Revista Brasileira de Geofísica
RBGf
Brazilian Journal of Geophysics

Revista Brasileira de Geofísica (2016) 34(2): 251-260

ISSN 0102-261X
www.scielo.br/rbg

SEISMIC TOMOGRAPHY USING METROPOLIS METHOD OF VELOCITY FIELDS PARAMETERIZED BY HAAR WAVELET SERIES

Alexsandro Guerra Cerqueira[1], Wilson Mouzer Figueiró[1] and Paulo Eduardo Miranda Cunha[2]

ABSTRACT. The representation of compressional seismic waves velocity fields from geological models through numerical parameters has a strong geophysical importance, because, it makes possible to quantify such qualitative models, allowing its mathematical manipulation. In this way, the parameterization by Haar wavelet series may be seen as an attractive alternative. The pyramid algorithm is used to obtain a multi-scale wavelet series representation of such models. It is applied filters that assure an optimized parameterization of models with important parameter reduction without significative loss in the model representation. It is accomplished the parameterization of velocity field models in order to verify the capacity of the wavelet series to represent functions, in an acceptable way, with some degree of complexity, using coefficients provided by the pyramid algorithm. Then, target models are represented by means of linear combination of simpler functions and its coefficients are estimated by means of the inversion process, using traveltime data, defined by the Metropolis method. In this way, the inversion aims convergence to a target model previously proposed. In a brief, coefficients of Haar wavelet series are used as parameters of the model to be estimated by tomography inversion.

Keywords: parameterization, Haar wavelet series, pyramid algorithm, seismic tomography, seismic velocity field, traveltime data, Metropolis method.

RESUMO. A representação de campos de velocidades sísmicas compressionais, através de parâmetros numéricos, é de importância básica na geofísica, pois torna possível a quantificação de modelos, antes qualitativos, permitindo assim que sejam matematicamente manipulados. A parametrização por série ondaleta Haar pode ser vista como uma alternativa atrativa para quantificar tais modelos. O algoritmo piramidal pode ser utilizado para obtenção da série ondaleta multi-escala e, também, auxilia na aplicação de filtragens ou técnicas redutoras de coeficientes que garanta uma parametrização otimizada do modelo, com substancial redução de parâmetros sem prejuízos importantes na representação do modelo. Neste trabalho é realizada a parametrização de alguns modelos já conhecidos na geologia para verificar se a série ondaleta, utilizando os coeficientes fornecidos pelo algoritmo piramidal, cumpre de forma aceitável o seu papel de representar funções, com um certo grau de complexidade, através de combinações lineares de funções mais simples. Após a certificação da possibilidade de parametrização com um pequeno número de coeficientes, faz-se a modelagem de dados de tempos de trânsito no modelo corrente parametrizado, utilizando a técnica de traçamento de raios, dentro de um processo de inversão de tempos de trânsito definido pelo método Metropolis, objetivando convergência para um modelo alvo definido. Para isto são utilizados os coeficientes da série ondaleta Haar 1D como parâmetros a serem estimados dentro do contexto da inversão tomográfica.

Palavras-chave: parametrização, série ondaleta Haar, inversão sísmica tomográfica, campo de velocidade sísmica, dados de tempo de trânsito, método Metropolis.

[1] Universidade Federal da Bahia, Salvador, BA, Brazil – E-mails: alexleogc@gmail.com; figueiro6@gmail.com
[2] PETROBRAS, Brazil – E-mail: pemcunha@gmail.com

INTRODUCTION

A mathematical problem of interest, not only theoretically, but also with respect to applications, is the one concerned about the representation (or decomposition) of functions with high degree of complexity in a linear combination (series) of simpler functions (Cerqueira & Figueiró, 2012), (Santos & Figueiró, 2006) and (Bastos, 2013). Like the other series already known (polynomial, Santos & Figueiró, 2011; trigonometric, Santos & Figueiró, 2006; and splines, Santana & Figueiró, 2008), the wavelet series (Morettin, 1999) is defined as a mathematical function capable of decomposing, represent or describe other functions. Differently of splines, and other kind of mathematical series, the wavelet series allows the analysis of such functions at different scales and it is used to the data compression and to the noise attenuation. It is applied consistently in areas of exact sciences, such as: physics, electrical engineering, geophysics, and etc (Lee & Yamamoto, 1994).

The first author to mention the term "wavelet" was Alfréd Haar (Haar, 1910) and the formal concept of wavelets was firstly proposed by Jean Morlet with the help of Grossman (Polikar, 1999). The wavelet series is similar to the Fourier series. This last proposes a decomposition of a periodic and continuous function by a sum of sinusoidal functions weighted by coefficients, and the first a decomposition of any function, with finite energy, by a sum of functions belonging to a wavelet basis.

The wavelet series can represent continuous or non-continuous functions through a linear combination of simpler functions, belonging to a basis, weighted by coefficients (parameters). In multi-scale version of wavelet series, it is possible to obtain its coefficients from the pyramid algorithm (Cunha, 2009), also known as discrete wavelet transform. In this article, it was accomplished the parameterization of compressional seismic velocity fields using the Haar wavelet series coefficients, according to different levels of scale, in order to have a control over how (and which) coefficients can be eliminated without significative loss in the quality of the representation, and to obtain a maximum reduction of the number of coefficients in each parameterization, using techniques such as the pyramid algorithm. Such coefficients have not physical meaning, they are just dimensionless weights.

