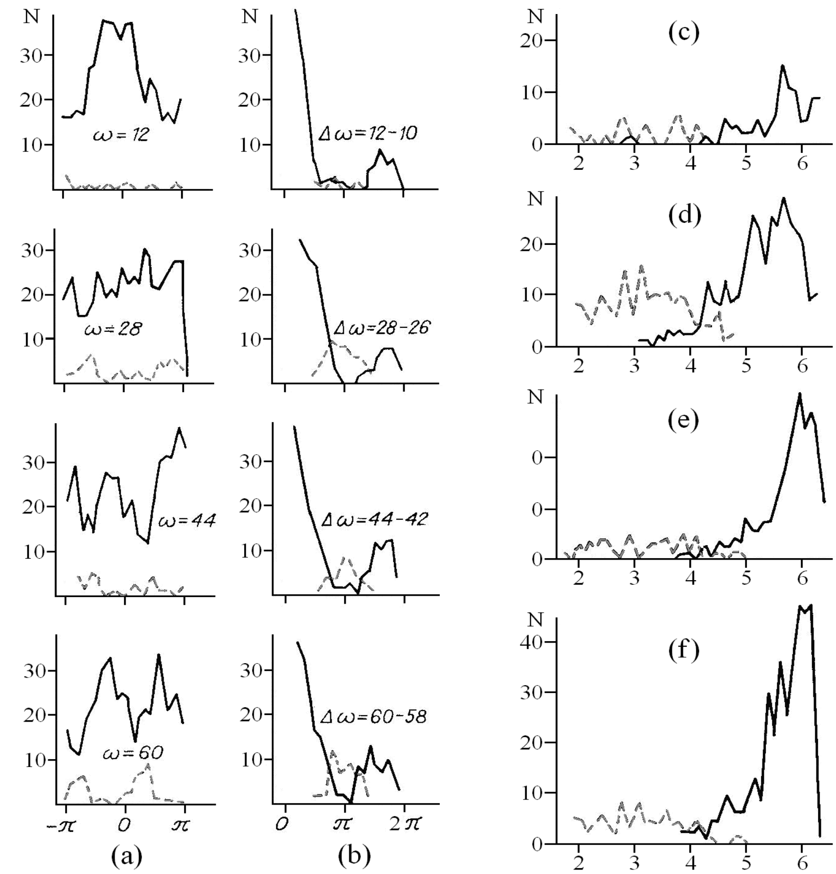

Figure 7 – Histograms of the phase spectra constructed by the initial values (a) and their difference (b), as well as aggregate histograms constructed by the difference of the phase spectra in the case of mathematical modeling (c), using windows with a small (d) and with a large smoothing (e) and a decrease in step frequency (f).

Based on real data, a model was prepared, containing several major intermediate boundaries and the target object (see Fig. 8). A fairly simple type of model that preserves the basic features of a real medium, was chosen deliberately, because allowed, while maintaining the global structure of the medium, to focus on the target object, located directly under one of the reference horizons named horizon A. Several techniques were used to calculate fields of the reflected waves, which were analyzed afterwards.

There were considered two modifications of the model (see Fig. 9). In the case of the simple model 1 the reservoir is directly adjacent to the horizon A (see Fig. 9(a)). The reservoir thickness was varied from 0 to 30 m and the pinch-out point lies at the given horizon. In the case of the more complex model 2 the reservoir, contained an oil-water contact (OWC), is located approximately 40 m below the horizon A (see Fig. 9(b)). Its thickness was varied from 0 to 20 m (at the highest point).

Figures 10 and 11 present the results of the analysis of reflected signals. These signals belong to the specified local areas of the general model, containing the pinching-out points of the target horizon. The reflected signals were selected from the zero-offset sections, taking into account changes in their arrival time. This choice of signals allowed to constrict the analyzed time interval.

