ASSESSING TIME-LAPSE FULL-WAVEFORM INVERSION STRATEGIES IN A BRAZILIAN PRE-SALT SETTING

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ABSTRACT. We conducted a comparative study of time-lapse full-waveform inversion (time-lapse FWI) strategies, considering a typical deep-water Brazilian pre-salt setting. This study employed a realistic P-wave model, ocean bottom nodes (OBN) acquisition, noisy data, and a Gaussian anomaly to represent time-lapse model changes. We evaluated the four most commonly used time-lapse FWI schemes. In the first, known as parallel time-lapse FWI, two independent FWI processes are performed from the same initial model, utilizing baseline and monitor datasets. In the second strategy, namely sequential time-lapse FWI, the retrieved baseline model serves as the starting model for inverting monitor data. In both cases, the time-lapse model is derived by subtracting the retrieved baseline model from the retrieved monitor model. The remaining two methods were double-difference and central-difference time-lapse FWI. Our findings demonstrate that all these schemes can detect model variations of 3%. Remarkably, the central-difference time-lapse FWI method demonstrated superior accuracy in producing time-lapse models and, as such, presents itself as a promising strategy for implementation in time-lapse studies within Brazilian pre-salt regions.

Keywords: Time-lapse FWI; Brazilian pre-salt; ocean bottom nodes (OBN); Central-difference FWI; Seismic reservoirs
INTRODUCTION

Characterizing and monitoring reservoirs are vital in strategically planning oil and gas production operations. Time-lapse studies serve as a critical tool for discerning nuanced variations within seismic reservoir properties (Sambo et al., 2020). These variations can be attributed to many factors, including the dynamic processes associated with fluid injection and extraction. Time-lapse investigations delve into the meticulous analysis of geophysical data acquired through two or more distinct seismic surveys conducted at varying temporal intervals within the same geographical area (Lumley, 2001). Through these comparative examinations, geoscientists can understand how subsurface physical properties evolve over time, enabling them to optimize production strategies, enhance hydrocarbon recovery, and make informed decisions regarding reservoir management and environmental impact assessments (Nguyen et al., 2015; Cardoso et al., 2022).

Time-lapse approaches operate under the premise that features within the target area exhibit more significant variations from one seismic acquisition to another when contrasted with the surrounding region. Within this context, alterations in the overburden are obtained by analyzing the disparities among each survey conducted in the time-lapse domain. This approach is grounded in the reliable assumption that geological attributes, which remain constant over time and contribute to the seismic image, including factors like lithology, porosity, and shale content tend to cancel out. Consequently, the remaining changes observed (time-lapse model) are primarily associated with the time-varying dynamic properties associated with, for instance, fluid flows. These dynamic properties encompass alterations in fluid saturation and pore pressure, which are crucial factors to monitor and understand when evaluating the subsurface environment.

However, the issue of non-repeatability (NR) can introduce a significant challenge in obtaining suitable time-lapse models (Borges et al., 2021). NR issues can cause false time-lapse anomalies, which may be mistakenly interpreted as alterations in the physical characteristics of the subsurface (Zhou and Lumley, 2021b). To address this challenge, the deployment of ocean bottom node (OBN) surveys has gained prominence, representing a practical solution to mitigate NR errors (Yang et al., 2016). This shift towards OBN surveys has been driven by the inherent difficulties associated with streamer surveys in managing NR, azimuth illumination, fold and longer offsets concerns Cypriano et al. (2019). A case in point is the Tupi Nodes pilot project, a study conducted by Cruz et al. (2021), which underscores the advantages of OBN technology in the context of deep-water Brazilian pre-salt reservoirs. The Tupi Nodes pilot project demonstrated a highly favorable time-lapse response by leveraging full-waveform inversion (FWI) (Virieux and Operto, 2009) as an integral component of the time-lapse seismic processing toolkit. FWI enables precise estimation of rock property changes (Warner et al., 2013; Górszczyk et al., 2021; da Silva et al., 2024), such as P-wave velocity alterations, further enhancing subsurface analysis accuracy in dynamic geological environments. Consequently, operating the FWI technique to analyze OBN data can yield significantly more precise subsurface models.