In a subsequent step, it is made the modeling synthetic data. The ray tracing technique, proposed by (Červený, 2001), was implemented in order to enable the calculation of traveltime that a given wave spends to travel from a source to a receiver (Perin & Figueiró, 2010) and (Santos, 2009) thus, generating a traveltime profile that simulates, approximately, a seismic acquisition. Generally, velocity fields are not considered continuous functions, then it is necessary to implement interpolation and derivative techniques in Fortran.

The current models, parameterized by Haar wavelet series, undergo an inversion process that aims to minimize the difference between the observed and calculated data. A global search inversion method, known as Metropolis, was used to obtain a model that best fits the problem of minimizing the distance between the observed and calculated data serving as a stopping criterion. The goal is to perform the inversion of traveltime data with the coefficients of the Haar wavelet series representing the velocity fields in a satisfactory way, using only one global scope algorithm without combining any other type of inversion method.

THEORETICAL REVISION

Parameterization by Haar Wavelet

The Haar wavelet function (ψ) is defined as:

$$\psi(t) = \begin{cases} 0, & -\infty < t < 0 \\ 1, & 0 \leq t < \dfrac{1}{2} \\ -1, & \dfrac{1}{2} \leq t < 1 \\ 0, & 1 \leq t < \infty. \end{cases} \quad (1)$$

The Haar wavelet basis is expressed as follows:

$$\psi_{j,k}(t) = 2^{j/2}\psi(2^j t - k), \quad \forall t\colon \frac{k}{2^j} \leq t < \frac{k+1}{2^j}. \quad (2)$$

The Haar wavelets indicatrix function, $\phi(t)$, and the wavelet, $\psi(t)$, can be written as:

$$\phi_{j,k}(t) = \begin{cases} 2^{j/2}, & 2^{-j}k \leq t < 2^{-j}(k+1) \\ 0, & \text{otherwise} \end{cases}, \quad (3)$$

and

$$\psi_{j,k}(t) = \begin{cases} 2^{j/2}, & 2^{-j}k \leq t < 2^{-j}(k+1/2) \\ -2^{j/2}, & 2^{-j}\,(k+1/2) \leq t < 2^{-j}(k+1) \\ 0, & \text{otherwise.} \end{cases} \quad (4)$$

Making the change $j = -l$ in the scale index, it follows the basis:

$$\phi_{l,k}(t) = 2^{-l/2}\phi(2^{-l}t - k) = \begin{cases} 2^{-l/2}, & 2^l k \leq t < 2^l(k+1) \\ 0, & \text{otherwise,} \end{cases} \quad (5)$$

and

$$\psi_{l,k}(t) = 2^{-l/2}\psi(2^{-l}t-k) = \begin{cases} 2^{-l/2}, & 2^l k \le t < 2^l(k+1/2) \\ -2^{-l/2}, & 2^l\,(k+1/2) \le t < 2^l(k+1) \\ 0, & \text{otherwise.} \end{cases} \quad (6)$$

A function $f(t)$ can be represented as:

$$f(t) = \sum_k d_{lo,k}\phi_{lo,k}(t) + \sum_{l\ge lo}\sum_k c_{l,k}\psi_{l,k}(t). \quad (7)$$

The Equation (7) is called wavelet series and the coefficients are given by:

$$c_{j,k} = \langle f, \psi_{j,k}\rangle = \int_{-\infty}^{\infty} f(t)\psi_{j,k}(t)dt, \quad (8)$$

and

$$d_{jo,k} = \langle f, \phi_{jo,k}\rangle = \int_{-\infty}^{\infty} f(t)\phi_{jo,k}(t)dt. \quad (9)$$

Pyramid Algorithm

The Haar pyramid algorithm (also known as discrete wavelet transform) calculates, recursively, the coefficients of expansion represented by Equations (8) and (9), using the scaling coefficients of the $\phi_{j,k}(t)$ basis, beginning with $l = 0$, for subsequent scales by the following equation:

$$\phi_{l,k}(t) = \frac{1}{\sqrt{2}}\left[\phi_{l-1,2k}(t) + \phi_{l-1,2k+1}(t)\right]. \quad (10)$$

As this relation is valid for all $l \in Z$, it can be rewrite by displacing l to $l+1$:

$$\phi_{l+1,k}(t) = \frac{1}{\sqrt{2}}\left[\phi_{l,2k}(t) + \phi_{l,2k+1}(t)\right]. \quad (11)$$

Haar wavelet function $\psi(t)$ is written as:

$$\psi(t) = \phi(2t) - \phi(2t-1), \quad (12)$$

and this allows to write:

$$\psi_{l+1,k}(t) = \frac{1}{\sqrt{2}}\left[\phi_{l,2k}(t) - \phi_{l,2k+1}(t)\right]. \quad (13)$$

Applying the operator $\langle f, \cdot\rangle$ in all terms of Equations (11) and (13), we have:

$$\langle f(t), \phi_{l+1,k}(t)\rangle = \frac{1}{\sqrt{2}}\left[\langle f(t), \phi_{l,2k}(t)\rangle + \langle f(t), \phi_{l,2k+1}(t)\rangle\right], \quad (14)$$

and,

$$\langle f(t), \psi_{l+1,k}(t)\rangle = \frac{1}{\sqrt{2}}\left[\langle f(t), \phi_{l,2k}(t)\rangle - \langle f(t), \phi_{l,2k+1}(t)\rangle\right]. \quad (15)$$

