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# SHALLOW WATERS INTERFERENCE PATTERN FEATURES AND APPLICATIONS

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**ABSTRACT.** Studies in shallow waters pointed out that the acoustic wave interference patterns have potential application in a broad range of problems in underwater acoustics such as: passive sonar, active sonar, array processing, time-reverse mirrors and geoacoustic inversion. For shallow waters underwater sound waveguides, the main features of this research is interference pattern or "striations" obtained by means of Frequency-time or Frequency-distance spectrograms. This work aims to characterize these interference pattern or striations related to "invariant" parameter  $\beta$ . The theory is developed based on real underwater waveguide. The present results confirm that the parameter  $\beta$  is frequency dependent and at high frequencies it tends to the unity. In another words, far from the waveguide cut-off frequency the present real waveguides could be treated by an ideal waveguide. Besides, in this approximation it is possible to make source-receiver passive distance predictions based on the related interference pattern.

Keywords: underwater acoustic, waveguide, interference pattern.

**RESUMO.** Estudos em águas rasas apontam que os padrões de interferência de ondas acústicas têm aplicação potencial em uma ampla gama de problemas de acústica submarina, tais como: sonar passivo, sonar ativo, processamento de *arrays*, espelhos de reversão temporal e inversão geoacústica. O foco principal desta pesquisa é o padrõe de interferência ou estrias obtidas por meio de espectrogramas e gráficos no plano de Frequências e Distâncias. Este trabalho tem como objetivo caracterizar estes padrões de interferência ou estrias com o parâmetro invariante  $\beta$ . A teoria é desenvolvida com base em um guia de onda real. Os resultados deste estudo confirmam que o parâmetro  $\beta$  é dependente da frequência e, em altas frequências, tende à unidade. Em outras palavras, longe da frequência de corte, o comportamento do guia de onda real é semelhante ao guia de onda ideal. Além disso, nesta aproximação, é possível fazer previsões de distância passiva entre o receptor e a fonte com base no padrão de interferência.

Palavras-chave: acústica submarina, guia de ondas, padrão de interferência.

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# INTRODUCTION

Interference is a natural wave propagation phenomenon that occurs in optics, electromagnetic, acoustic and guantum mechanics (matter waves) as well. Few decades ago, Chuprov (Chuprov, 1982) apply this wave feature in underwater acoustic. In this pioneering work, Chuprov (Chuprov, 1982) applying the invariant adiabatic theory (Brekhovskikh & Godin, 1990; D'Spain & Kuperman, 1999; Sell & Culver, 2011; Harrison, 2011; Rouseff & Spindel, 2002; Cockrell, 2011; Jensen et al., 2011) shown that it is possible to use underwater sound interference to obtain a spectrogram of source-receiver range in frequency domain. Nowadays (Brekhovskikh & Godin, 1990; D'Spain & Kuperman, 1999; Sell & Culver, 2011; Harrison, 2011; Rouseff & Spindel, 2002; Cockrell, 2011; Jensen et al., 2011), studies pointed out that these interference patterns have a potential application in a broad range of problems in underwater acoustics, such as: passive sonar, active sonar, array processing, time-reverse mirrors and geoacoustic inversion (Cockrell, 2011).

In the present work, the interest is use these ideas to characterize underwater sound propagation in Brazilian shallow waters in order to develop a passive detection and classification of ships. More specifically, it's well known that propeller noise from ships produces a sound pressure with a great variety of frequency and amplitudes, the intensity of sound generated a striation in some conditions. Such striations are visualized in spectrograms frequency-time or frequency-distance plots.

The shift in the structure of the interference pattern as a function of frequency and range is a robust feature of waveguide propagation and is described by a scalar parameter refereed to as waveguide invariant  $\beta$  (Jensen et al., 2011).

This article describe the interference pattern and  $\beta$  invariant theory, apply it to a real environment scenario in order to make source-receiver distance estimation.

#### THEORY

The wave equation in the frequency domain is the Helmholtz Equation Eq. (1) (Jensen et al., 2011; D'Spain & Kuperman, 1999). Solving it it is possible do find the pressure sound field for a point source. The potential displacement  $\psi$  proportional to the pressure field P in the relation  $P = -K \bigtriangledown^2 \psi$  where  $K = \rho c^2$  is the Bulk modulus (Jensen et al., 2011),  $\rho$  is the medium density and c the sound speed. The parameter  $\omega$  is the angular frequency,  $S_{\omega}$  is the power source, z and  $z_s$  are the receiver and source depth, respectively, H is the maximum waveguide depth, c(z) is the SSP (Sound Speed Profile) as function of depth z.

$$\frac{d^2\psi}{dz^2} + \left(\frac{\omega^2}{c^2(z)} - \kappa_r^2\right)\psi = S_w\delta(z - z_s) \qquad (1)$$

To plot the waveguide invariant frequency-range striations (Fig. 1) one must transform the time-domain wave equation into the frequency domain. Then transform that frequency-domain Eq. (1) into the horizontal wavenumber domain, and then separate it into a depth-dependent equation and range-dependent equation Eq. (2).

$$P = \sum_{m=0}^{m_{\text{max}}} \varphi_n(z_s) \varphi_n(z) H_0(\kappa_{rm} r)$$
(2)

where m is the mode number and  $m_{\max}$  is the maximum number of modes in the waveguide.