This work essays a comprehensive comparative analysis of time-lapse FWI methodologies within the context of a typical deep-water Brazilian pre-salt geological setting. Specifically, we consider an OBN acquisition geometry to determine the most effective time-lapse FWI seismic monitoring technique for identifying changes in the properties of ultra-deep reservoirs while contending with noisy data. It is important to emphasize that, owing to the inherent nonlinear characteristics of FWI, our investigation also delves into the nonlinear artifacts introduced by the data inversion process. These artifacts can introduce subsurface model changes unrelated to...
reservoir variations, as highlighted in prior researches (Yang et al., 2015; Zhou and Lumley, 2021b; da Silva et al., 2023). The choice to employ FWI is rooted in its standing as a robust seismic inversion method that leverages the comprehensive physical principles embedded within a wave equation (Virieux and Operto, 2009). From a practical standpoint, FWI is usually formulated as a local optimization problem, where the primary objective is to minimize the sum of squared differences between the modeled data (derived from the wave equation solution) and the observed seismic data (Fichtner, 2010). Utilizing the entire waveforms, rather than solely travel times or amplitudes, enables a comprehensive evaluation of the propagation of waves that illuminate the subsurface.

Time-lapse seismic methodologies entail the implementation of two distinct seismic surveys within the same geographical area. In the initial survey, known as the baseline data acquisition, baseline data is recorded, while the follow-up surveys record monitor data. In this work we explore the applicability of the four most widely employed time-lapse FWI strategies, considering one monitor data acquisition. The first strategy, **parallel time-lapse FWI** (Lumley, 2001), consists of conducting two independent FWI processes starting from the same initial model. The baseline and monitor models are constructed using the respective baseline and monitor data sets. The time-lapse model is subsequently ascertained by subtracting the retrieved baseline model from the retrieved monitor model. In the second strategy, **sequential time-lapse FWI** (Routh et al., 2012), the retrieved baseline model serves as the starting point for inverting the monitor data. The time-lapse changes are calculated by subtracting the retrieved baseline model from the new retrieved monitor model obtained through the inversion of the monitor data. In the third time-lapse FWI approach, double-difference time-lapse FWI (DDWI) (Yang et al., 2015), the retrieved baseline model serves as the initial model for inverting the difference between the baseline and monitor data sets. The time-lapse changes are derived by subtracting the retrieved baseline model from the new model acquired through this inversion process. Finally, in the fourth time-lapse FWI approach considered in this work, **central-difference time-lapse FWI** (CFWI) (Zhou and Lumley, 2021a), the retrieved baseline and monitor models are employed as initial models for a new FWI application. In particular, the retrieved baseline model is used as the starting model for inverting the monitor model, producing a second monitor model. Simultaneously, the retrieved monitor model is employed as the initial model for inverting the baseline data, creating a second baseline model. The resulting time-lapse model is then computed by subtracting the baseline models’ arithmetic mean from the monitor models’ arithmetic mean.

The structure of this work is as follows. In the subsequent section we briefly present the main ingredients of FWI and the time-lapse schemes employed in this work. Then, in the numerical experiments section, we present our implementation of a 2D acoustic FWI in the time-domain employing a finite difference computational algorithm. Notably, our focus is examining the sensitivity of the aforementioned time-lapse methodologies employing FWI and OBN. Finally, in the last section we discuss and present our concluding remarks, outlining the best time-lapse strategies and prospects for future research endeavors.
METHODS

Full-waveform inversion (FWI)

Full-waveform inversion (FWI) is a powerful technique that aims to retrieve a high-resolution subsurface model iteratively (Virieux and Operto, 2009). In this approach, an initial model is considered, and seismic waveforms are modeled based on this model by numerically solving a wave equation. In this work we assume the premise that the following acoustic wave equation describes the wavefields:

\[ \nabla^2 p_s(\vec{x}, t) - \frac{1}{m^2(\vec{x})} \frac{\partial^2 p_s(\vec{x}, t)}{\partial t^2} = f_s(\vec{x}_s, t), \]  

where \( p_s \) denotes the modeled wavefield, \( m \) represent the P-wave velocities (model parameters) to be estimated, and \( f_s(\vec{x}_s, t) \) is a seismic source fired at the position \( \vec{x} = \vec{x}_s \), with \( \vec{x} \) and \( 0 \leq t \leq T \) representing, respectively, the spatial coordinate and the time; \( T \) is the maximum recording time.