Equations (8) and (9) allow to rewrite Equations (14) and (15) as:

$$d_{l+1,k} = \frac{1}{\sqrt{2}}(d_{l,2k} + d_{l,2k+1}), \quad (16)$$

and,

$$c_{l+1,k} = \frac{1}{\sqrt{2}}(d_{l,2k} - d_{l,2k+1}). \quad (17)$$

Seismic Modeling

Ray tracing, used in seismic modeling of velocity fields, is based on the seismic ray theory that comes from the study of the propagation of seismic waves in heterogeneous media. The ray theory results from the assumption of the existence of high frequency solution to elastic-dynamic equation, and thus, it does not provide an exact solution to the equation of the wave propagation in elastic media (Martinez, 2012). However, it can reproduce the ray trajectory with good accuracy when it comes to the problems related to reservoir exploration.

The ray tracing uses a particular solution of the ray system of equations (Červený, 2001):

$$\begin{cases} \dfrac{d\boldsymbol{X}(u)}{du} = (p_k p_k)^{\frac{n}{2}-1}\boldsymbol{P}(u) \\ \dfrac{d\boldsymbol{P}(u)}{du} = \dfrac{1}{n}\vec{\nabla}\left[\dfrac{1}{v^n}\right] \\ \dfrac{dT}{du} = (p_k p_k)^{\frac{n}{2}} = v^{-n}, \end{cases} \quad (18)$$

where $\boldsymbol{X}(u)$ is the position vector $(x(u),\ z(u))$, $\boldsymbol{P}(\tau) = (p_1(u), p_2(u))$ is the slowness vector (that is perpendicular to the wavefront in a given isotropic media).

Equations (18) allow to write new equations in terms of different variables u corresponding to different values of n. In this paper, we use the value $n = 2$ and the parameter u is replaced by τ, then:

$$\begin{cases} \dfrac{d\boldsymbol{X}(\tau)}{d\tau} = \boldsymbol{P}(\tau) \\ \dfrac{d\boldsymbol{P}(\tau)}{d\tau} = \dfrac{1}{2}\vec{\nabla}\left[\dfrac{1}{v^2(x,z)}\right] \\ \dfrac{dT}{d\tau} = \dfrac{1}{v^2(x,z)}, \end{cases} \quad (19)$$

where parameter $\tau = \int_0^T v^2(x,z)dT$, T is the traveltime along the ray trajectory and dT is the time integration element. There is no physical meaning for the τ variable, but it has $[L^2T^{-1}]$ dimension, or $[m^2s^{-1}]$ in the International System Units (SI).

Expanding Equations (19) by Taylor's series to the second order, we have that:

$$\boldsymbol{X}(\tau+\delta\tau) = \boldsymbol{X}(\tau) + \frac{d\boldsymbol{X}(\tau)}{d\tau}\delta\tau = \boldsymbol{X}(\tau) + \boldsymbol{P}(\tau)\delta\tau \tag{20}$$

$$\boldsymbol{P}(\tau+\delta\tau) = \boldsymbol{P}(\tau) + \frac{d\boldsymbol{P}(\tau)}{d\tau}\delta\tau = \boldsymbol{P}(\tau) + \frac{1}{2}\vec{\nabla}\left[\frac{1}{v^2(x,z)}\right]\delta\tau. \tag{21}$$

The slowness vector is updated constantly to satisfy the condition of the iconal equation in each polygonal node that represents the ray trajectory by:

$$\|\boldsymbol{P}\| = \sqrt{p_1^2 + p_2^2} = \frac{1}{v(x,z)}. \tag{22}$$

The traveltime is calculated iteratively during the ray tracing process as the wave travels along the ray path from a point (x_N, z_N) to another (x_{N+1}, z_{N+1}) and the time from source $S = (x_0, z_0)$ to receiver $R = (x_L, z_L)$ is calculated as follows:

$$T(x_{N+1}, z_{N+1}) = T(x_N, z_N) + \frac{1}{v_N}\sqrt{(x_{N+1}-x_N)^2 + (z_{N+1}-z_N)^2}. \tag{23}$$

Seismic Data Inversion Using Metropolis Algorithm

The Metropolis method is a global search inversion technique. It is a variant of the Monte Carlo method (Kalos & Whitlock, 2008) and it is an iterative method that starts from an initial model in search of a model that is a global minimum of an objective function (Perin, 2014) that involves the difference between the observed ($\boldsymbol{d}_{obs}$) and the calculated ($g(\boldsymbol{m})$) data. As an example of objective function, we have:

$$S(\boldsymbol{m}) = \|\boldsymbol{d}_{obs} - g(\boldsymbol{m})\|_2^2. \tag{24}$$

According to Press et al. (1997), the Metropolis algorithm is divided into the following steps:

1. A description of a possible system configurations (initial model);
2. A random generator disturbances of the parameters used in current models;
3. A probability function $P(\boldsymbol{m})$, that aims to reach its global maximum; and
4. The use of a stopping criterion.