Note, that the traces, contained the analyzed reflected signals, were located on the free surface of the original model at the points with coordinates of 1 km (trace No. 10) and further with the step 100 m. At the same time the first signal reflected from the pinching-out zone (in all the cases) corresponded to the fourteenth trace. The wave field was calculated in all of the experiments on the basis of the ray method using Ricker's wavelet. The shape of the pulse, corresponded to the reflection from the boundary outside of reservoir, is well represented at all the figures on the first of shown trace intervals.

Figure 10 presents the results of two experiments for the simple model 1. For the first of the presented experiments, the pulse frequency was 50 Hz (see Fig. 10(a)), and for the second

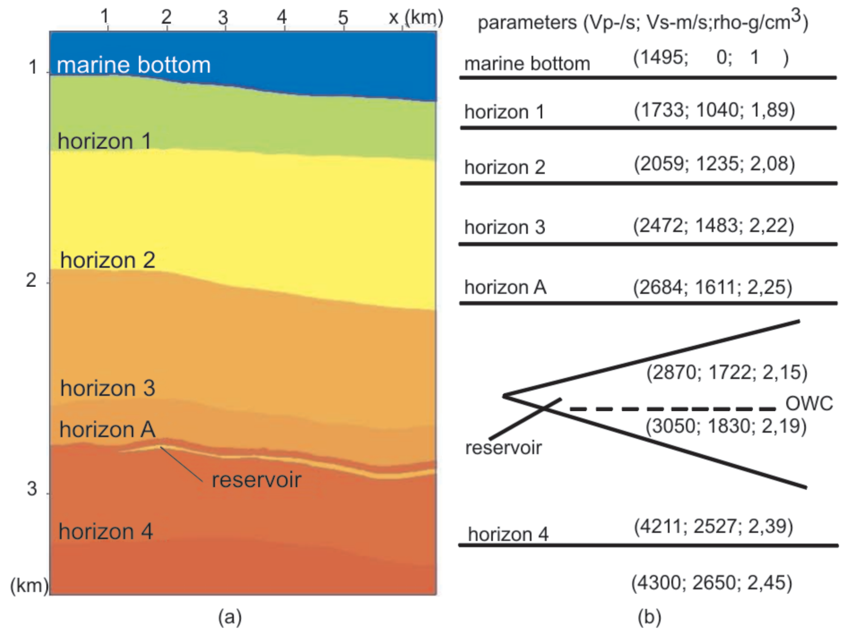

Figure 8 – Structure of general model of medium with the target reservoir (a), and the values of elastic parameters (b).

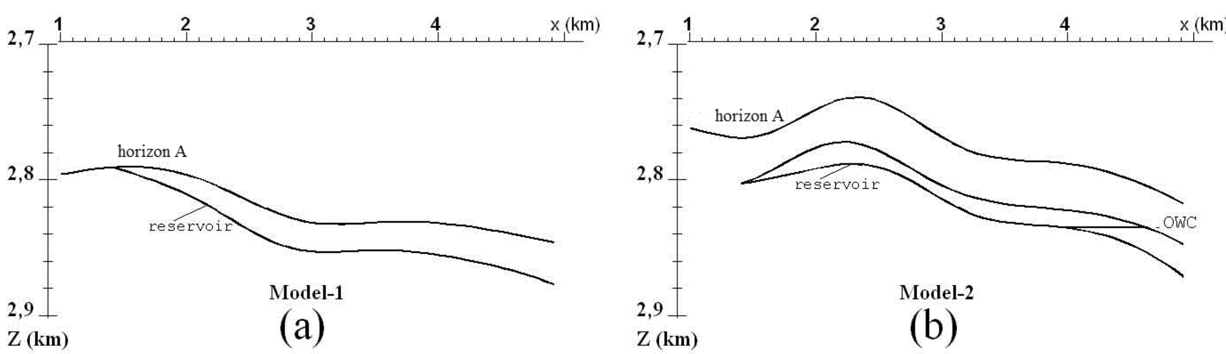

Figure 9 – Two models of reservoir pinch-out zone.

experiment it was equal to 75 Hz (see Fig. 10(b)). The original signals, reflected from the target object, are shown in both figures above. The amplitude component is shown in the middle of the figure, and the phase component – at the bottom.