Due to the FWI non-linearity, model recovery is performed iteratively (Virieux and Operto, 2009). In this work we consider a Fletcher-Reeves nonlinear conjugate gradient method (see, for example, Nocedal and Wright (2006)) to solve the FWI problem. We chose to use this nonlinear conjugate gradient algorithm because it has been shown to work well when analyzing real data from the pre-salt region of Brazil, as recently presented by da Silva et al. (2024). This optimization method involves updating the subsurface model by minimizing an objective function \( \phi \) in the following way:

\[ m_{i+1} = m_i - \alpha_i h(m_i), \quad \text{for} \quad i = 0, 1, 2, \cdots, N_{\text{iter}}, \]  

where for \( N_{\text{iter}} \) represents the maximum number of FWI iterations and \( \alpha_i > 0 \) denotes the step size (Nocedal and Wright, 2006), and

\[ h(m_i) = \begin{cases} 
\nabla_m \phi(m_0), & \text{if } i = 0 \\
\nabla_m \phi(m_i) + \zeta(m_i)h(m_{i-1}), & \text{for } i = 1, 2, \cdots, N_{\text{iter}} 
\end{cases} \]  

is the descent direction, \( \nabla_m \phi(m) \) denotes the gradient of the objective function, and

\[ \zeta(m_i) = \frac{\nabla_m \phi(m_i) \left( \nabla_m \phi(m_i) - \nabla_m \phi(m_{i-1}) \right) \nabla_m \phi(m_{i-1})}{\nabla_m \phi(m_{i-1}) \nabla_m \phi(m_{i-1})}. \]  

We compute the gradient of the objective function efficiently by applying the adjoint-state method (see, for instance, Plessix (2006)). In this regard, the gradient is obtained by cross-correlating the wavefield \( p_s \) with the
adjoint-wavefield $\lambda_s$, given by (Lailly, 1983):

$$\nabla_m \phi(m) = - \sum_s \int_0^T \lambda_s(T-t) \frac{\partial^2 p_s(m,t)}{\partial t^2} \, dt,$$

(6)

in which the adjoint-wavefield is obtained by solving

$$\nabla^2 \lambda_s(\vec{x},t) - \frac{1}{m^2(\vec{x})} \frac{\partial^2 \lambda_s(\vec{x},t)}{\partial t^2} = \sum_r \Gamma^\dagger_{s,r}(\Gamma_{s,r} p_s(m,t) - d_{s,r}(t)),$$

(7)

which is the adjoint-state wave equation, where $\dagger$ denotes the transpose.

### Time-lapse FWI strategies

In this section we present the time-lapse FWI frameworks considered in this work. Time-lapse analyses involve the conduction of two distinct seismic surveys operated within the same geographic region but at varying time intervals. The initial survey yields what we term as the baseline data, denoted as $d_b$, while the subsequent survey captures the monitor data, labeled as $d_m$. For a concise notation, we adopt the expression $\delta m$ to represent the retrieved time-lapse model.

In this work we explore four distinct time-lapse FWI schemes:

(i) **Scheme I:** In the first scheme, also known as the parallel time-lapse FWI (Lumley, 2001), we perform two independent FWI procedures from the same initial model, denoted as $m_0$, where the baseline and monitor seismic data are utilized in these detached inversions. The resulting time-lapse model is derived by subtracting the retrieved baseline model, $m_{b}$, from the retrieved monitor model, $m_{m}$, denoted as:

$$\delta m_{\text{par}} = m_{m} - m_{b}.$$  

(8)

(ii) **Scheme II:** In the second scheme, also known as the sequential time-lapse FWI (Routh et al., 2012), we start by obtaining the baseline model, $m_{b}$, inverting the baseline data, $d_b$, by starting from the initial model $m_0$. Subsequently, a new FWI is conducted, inverting the monitor data $d_m$ by using the retrieved baseline model $m_{b}$ as the initial model. The resulting time-lapse model is obtained from

$$\delta m_{\text{seq}} = m'_{m} - m_{b},$$

(9)

where $m'_{m}$ represents the retrieved monitor model when initiated from $m_{b}$.

(iii) **Scheme III:** In the third scheme, also known as the double-difference time-lapse FWI (DDWI) (Yang et al., 2015), we first obtain the baseline model, $m_{b}$, inverting the baseline data, $d_b$, from the initial model $m_0$. Subsequently, a new FWI is conducted, starting from the retrieved baseline model $m_{b}$ and inverting the difference between the double-difference: $\delta d(m_{dd},t) = [\Gamma_{s,r} p_{s_b}(m_{b},t) - \Gamma_{s,r} p_{s_{dd}}] - [d_{b}(t) - d_{m}(t)]$, where $\Gamma_{s,r} p_{s_b}(m_{b},t)$ and $\Gamma_{s,r} p_{s_{dd}}(m_{dd},t)$ are the modeled data from the retrieved baseline model, $m_{b}$, and
the model \( m_{dd} \) to be reconstructed, respectively. The resulting time-lapse model is then obtained through

\[
\delta m_{dd} = m_{dd} - m_b,
\]

with \( m_{dd} \) representing the resulting model from the inversion of the observed data difference, \( d_b(t) - d_m(t) \), to the modeled data difference, \( \Gamma_{s,r}p_{sb}(m_b, t) - \Gamma_{s,r}p_{sd} \), starting from the model \( m_b \).