The probability function $P(\boldsymbol{m})$ is given by:

$$P(\boldsymbol{m}) = \exp(-\|\boldsymbol{d}_{obs} - g(\boldsymbol{m})\|_2^2). \tag{25}$$

Auxiliary Tools

Compressional velocities fields, considered here, are two-dimensional functions, and the parameterization of such fields are made from Equation (7) to a function of one variable. To avoid this situation, a strategy is adopted to collect the velocities $v(x,z)$ of models that could not be parameterized as a function of a single variable. A grid of nodes is put on the seismic velocity domain, similar to Figure 1, and it can be arranged as a vector to apply the pyramid algorithm, and thus obtain the coefficients to represent the velocity function $v(x,z)$ by wavelet series.

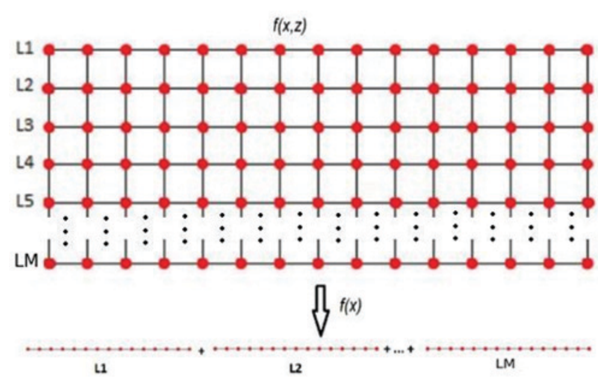

Figure 1 – 2D grid containing the seismic velocities used for the representation of the numerical model by a parametric one.

It is known that some coefficients have null or a little influence on the function representation by Haar wavelet series. The mean value parameter reduction technique supposes that a set of coefficients with close values can assume a single value that would be the weighted median of these coefficients. Given a set of coefficients $C = \{c_{j,0}, c_{j,1}, c_{j,2}, \ldots, c_{j,2^{n-1}}\}$, it can be replaced by a single value that is equal to their average value allowing that a full scale is not totally lost.

The ray tracing program requires the solution of a numerical derivative. A simple way to approximate numerically the derivative of a function is through the finite difference method, calculating the slope of a secant line next to the point where is wanted to calculate the value of the derivative. Given a function, $f(x)$, can be calculated its derivative in the following mode:

$$f'(x) \cong \frac{f(x+\Delta x) - f(x-\Delta x)}{2\Delta x}, \tag{26}$$

where Δx is small enough that represents a slight variation. It is known as centered difference approximation and it is more accurate than other definitions for numerical calculation of derivatives.

Inversion algorithms need some stopping criterion. The criterion adopted in the inversions procedures is the relative difference between the traveltimes (RDT) less than a tolerance suggested by the programmer. The RDT, expressed as a percentage, is calculated as follows:

$$RDT = \frac{\sum_{i=1}^{N} |t_{obs}(i) - t_{calc}(i)|}{\sum_{i=1}^{N} t_{obs}(i)} \times 100\%, \quad (27)$$

where N is the number of receivers used in modeling, $t_{obs}(i)$ and $t_{cal}(i)$ are, respectively, the observed and calculated traveltimes for the receiver R_i.

With the pyramid wavelet series parameterization, the coefficients series (or model parameters) are hierarchized in several levels. Although, without calculations of parameters correlations, intuitively, it is possible to say that: the increase of the level difference between two parameters, increases the parameter correlation between them. But, as the studied problem is highly nonlinear, such correlation must not be close to 1 or –1. This subject is not extended here, because correlation analysis is not the focus of this work.

In the seismic exploration practical activities, velocity fields has, usually, a maximum ($v_{\max} \cong$ 8.0 km/s) and a minimum ($v_{\min} \cong$ 1.0 km/s) values. Considering the particularities of the problem being, here, treated, it is reasonable to propose the following parameters constraints:

$$d_{J,0} 2^{-J/2} + \sum_{j \le J} \sum_{k} |c_{j,k}| 2^{-j/2} \le v_{\max}, \quad (28)$$

and

$$v_{\min} \le d_{J,0} 2^{-J/2} - \sum_{j \le J} \sum_{k} |c_{j,k}| 2^{-j/2}. \quad (29)$$

Then,

$$(v_{\min} - \varepsilon) 2^{J/2} \le d_{J,0} \le (v_{\max} - \varepsilon) 2^{J/2}, \quad (30)$$

and

$$\sum_{j \le J} \sum_{k} |c_{j,k}| 2^{-j/2} \le \frac{v_{\max} - v_{\min}}{2} + \varepsilon, \quad (31)$$

where $\varepsilon > 0$ is a small tolerance.

RESULTS

FOUR HOMOGENEOUS LAYERS WITH HORIZONTAL INTERFACES MODEL (M_{1N})

Figure 2 shows a numerical seismic velocity field model with four horizontal layers (M_{1N}). The physical property considered is the propagation velocity of compressional waves, that is represented, in numerical terms, by a matrix and it is used as a reference for representing the Haar wavelet series.

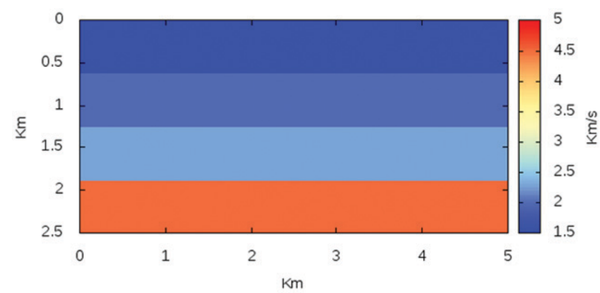

Figure 2 – Numerical model, M_{1N}, using 128 discretized values of seismic velocities along the z axis in order to accomplish the parameterization.

