# Improving velocity model building with Physics Informed Neural Networks and Fourier Neural Operators

Bernardo Fraga (CBPF), Ana Paula Müller (Petrobras), Clécio de Bom (CBPF)



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**Full-waveform Inversion (FWI) is one of the most commonly used methods in the industry for velocity model building. This inversion problem is ill-posed, and moreover the conditions on which seismic data are taken can be less than ideal, with problems such as the absence of low frequencies and limited offsets; the success of the FWI depends heavily on the initial velocity assumption. Machine Learning (ML) methods are now widely used in different fields, including seismic, and ML methods have been used to improve or accelerate FWI. Some substitute it altogether by a supervised neural network, while others approach specific FWi issues. One approach that shows promise is to build a Physics Informed Neural Network (PINN) by using the gradients of the FWI to update the weights of the neural network; this method does not require an input to produce velocity models, relying only on the FWI weights, and converges to satisfactory results faster than using a regular supervised model. In this work, we propose the use of Fourier Neural Operators (FNO), which can learn mappings in function space, instead of normal Euclidean space, making them faster and more efficient. We build a UNet where the convolutional layers are substituted by Fourier layers, using the gradients of the FWI to update the weights. We compare the UNet with FNOs with a traditional UNet, and find that networks using FNOs are lighter and produce similar results.**

Traditional neural networks (NNs) learn a map between a finite dimensional Euclidean space and a set of classes (as in image classification) or between two finite dimensional Euclidean spaces (e.g. segmentation). Neural operators were originally built to learn mappings between functions in order to solve Partial Differential Equations (PDEs), to try to circumvent the problems faced by traditional PDE solvers, such as finite difference or finite element methods. One of the main advantages of neural operators is their mesh-invariance, so that these models can learn from one less accurate discretization but still can give high resolution predictions.

FNOs are a specific type of neural operators that transform the input to Fourier space; convolutions then become pointwise multiplication, making the operation faster. A Fourier layer basically consists of a Fourier transform followed by a linear transformation, and an inverse Fourier transform to bring the data back to the spatial domain. FNOs have been used in supervised learning approaches to map from shot gathers to velocity models, showing superior results when compared to standard convolutional neural networks (CNNs).

In this work, we combine the use of FNOs with a PINN using the gradient of the FWI; the FNO network takes the acquired seismic data as input and produces an initial velocity model, which is used by a wave propagator to generate the corresponding synthetic data, which is then compared to the acquired data to evaluate the misfit. The misfit is back-propagated by the wave simulator, generating the FWI gradient, which is in turn back-propagated through the network, updating its weights.

We build a UNet where the convolutional layers are substituted by FNOs, and find out that it has similar performance to a standard UNet but with fewer trainable parameters when testing on benchmark models.