It is seen that at a frequency of 50 Hz for the initial section (see Fig. 10(a)) we can determine the change in the waveform associated with the wedging-out of horizon from traces No. 21-22, when a widening in the impulse shape is observed. We perhaps can make a similar identification of the horizon pinching-out zone by analyzing variations of the energetic characteristic, and also the amplitude spectrum. At the same time, by using the phase spectrum we already are able to clearly identify changing in the pulse shape for the fourteenth signal. But the most interesting thing does happen when we turn to the analysis of changes in the waveform associated with the wedging-out of the horizon, for a frequency of 75 Hz (see Fig. 10(b)). In this case, change in the waveform is now determined for the seventeenth trace. At the same time, on the amplitude spectrum of this section we can see such changes starting with fifteenth trace, and on the phase spectrum again starting with fourteenth trace. It is important that the structure of the phase spectrum for pulses with a frequency of 50 Hz and 75 Hz is very close (compare the relevant parts of Figs. 10(a) and 10(c)). Thus, analysis of the phase spectrum for the signal with a lower frequency allows us to get the same results as for the signal with higher frequency. The result is based on the fact that the phase spectrum, in contrast to the amplitude spectrum, is less

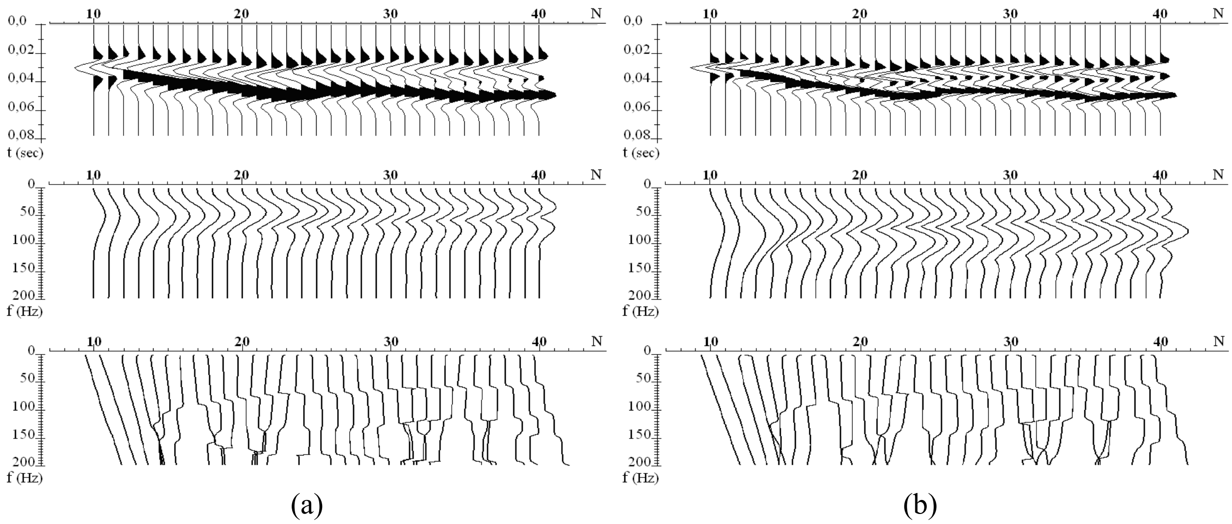

Figure 10 – The results of the model experiment for the simple model 1 with fundamental frequency of the incident impulse: (a) 50 Hz and (b) 75 Hz.