Parameterization

The parameterized model, M_{1P} (Fig. 3), is obtained from M_{1N} with the assistance of the pyramid algorithm. The purpose of such algorithm is to generate the coefficients required to represent the seismic velocity field in an organized multi-resolution fashion. The parameterization of this and the next models are performed using the wavelet series, Equation (7). The model M_{1P} is represented by the wavelet series with the following four coefficients values: $d_{7,0} = 29.4156, c_{7,0} = -9.0510, c_{6,0} = -1.6000$ and $c_{6,1} = -8.800$. Such coefficients are obtained through the pyramid algorithm, from the $v_{1N}(z)$ representation using only the zero scale of function ϕ.

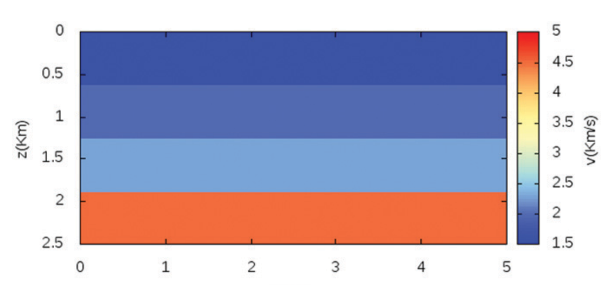

Figure 3 – Model parameterized, M_{1P}, with indices l and k ranging from: $l_o = 1$ to $l_f = 7$, and $k_o = 0$ to $k_f = 63$, using 4 coefficients. As target model for inversion, this model is renamed to M_{1T}.

Forward Modeling

The ray tracing in the model parameterized (Fig. 4) is used to collect the traveltime of the first arriving waves. The source, S, is placed on the surface, $S = (0, 0)$, where it is generate ray trajectories to arrive at the receiver positions. The parameterized model is used as a target model inside an iterative inversion process.

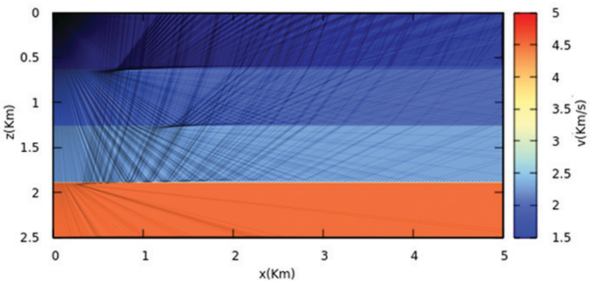

Figure 4 – Ray tracing in the parameterized model M_{1P}, with 1,500 rays departing from a source position located on the surface in $S = (0, 0)$.

Inversion

The relative difference between traveltimes (RDT) obtained and calculated, respectively, on the target and initial models, was of 54.30%. This last one, used by the inversion iterative process, is called M_{1O} and it is shown in the Figure 5.

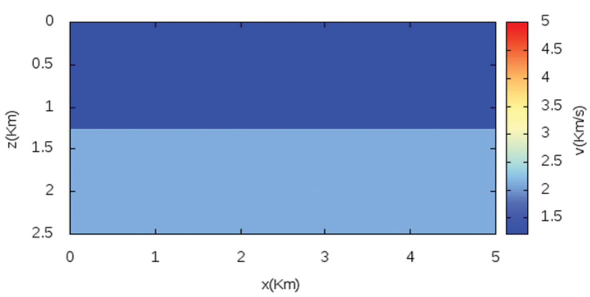

Figure 5 – Initial model, M_{1O}, used by the Metropolis process of inversion.

The Metropolis iterative process of inversion needed 10 iterations for convergence. The stopping criterion evaluated to obtain the inverted model, M_{1I}, was a current model that made the RDT less than 10%. For the inverted model (Fig. 6), such difference was equal to 7.02%.

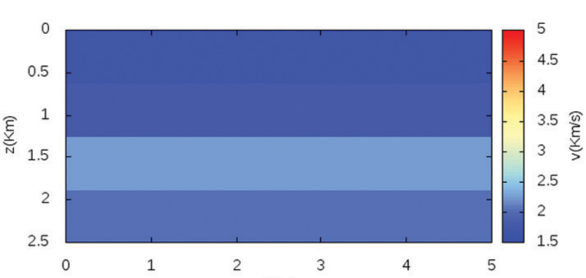

Figure 6 – Inverted model, M_{1I}, parameterized with 4 coefficients and obtained by the Metropolis inversion method.

The Figure 7 shows RMD, the relative velocity difference between the models M_{1I} and M_{1T}, and Table 1 shows the coefficients used in the representation of models: initial, inverted and target.

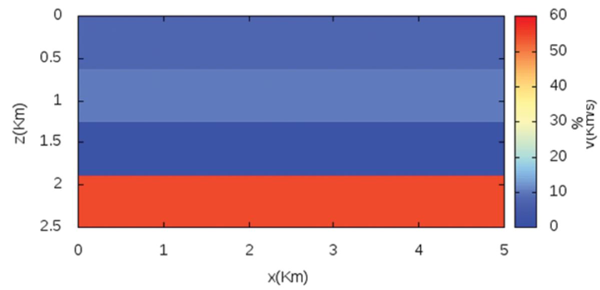

Figure 7 – Relative model difference (RMD) between seismic velocities of M_{1I} and M_{1T}.