Figure 1 – Interference Pattern.

This intensity of sound Eq. (3) produced by an acoustic source is proportional to the mean square pressure:

$$I = \frac{|P|^2}{\rho c} \tag{3}$$

where  $\rho$  is the density of the medium and c the velocity of sound. The intensity of sound received could be decomposed in discrete normal modes (Jensen et al., 2011).

The range-dependent equation provides the  $e^{i\kappa_{rm}r}$  terms that interfere with another (Cockrell, 2011). The depth-dependent equation  $\varphi_n(z_s)\varphi_n(z)$  provides the  $\kappa_{rm}$  which depends on  $\omega$ . The difference between pairs of horizontal wavenumber  $\Delta k_{mn} = k_{rm} - k_{rn}$  in Eq. (4) and how that difference depends on  $\omega$ , determines the interference pattern or striations.

$$I(r,\omega) \equiv I_0 \sum_n B_n^2 + 2 \sum_{m \neq n} B_m B_n \cos(\Delta k_{mn(\omega)} r)$$
 (4)

The next subsection contain explanation about WKB solution for the  $\Delta k_{mn}$  term in Eq. (2) and the interference wavelength, parameters used to plot striations and to estimate distance.

# Wentzel-Kramers-Brillouin (WKB) Approximation to Solve Horizontal Wavenumber ( $\kappa_r$ )

The WKB approximation is a general mathematical technique for approximating the solution of certain types differential equations in applied math, quantum mechanics (Sakurai, 1985), electromagnetism and underwater acoustics (Brekhovskikh & Godin, 1990).

In this work is used to evaluate the horizontal wavenumber of modes in ocean acoustic waveguides. Under the WKB approximation horizontal wavenumber are evaluate solving  $\kappa_r$  in the Eq. (5) (Lee et al., 1988; Cockrell & Schmidt, 2010):

$$m = \phi(\kappa_r, \omega) + \Delta \phi_{dn}(\kappa_r, \omega) + \Delta \phi_{up}(\kappa_r, \omega) + 1$$
  
(5)  
$$m = 1, 2, 3, \dots \text{(mode number)}$$

where:

$$\phi(\kappa_r, \omega) = \frac{1}{\pi} \int_{z_1}^{z_2} \kappa_z(z, \kappa_r) dz$$

$$\kappa_z = \sqrt{\left(\frac{\omega}{c(z)}\right)^2 - \kappa_r^2}$$
(6)

 $\kappa_z$  is the vertical wavenumber,  $z_1$  and  $z_2$  are the sea surface (z = 0) and bottom (z = H) or the turning point  $\kappa_z(z) = 0$  and:

$$\Delta \phi_{up}(\kappa_r, \omega) = \begin{cases} -\frac{1}{4} \text{ if } \kappa_r > \kappa(0) \text{ mode has an} \\ \text{upper turning point} \\ -\frac{1}{2} \text{ if } \kappa_r < \kappa(0) \text{ modes reflect} \\ \text{from surface} \end{cases}$$
$$\Delta \phi_{dn}(\kappa_r, \omega) = \begin{cases} -\frac{1}{4} \text{ if } \kappa_r > \kappa(z) \text{ mode has a} \\ \text{lower turning point} \\ \phi_b \text{ if } \kappa_r < \kappa(z) \text{ modes reflect} \\ \text{from hottom} \end{cases}$$

where  $\phi_b(\kappa_r,\omega)$  is:

- Bottom half-space  $\phi_b(\kappa_r, \omega) = \frac{1}{2\pi} \times$  the phase angle of the bottom halfspace reflection coefficient;
- Bottom fluid  $-\frac{1}{2} \leq \phi_b(\kappa_r,\omega) \leq 0$  and;

• Bottom vacuum (Ideal waveguide)  $\phi_b(\kappa_r,\omega) = -\frac{1}{2}$ 

For instance, in an ideal waveguide  $\Delta \phi_{up} = -\frac{1}{2}$  and  $\Delta \phi_{dn} = -\frac{1}{2}$  and  $\kappa_{zm} = \frac{m\pi}{H}$ . Consequently,

$$\kappa_{rm} = \sqrt{\kappa^2 - (m\pi/H)^2}.$$

#### $\beta$ Invariant

Striations width interfere in the calculation value of the invariant  $\beta$ , that makes uncertainties in the distance estimation Eq. (7) (Chaves et al., 2013).