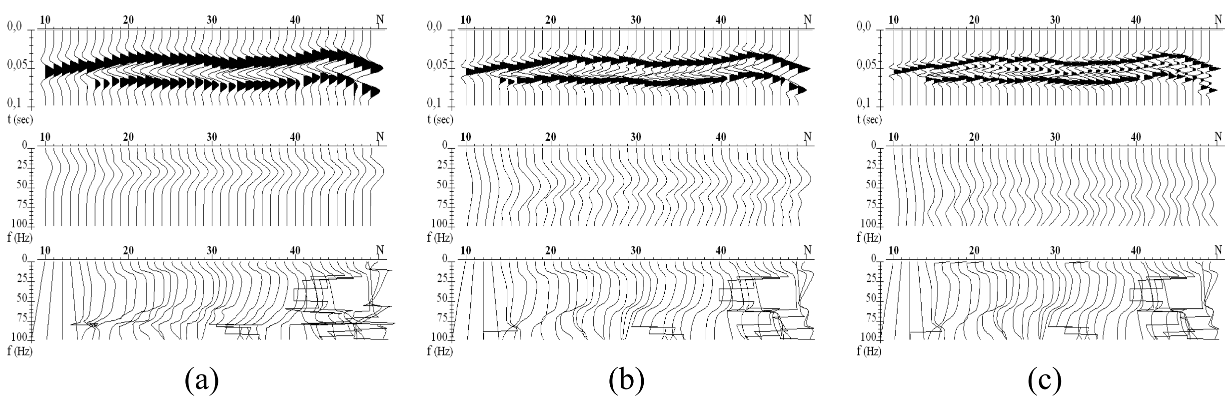

Figure 11 – The results of the model experiment for the model 2 with fundamental frequency of the incident impulse: (a) 25 Hz, (b) 50 Hz, and (c) 75 Hz.

dependent on the fundamental frequency of the incident pulse. Therefore, the special features, associated with the wedging-out of the horizon may appear more significant at a relatively low frequency signal.

Experiments, carried out with the model 2 (see Fig. 11), confirm a much lower dependence of the structure of the phase spectrum of the fundamental frequency of the incident pulse, than it may appear in the structure of the reflected signal and its amplitude spectrum. A comparison of Figures 11(a)-(c) shows that the phase spectrum is changed. At the same time, the amplitude spectrum has significant variations and these changes are mainly connected with the redistribution of energy. Obviously, it gives an opportunity to localize more accurately the pinching-out zone of the target horizon. But the phase spectrum can do it in the case of a low fundamental frequency of the incident pulse. Analysis of the results presented in Figure 11 points to another important feature, associated with a significant reaction of the phase spectrum to modifications of the structure of the medium. Thus, the OWC is manifested in traces with numbers greater than 41 by means of a drastic change in the structure of the phase spectrum. A weak dependence of these manifestations in the phase spectrum, depending on the frequency of incident pulse, allows to determine the OWC for the frequency 25 Hz (see Fig. 11(a)). Such identification cannot be done using both the original reflected signals and the amplitude spectrum.

The results of the analysis of phase spectra for the signals (with relatively low fundamental frequency), reflected from the real target, can serve as the confirmation of the results obtained in model experiments. These results are presented in Figure 12, which shows a part of the time section, where there are reflections