HIGH-VELOCITY INTRUSION MODEL (M_2)

A numerical model representation of a high-velocity intrusion, M_{2N}, is shown in Figure 8. It is a model with a higher degree of complexity relatively to M_{1N}. A grid having 32 rows and 64 columns (2,048 samples of seismic velocities) was created according to the methodology previously described, with the objective of using the pyramid algorithm to obtain a set of Haar wavelet coefficients in the sense of multi-scale representation.

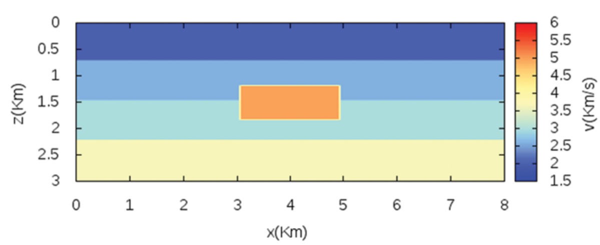

Figure 8 – Numerical model, M_{2N}, with 2,048 discretized values (32 rows and 64 columns) of seismic velocities in the nodes of a rectangular mesh placed over the model.

Parameterization

From M_{2N}, we got the parameterized velocity field, M_{2P}, shown in Figure 9. The parameterized model, M_{2P}, differs from M_{2N}, because it is generated by the concatenation of grid lines as shown in Figure 1. The model was organized as an array with 2,048 values of seismic velocities. The reduction of parameters was applied in M_{2P} to decrease the number of coefficients used in the representation of M_{2N}. Initially, it was necessary 28 coefficients to parameterize M_{2N}. After the reduction of parameters, this amount falls to 7 coefficients, as shown in Figure 10.

Forward Modeling

The ray tracing technique was applied on the parameterized model, M_{2P}. To improve the field illumination, three sources

Table 1 – Coefficients used (or estimated) in the representation of models: M_{1O}, M_{1I} and M_{1T}.

Model	$d_{7,0}$	$c_{7,0}$	$c_{6,0}$	$c_{6,1}$
M_{1O}	19.0000	–5.0000	0.0000	0.0000
M_{1I}	22.0789	–2.1858	–0.2858	0.8426
M_{1T}	29.4156	–9.0510	–1.6000	–8.8000

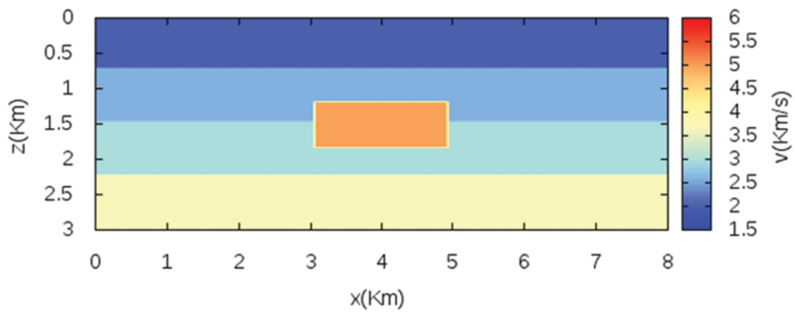

Figure 9 – Parameterized model, M_{2P}, with indices l and k ranging from: $l_o = 1$ to $l_f = 11$, and $k_o = 0$ to $k_f = 1,023$, using 28 non-zero coefficients.

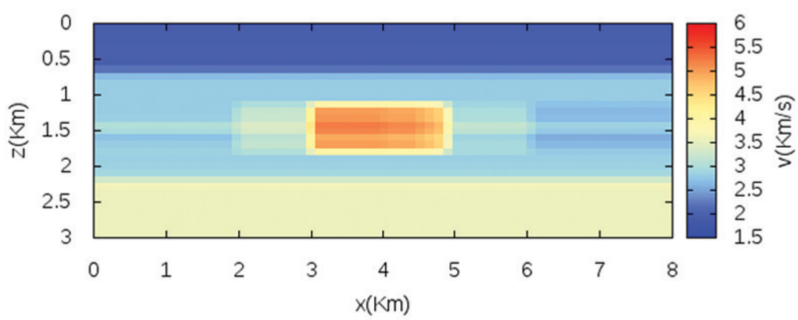

Figure 10 – Target model, M_{2T}, with indices l and k ranging from: $l_o = 1$ to $l_f = 11$, and $k_o = 0$ to $k_f = 1,023$, using 7 non-zero coefficients. It was applied the reduction parameters technique.