(iv) **Scheme IV**: In the last scheme, also known as the central-difference time-lapse FWI (CFWI) (Zhou and Lumley, 2021a), we conduct two independent FWI processes starting from the model \( m_0 \), using the baseline and monitor seismic data to retrieve the baseline \( m_b \) and monitor \( m_m \) models, in a manner akin to the parallel time-lapse FWI strategy. This is the first step. The second step also involves two FWI procedures, closely resembling the sequential time-lapse FWI strategy, with one using the monitor data \( d_m \) and initiating from \( m_b \) to establish a new monitor model \( m'_m \). The other utilizes the baseline data \( d_b \) and starting from \( m_m \) to generate a new base model \( m'_b \). Then, the resulting time-lapse model is obtained from the average of the differences between the monitor and baseline models from both steps, specifically

\[
\delta m_{cd} = \frac{m_m + m'_m}{2} - \frac{m_b + m'_b}{2}.
\]

Figure 1 provides a graphical representation summarizing these time-lapse strategies, shedding a comprehensive understanding of the workflows of these time-lapse FWI schemes.
NUMERICAL EXPERIMENTS

In order to conduct a comparative assessment of the time-lapse FWI schemes, we consider a realistic Brazilian pre-salt P-wave velocity model, as initially modified from Karsou (2020), which is depicted in Fig. 2(a). This model encompasses a geological structure featuring a deep-water layer, with an average depth of 2 km, underlying post-salt marine shales and rock layers, a substantial salt body, a pre-salt oil reservoir, and bedrock below. We discretize this P-wave model into a regular grid comprising 840 × 280 points, with each cell measuring 25 × 25 m. We adopt an OBN geometry for data acquisition in all numerical experiments. This geometry comprises 23 nodes situated on the ocean floor, spaced at intervals of 400 m (indicated by white triangles in Figure 2(a)), along with 257 seismic sources positioned at a depth of 10 m (marked by the green line in Figure 2(a)), and separated by 50 m apart. The seismic source employed was a Ricker wavelet with a peak frequency of 5 Hz. We set to 7 s the acquisition time.

Using the P-wave model presented in Fig. 2(a), we generate the baseline data set $d_b$ by employing the

![Figures 2 and 3](image)

**Figure 2**: (a) Typical Brazilian pre-salt P-wave velocity model and OBN acquisition, where the white triangles represent the nodes and the green line are the shot points. (b) Initial model $m_0$. (c) True time-lapse model changes.

**Figure 3**: Observed seismograms. (a) Baseline data, $d_b$. (b) Monitor data, $d_m$. (c) Difference between monitor and baseline data, $d_m - d_b$. 

2D time-domain acoustic wave equation (1). Subsequently, we introduce Gaussian noise with a signal-to-noise ratio (SNR) of 10dB to mimic real-world conditions. Moreover, we perturb the baseline model by introducing a bivariate Gaussian anomaly to construct the monitor model, as depicted in Fig. 2(c). In this context, we simulate a time-lapse model featuring a maximum P-wave velocity reduction of 3% at the reservoir level. While the differences between the monitor and baseline true models might not be readily discernible to the naked eye, denoting it as the true time-lapse model. With the monitor model in place, we then generate the monitor data $d_m$ using the 2D time-domain acoustic wave equation (1) and subsequently apply Gaussian noise with an SNR of 10dB. In Fig. 3, Panels (a) and (b) depict receiver-gathers corresponding to the first node from the baseline and monitor models, respectively, while Panel (c) highlights the difference between these seismograms.

To solve the FWI problem, we consider a scaling factor of 0.05, following the recommendation by Köhn (2011). We set the stopping criterion for our numerical simulations at 50 iterations. To avoid drastic effects caused by cycle-skipping issues (Hu et al., 2018), we consider that the initial model $m_0$ was well determined from a kinetic point of view. In particular, we generate the initial model $m_0$ by smoothing the true model (Fig. 2(a)) with a Gaussian operator with a standard deviation of 250m. The model $m_0$ is depicted in Fig. 2(b). Figure 4 shows the retrieved FWI models. The FWI results exhibit remarkable similarity; thus, the minimal dissimilarities are only discernible in the time-lapse domain. Figure 5 shows the resulting time-lapse models associated with the four time-lapse FWI schemes. Within this visual representation, a notable observation emerges regarding Scheme II, where the resulting time-lapse model is predominantly marked by artifacts throughout the model, as showcased in Fig. 5(b). Unfortunately, this outcome falls short of expectations, as it deviates significantly from the true time-lapse model (Fig. 2(c)). Scheme III also exhibits several artifacts, albeit on a smaller scale, particularly near the upper regions of the salt layer. In contrast, Scheme I and Scheme IV demonstrate a more effective mitigation of the time-lapse noises from the ocean floor and the uppermost salt layer. Furthermore, it is crucial to recognize that, within Scheme I and Scheme IV, certain artifacts do exist; however, their presence is unrelated to geological structures, as seen in Scheme II and Scheme III. Instead, these artifacts are primarily associated with noise in the central portion of the model, spanning distances between 5 and 15 km, and mark the wavepaths at the model’s extremities where seismic illumination is practically absent.