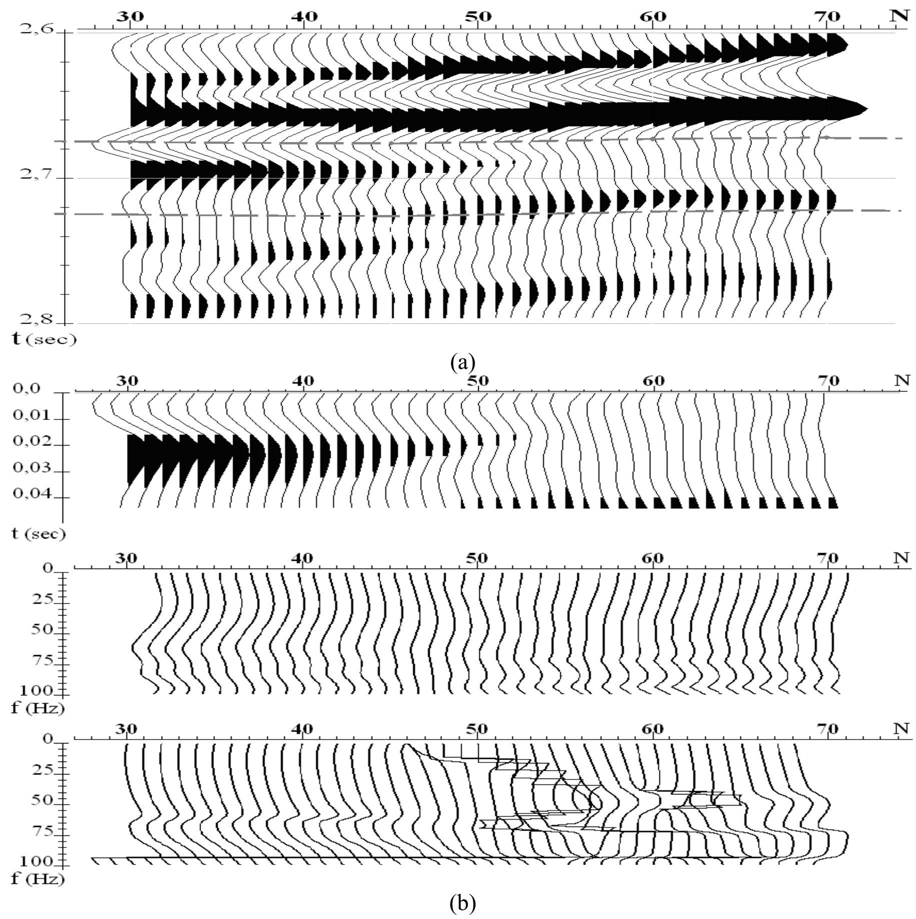

Figure 12 – Real data processing corresponded to the area of a possible pinch-out zone of the target object: (a) – original data, (b) – results obtained using the amplitude and phase spectra.

from the target object (see Fig. 12(a)). In this case, the dotted lines marked a target interval in order to clarify the pinching-out point of the object. Figure 12(b) shows the corresponding interval and the amplitude (middle part of the figure) and phase (lower figure) spectra calculated for this interval. Analysis of these spectra indicates that the original signals, as well as the values of the amplitude spectrum, can determine the pinching-out zone of the target horizon up to trace No. 52. The accuracy of the resulting estimates may be low, because the level of the amplitude variations is very low. At the same time, the values of the phase spectrum of this region can be adjusted up to trace No. 61 with a rather high degree of reliability, due to the fact that there are significant variations in the corresponding values of phase component.

CONCLUSIONS

The results presented in this work indicate on the importance and perspective of using the phase spectrum of seismic signals in the analysis of fine-grained structural features of the medium: pinching-out points, thin-bed effect, and others. However, this requires using the improved procedure in determining the phase spectrum: defined in the interval $(-\infty, \infty)$, but not in the interval $[-\pi, \pi]$. Otherwise we will lose or significantly distort information about the phase spectra in their processing. Special investigations of the discontinuities of the phase spectrum and its statistical characteristics, connected with the presence of various types of noise in real data, are important for the single-valued determination of the phase spectrum in the interval $(-\infty, \infty)$. These studies have allowed us to explain the behavior of sets of phase spectra calculated for the real signals. In addition, they provided an opportunity to formulate a criterion for the continuous extension of the phase spectrum, which ensures the efficient determination of the values of this component in the joint processing of large sets of phase spectra.

The effectiveness of using the phase spectra is confirmed by studying the phase spectra obtained for pinching-out zones of target horizons. It is shown that the phase spectrum can be substantially more informative than the amplitude spectrum. The pinching-out point can be more reliably determined by analyzing the phase spectra, reducing the area of uncertainty. The main advantage of phase spectra in this case is that the influence of structural features of the target horizon on the structure of the phase spectrum is weakly dependent on the frequency of the incident impulse. Therefore, at low frequencies one can obtain the results similar (by accuracy) to the results, that could be obtained at a high frequency.

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