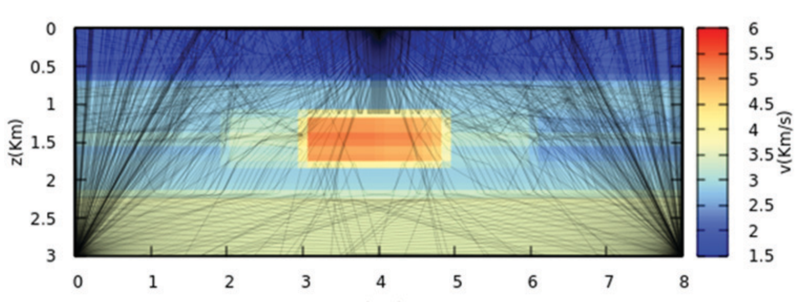

Figure 11 – Ray tracing in the parameterized model, M_{2P}, with 1,500 rays departing from sources located on the surface in the position $S_3 = (4,0)$ and two others, in wells, at positions $S_1 = (0,3)$ and $S_2 = (8,3)$.

were positioned on the target model. Two on wells at: $S_1 = (0,3)$ and $S_2 = (8,3)$, and other on the observation surface at $S_3 = (4,0)$. Ray tracing on the model M_{2P} is presented in Figure 11.

Inversion

The seismic inversion process is applied to the target model M_{2T}, on which has been used the parameters reduction technique. In order to accelerate the inversion process, it is necessary to parallelize the ray tracing inversion algorithm to reduce the processing time.

The relative difference between traveltimes (RDT) of the initial model, M_{2O} (Fig. 12), used in the iterative process, and the target model M_{2T}, was 79.25%. The final product of the inversion process is an inverted model (M_{2I}) represented by Figure 13. The Metropolis algorithm used 166 iterations to achieve its convergence, and it was relatively well succeeded in retrieving the high-velocity intrusion region of the model. For its other parts, it is not possible to say that inversion was good, except in regions near the deep sources. The RDT between M_{2I} and M_{2T} was 1.21%.

Figure 14 shows the relative model difference between M_{2I} and M_{2T} and, the Table 2 shows the coefficients used in the representation of models: inverted, initial and target.

See Cerqueira (2015) for details about relations between parameters c_m, presented in Table 2, and the series coefficients $c_{j,k}$.

It does not make sense, in this work, to use an initial model so much close to the target model, because it is used a global search inversion method and its application is more appropriated for initial distant models.

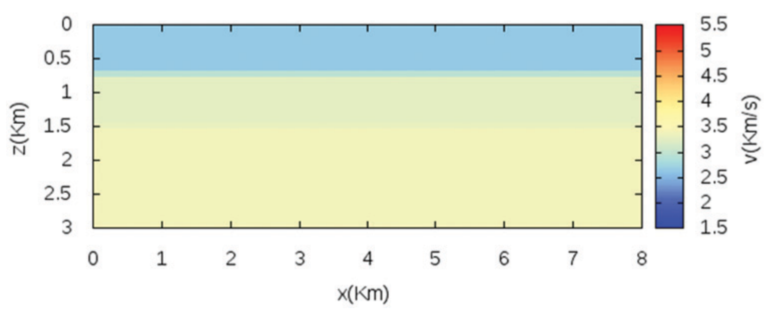

Figure 12 – Initial model, M_{2O}, used by the Metropolis inversion process.

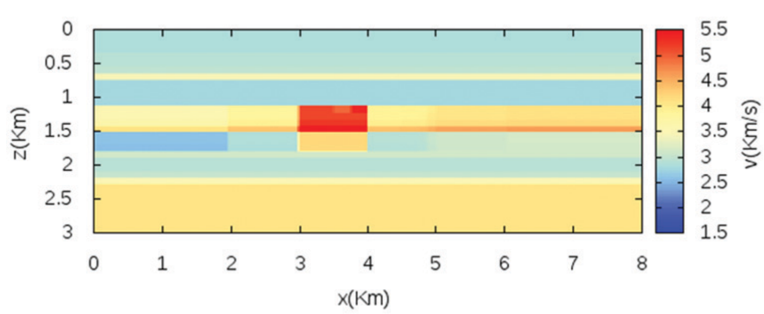

Figure 13 – Inverted model, M_{2I}, parameterized with 7 coefficients obtained by the Metropolis inversion method.

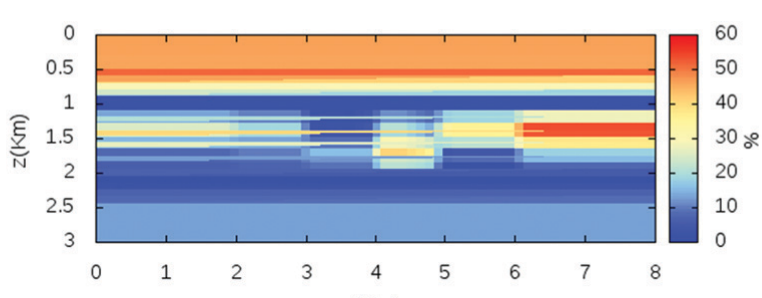

Figure 14 – Relative model difference (RMD) between seismic velocities M_{2I} and M_{2T}.

Table 2 – Coefficients used (or estimated) in the representation of models: M_{2O}, M_{2I} and M_{2T}.

Model	$d_{11,0}$	c_1	c_2	c_3	c_4	c_5	c_6
M_{2O}	144.0000	0.0000	0.0000	0.0000	0.0000	0.0000	–10.0000
M_{2I}	154.4846	–0.0653	–2.6614	–0.3497	–2.4242	–15.9243	–8.2091
M_{2T}	135.2872	1.0231	–3.6499	3.2040	–1.7636	–7.1088	–17.8931

CONCLUSIONS

The global search algorithm, Metropolis, proved to be efficient for the four layers and high-velocity intrusion model. An important point that made possible the development of these two inversions was a small number of coefficients in the parameterization of these models.