Figure 6 shows P-wave vertical-velocity profiles depicting model changes arising from the time-lapse FWI schemes against the true time-lapse model represented by the black curve. These profiles are observed at distinct distances: 10.25km, 10.50km (the central region of the Gaussian anomaly), and 10.75km. All the schemes can detect time-lapse changes within the pre-salt reservoir. Nevertheless, Scheme II exhibits heightened discrepancies when evaluating regions beyond the primary target area. This is evident from larger amplitude variations around $\delta m = 0$, which suggests pronounced errors relative to Schemes I, III, and IV. Furthermore, Scheme II appears to overreach in estimating time-lapse changes, as indicated by the purple curve. On a brighter note, the efficacy of Schemes I, III, and IV is highlighted by their adeptness at identifying the time-lapse changes, particularly within depths ranging from 5.5 to 6.0km. Notably, Scheme IV outperforms the rest in its remarkable precision within the area of peak seismic illumination (central segment of the P-wave model). This prowess is noted through the almost perfect alignment of the blue curve (representing Scheme IV) with the benchmark black curve in the pre-salt domain, as showcased in Fig. 6(b).
CONCLUSION

In this work we have compared the most used time-lapse FWI methodologies in the literature, drawing inspiration from the challenging ultra-deep reservoirs from the Brazilian pre-salt oil region. Our findings have unveiled the potentialities and difficulties of these time-lapse strategies in detecting subtle changes in P-wave velocity within a typical Brazilian pre-salt oil reservoir, all while utilizing cutting-edge OBN technology. Importantly, it is worth noting that the inversion artifacts exhibited distinctive behaviors across the various time-lapse FWI

Figure 4: FWI resulting models. (a) Retrieved baseline model, $m_b$, and (b) retrieved monitor model, $m_m$. Retrieved (c) monitor model $m'_m$ from baseline model $m_b$, and (d) retrieved baseline model $m'_b$ from monitor model $m_m$. (e) Retrieved DDWI model $m_{dd}$ from baseline model $m_b$ using double-difference data.

Figure 5: Recovered time-lapse model changes from: (a) Scheme I, (b) Scheme II, (c) Scheme III (DDWI), and (d) Scheme IV (CFWI).
Within the Brazilian pre-salt case study, Scheme I (parallel time-lapse FWI) and Scheme IV (central-difference time-lapse FWI) have emerged as promising strategies, providing robust and accurate time-lapse responses, even when analyzing noisy data. These two strategies have effectively represented the expected time-lapse model changes, as depicted in Fig. 5. On the other hand, while yielding satisfactory results, Scheme III has shown a tendency to introduce significant artifacts at the top of the salt layer. These artifacts, while intriguing, could potentially lead to misleading geophysical interpretations. Scheme II, however, has presented some limitations, as it introduces time-lapse changes across the entire P-wave model, not solely related to reservoir production. This scheme led to a higher incidence of changes in the overburden and possibly spurious correlations linked to noise. From a computational perspective, the complexity of these schemes is intricately related to the number of inversions conducted within each proposed workflow. Scheme I, Scheme II, and Scheme III exhibit comparable computational efforts, with the additional misfit data in Scheme III constituting a computationally trivial aspect. Conversely, Scheme IV entails two extra inversions compared to the previous schemes, necessitating more substantial computational resources.

Our future endeavors are poised to explore the resilience of these four strategies concerning non-repeatability (NR) effects, encompassing factors such as water velocity variations and errors in shot-receiver positioning. Furthermore, we aim to model the complexities of wave physics, incorporating effects related to density and shear velocity. These endeavors will undoubtedly contribute to a more comprehensive understanding of time-lapse FWI methodologies and their adaptability in challenging geological settings.
AUTHOR CONTRIBUTIONS

All authors contributed equally to this work.

CONFLICTS OF INTEREST

The authors declare no conflict of interest.

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