$$r = \frac{\beta \times \omega \times dr}{d\omega} \tag{7}$$

The parameter  $\beta$ , also could be thought, as a variation of the phase velocity v Eq. (9) and group velocity u Eq. (10) (Clay & Medwin, 1977), from adjacent modes as in Eq. (8) (Chuprov, 1982) (Brekhovskikh & Godin, 1990):

$$\beta = \left(\frac{u}{v}\right)^2 \frac{dv}{du} \tag{8}$$

$$v = \frac{\omega}{\kappa_r} \tag{9}$$

$$u = \frac{d\omega}{d\kappa_r} \tag{10}$$

#### Interference Wavelength

The Interference wavelength between to adjacent modes is defined in Eq. (11) (Clay & Medwin, 1977):

$$\Lambda_{mn} = \frac{2\pi}{\kappa_{rn} - \kappa_{rm}} \tag{11}$$

### SCENARIO

where:

In this work, the scenario is the real waveguide with a isovelocity sound speed profile, also named Pekeris Waveguide (Worzel et al., 1948). In the Pekeris waveguide the acoustic impedances,  $\rho c$  between the water and the sediment, are used in order to calculated the phase shift.

All parameters used in this work to calculated  $\beta$  and the interference pattern are from the classical Long Island experiment (Clay, 1959, 1964; Clay & Medwin, 1977).

This scenario was chosen because only two modes are propagating in this waveguide and the environmental configuration is well known, in order to allow precise propagation constants calculation. For the scenario in the Figure 2, the waveguide depth is H = 22.6 meters, source depth  $Z_S = 10.4$  m, receiver depth  $Z_R = 19.8$  m, sound speed in the water is  $c_0 = 1508$  m/s and water density is  $\rho = 1033$  kg/m<sup>3</sup>.

The sediment sound speed is  $c_1=$  1689 m/s (Clay, 1964) and the density is  $\rho_1=$  2066 kg/m^3.





# RESULTS

Figure 1 shows the interference pattern or striation evaluated using Eq. (4) with the propagations constants for adjacent modes, in this scenario, modes 1 and 2. The maximum for each frequency is plotted resulting a plot with an interference pattern. At high frequencies the interference wavelength is higher than frequencies approaching the cut-off.

Figure 3a represents a simulated spectrogram received from one hydrophone, where it is possible to see the interference pattern. The source-receiver velocity is 2 m/s. The only real data available is at the 147.8 Hz frequency. Others frequencies are calculated using Eq. (4).

Applying Eq. (5) described in the Section Scenario, it is possible to evaluate  $\kappa$ ,  $\kappa_z$  and  $\kappa_r$  for each mode and frequency. The  $\beta$  value for each frequency was calculated by Eq. (8) and plotted in Figure 3b. It could be noted in that the  $\beta$  value far from the waveguide cut-off frequencies approach the unity (Fig. 3d). Approaching this frequency, the  $\beta$  value increase. Ideal waveguides have  $\beta = 1$  for all band of frequencies (Chuprov, 1982).

Figure 3c shows the interference wavelength as function of frequency, this value was used to calculate the distance using Eq. (7).

Table 1 shows the distance calculated by the interference pattern and the real distance from the scenario. The error is about 3.4% meaning a good agreement.

### CONCLUSION

Based on the classical Long Island Experiment this work analyzed some waveguide features in order to apply some passive distance techniques. Preliminary results in passive distance estimation using the interference pattern and the invariant  $\beta$  suggest that this kind of technique is possible. The percent error in this first distance estimative is about 3.4%.

The interference pattern or striations are an interesting underwater acoustic feature. In shallow waters environment, like used in this work, it could be possible to calculated the  $\beta$  parameter as function of frequency and to plot the striations in the frequency-distance plane.

Additionally, the value of  $\beta$  far from the cut-off waveguide approach the unity. So, the result suggests that in these frequencies the real waveguide could be treated by an ideal waveguide to use  $\beta$  to estimated source-receiver distances.

At high frequencies, far from cut-off, additional propagation modes contribute to modal summation. At distance much larger than the wavelength these frequencies tend to attenuate more rapidly due additional interaction with boundaries. In range dependents scenarios the solution of the WKB approximation is an hyper geometric function.

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Figure 3 – Passive distance technique. (a) Spectrogram. (b) Frequency-distance plane. (c) Interference Wavelength. (d)  $\beta$ .

	Frequency (Hz)	$\beta$	Distance (m)	Distance Calculated (m)	Error (%)	
ĺ	146.1	1.3619	2992	3095	3.4	

Table 1 Distance recults

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