An interesting observation about the velocity field models is that the initial models are very different from the target models, demonstrating the applicability of the Metropolis method when it comes to global search inversion of simple models.

The model M_{1I} failed to recover the last layer because the algorithm developed only considers the first traveltime at the receivers, it means that there is not enough information about the last layer.

In the final product of inversion of the high-velocity intrusion model was indentified the high-velocity zone, however, the inversion process was unable to determine with good accuracy the parallel plane layers. In general, the relative differences between seismic velocities were lower than 30% with higher error values in isolated regions.

The tomographic inversion of seismic velocity fields parameterized by Haar wavelet series using the Metropolis method is not very accurate. Local search methods, such as Gauss-Newton and others, can be used in connection in future studies to improve inverted models accuracy.

ACKNOWLEDGMENTS

The authors thank to: UFBA (Universidade Federal da Bahia, Brazil); CPGG-UFBA (Research Center of Geophysics and Geology of Universidade Federal da Bahia, Brazil); CNPq (Conselho Nacional de Desenvolvimento Científico e Tecnológico, Brazil), and Applied Geophysics Thematic Studies Network of Petrobras.

REFERENCES

BASTOS IP. 2013. Parametrização Unidimensional de Campos de Velocidades Sísmicas por Ondaleta de Morlet. Undergraduate Final Project, Geophysics, UFBA, Salvador, BA, Brazil. 52 pp.

CERQUEIRA AG. 2015. Inversão Sísmica Tomográfica de Campos de Velocidades Parametrizados por Série Ondaleta Haar usando o Método Metropolis. Master Dissertation, Geofísica, UFBA, Salvador, BA, Brazil. 51 pp.

CERQUEIRA AG & FIGUEIRÓ WM. 2012. Otimização Empírica de Coeficientes na Parametrização por Série de Ondaleta Haar de Campos de Velocidades Sísmicas. In: Resumos Expandidos, V SimBGf (Simpósio Brasileiro de Geofísica), Salvador, BA, Brazil. CD-ROM.

ČERVENÝ V. 2001. Seismic Ray Theory. Cambridge University Press, Cambridge. 722 pp.

CUNHA PEM. 2009. Uma Introdução às Bases Teóricas para Séries e Transformada de Wavelets. Internal document, CENPES, PETROBRAS, Rio de Janeiro, RJ, Brazil.

HAAR A. 1910. Zur Theorie der Orthogonalen Funktionesysteme. Mathematische Annalen, p. 331–371.

KALOS MH & WHITLOCK PA. 2008. Monte Carlo Methods. WILEY-VCH, Weinheim. 203 pp.

LEE DTL & YAMAMOTO A. 1994. Wavelet Analysis: Theory and Applications. Hewlett-Packard. 44–52 pp.

MARTINEZ AC. 2012. Modelagem Sísmica de Tempos de Trânsito para Campos Bidimensionais de Velocidades Parametrizados por Ondaleta Haar. Undergraduate Final Project, Geophysics, UFBA, Salvador, Brazil. 49 pp.

MORETTIN PA. 1999. Ondas e Ondaletas: da Análise de Fourier à Análise de Ondaletas. Editora da Universidade de São Paulo, EDUSP, São Paulo, SP. 320 pp.

PERIN HM. 2014. Tomografia Sísmica usando Parametrização por Série Ondaleta Haar de Campos de Velocidades. Doctorate Thesis on Geophysics, UFBA, Salvador, BA, Brazil. 99 pp.

PERIN HM & FIGUEIRÓ WM. 2010. Parametrização de campo unidimensional de velocidades sísmicas usando ondaletas. In: Anais do IV Simpósio da Sociedade Brasileira de Geofísica, SBGf, Brasília, DF, Brazil. CD-ROM.

POLIKAR R. 1999. The Story of Wavelets. Rowan University, Dept. of Electrical and Computer Engineering & the Biomedical Engineering Program.

PRESS WH, TEUKOLSKY SA, VETTERLING WT & FLANNERY BP. 1997. Numerical Recipes in Fortran 77: The Art of Scientific Computing. Vol. 1. 1003 pp.

SANTANA JLS & FIGUEIRÓ WM. 2008. Diferentes parametrizações do campo de velocidades sísmicas do modelo geológico da quebra da plataforma continental. In: Resumos Expandidos, III Simpósio Brasileiro de Geofísica, SimBGf, Belém, PA, Brazil, CD-ROM, ISBN: 978-85-88690-13-4.

SANTOS VGB. 2009. Inversão Sísmica Tomográfica usando Norma de Integral de Função. Doctorate Thesis, Universidade Federal da Bahia, Salvador, Brazil. 107 pp.

SANTOS RHM & FIGUEIRÓ WM. 2006. Modelagem acústica bidimensional usando diferentes parametrizações de campos de velocidades. Brazilian Journal of Geophysics, 24(1): 103–115.

SANTOS VGB & FIGUEIRÓ WM. 2011. Seismic ray tomography using L_1 integral norm. Brazilian Journal of Geophysics, 29(2): 347–358.

Recebido em 11 novembro, 2015 / Aceito em 25 maio, 2016
Received on November 11, 2015 / Accepted on May 25, 2016